Dark Matter Questions
Sutton Trust Summer School, July 2015

Part I: the evidence for dark matter

In this question we are going to prove that most of the mass of the Milky Way must be in the form of dark matter

(a) Show that the rotation speed of a mass m in a circular orbit of radius r around a much larger mass M has the expression,

$$v = \sqrt{\frac{GM}{r}} \tag{1}$$

where G is the gravitational constant. To derive this formula, equate the expressions for the gravitational force between the two masses and the centripetal force on the smaller mass that is required to keep it in circular motion.

(b) Astronomers have observed that the speed of a star in the very outermost regions of the Milky Way is $v = 270$ km/s. Given that the radius of the Galaxy is roughly $r = 1.8 \times 10^{21}$ m estimate the *total mass* of the Milky Way. The gravitational constant is $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}.

(c) We know that the Milky Way contains around 10^{11} stars, given that the mass of the Sun is $M_\odot = 2 \times 10^{30}$ kg and that the Sun is a fairly average star, estimate the total mass of *stars* in the Milky Way.

(d) Combining your answers to parts (b) and (c) what fraction of the mass of the Milky Way is stars, and what hence what fraction must be dark matter?

This question is just a rough estimate for how much dark matter there should be in the Milky Way, if you have some time to spare, think about (but without calculation) what you would need to consider to get a more accurate estimate.

Part II: dark matter interactions

In this question I want you to see how rarely dark matter interacts, so we're going to estimate the number of collisions dark matter has with the human body in a year. The rate of dark matter collisions with a target made of an element with mass number A is (you don't need to derive this formula),

$$R = NnA^2\sigma \langle v \rangle \tag{2}$$

where n is the number density of dark matter in the Galaxy, N is the number of nuclei that your target is made of and $\langle v \rangle = 220,000$ m s^{-1} is the average speed of dark matter in the Galaxy. The constant σ is called the interaction cross-section and is a number that is used to describe how likely a particle interaction is.

(a) We don't know the precise mass of the dark matter particle yet, but a good guess would be something like 50 times the mass of the proton. The observed density of dark matter in the Galaxy is 5.3×10^{-22} kg m^{-3}, calculate the number density n of Galactic dark matter in m^{-3}. The mass of the proton is $m_p = 1.67 \times 10^{-27}$ kg.

(b) The human body is mostly made of water and because oxygen is a much larger nucleus than hydrogen the dark matter will mostly interact with oxygen. Estimate how many oxygen nuclei there are in an average human body.

(c) Dark matter doesn't interact very much at all so it has a very small cross-section $\sigma = 10^{-46}$ m^2. Use your answers for parts (a) and (b) in the above formula to calculate the rate of dark matter collisions. The answer will be in units of s^{-1}.

(d) How many dark matter collisions will an average human body undergo in one year?

This question was just an exercise, but essentially the same calculation was the subject of a paper in 2012 by two scientists, Katherine Freese and Christopher Savage.