



THE UNIVERSITY OF
SYDNEY



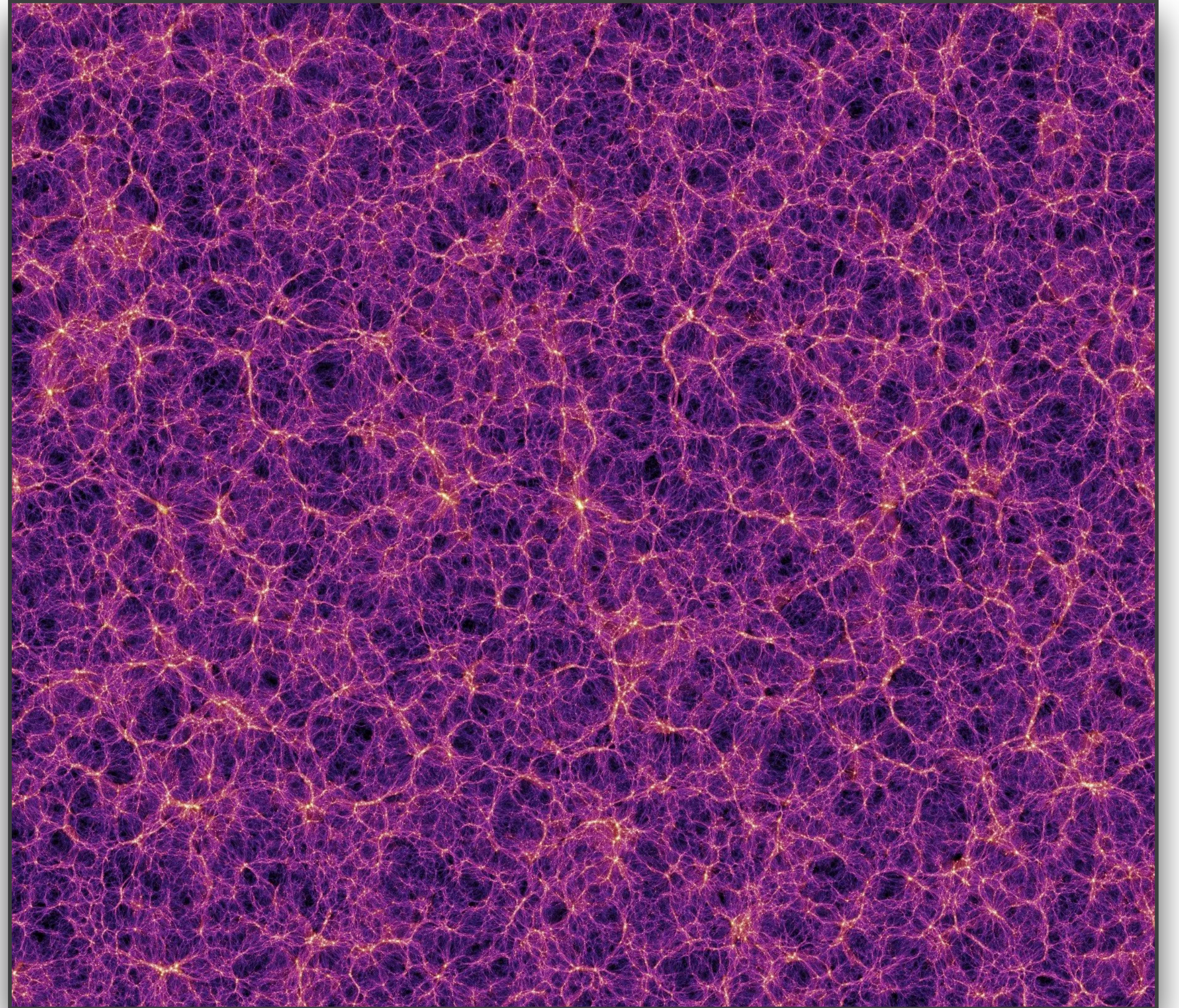
Axion cosmology and dark matter

Ciaran O'Hare
University of Sydney

Outline

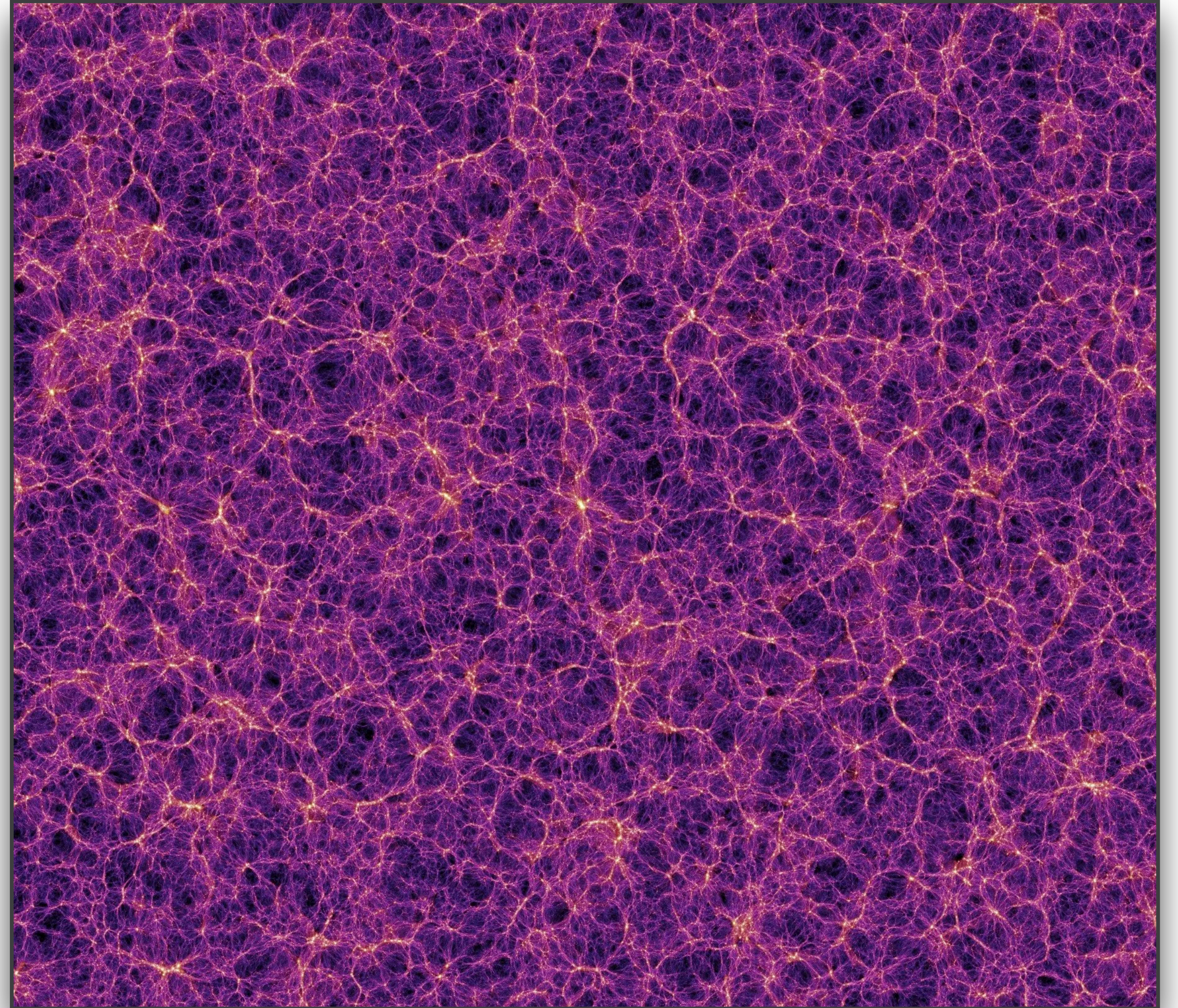
1. Cosmology basics
2. How cosmology can tell us where to look for the axion
3. How cosmology can tell us where *not* to look
4. “Near-field cosmology” a.k.a direct detection

How well do you know your Universe?



How well do you know your Universe?

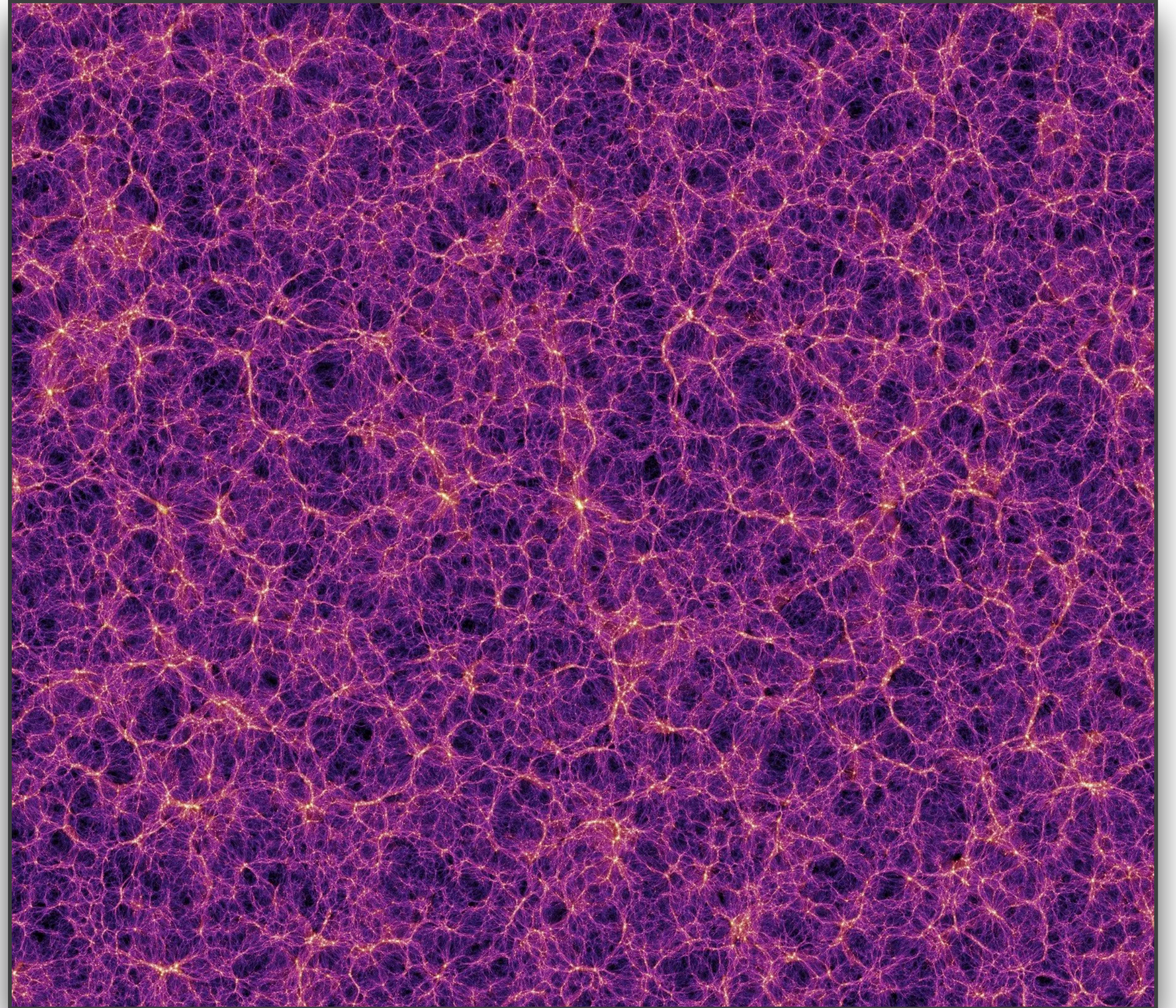
Age of Universe



How well do you know your Universe?

Age of Universe

→ 13.8 Gyr

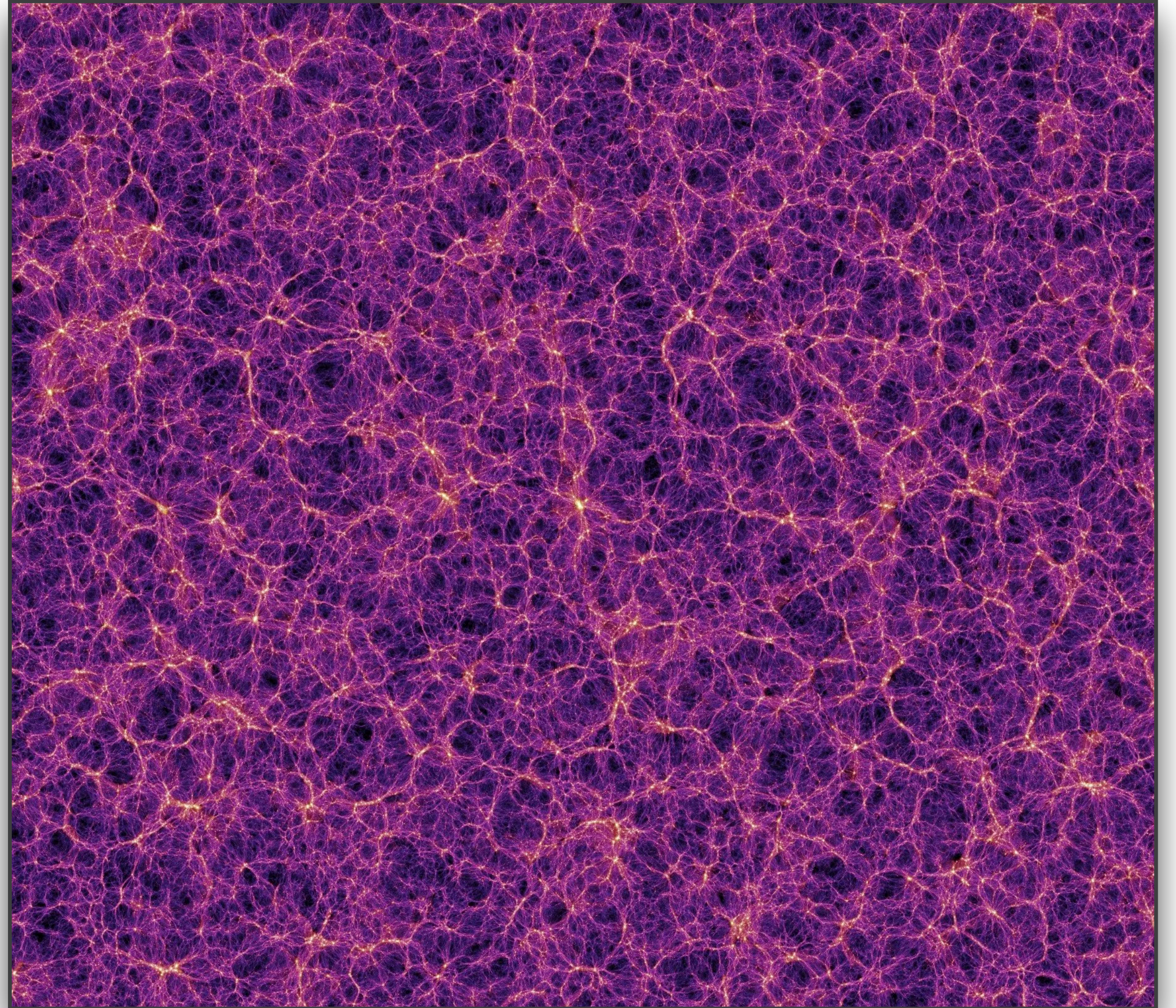


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Radius of *observable* Universe



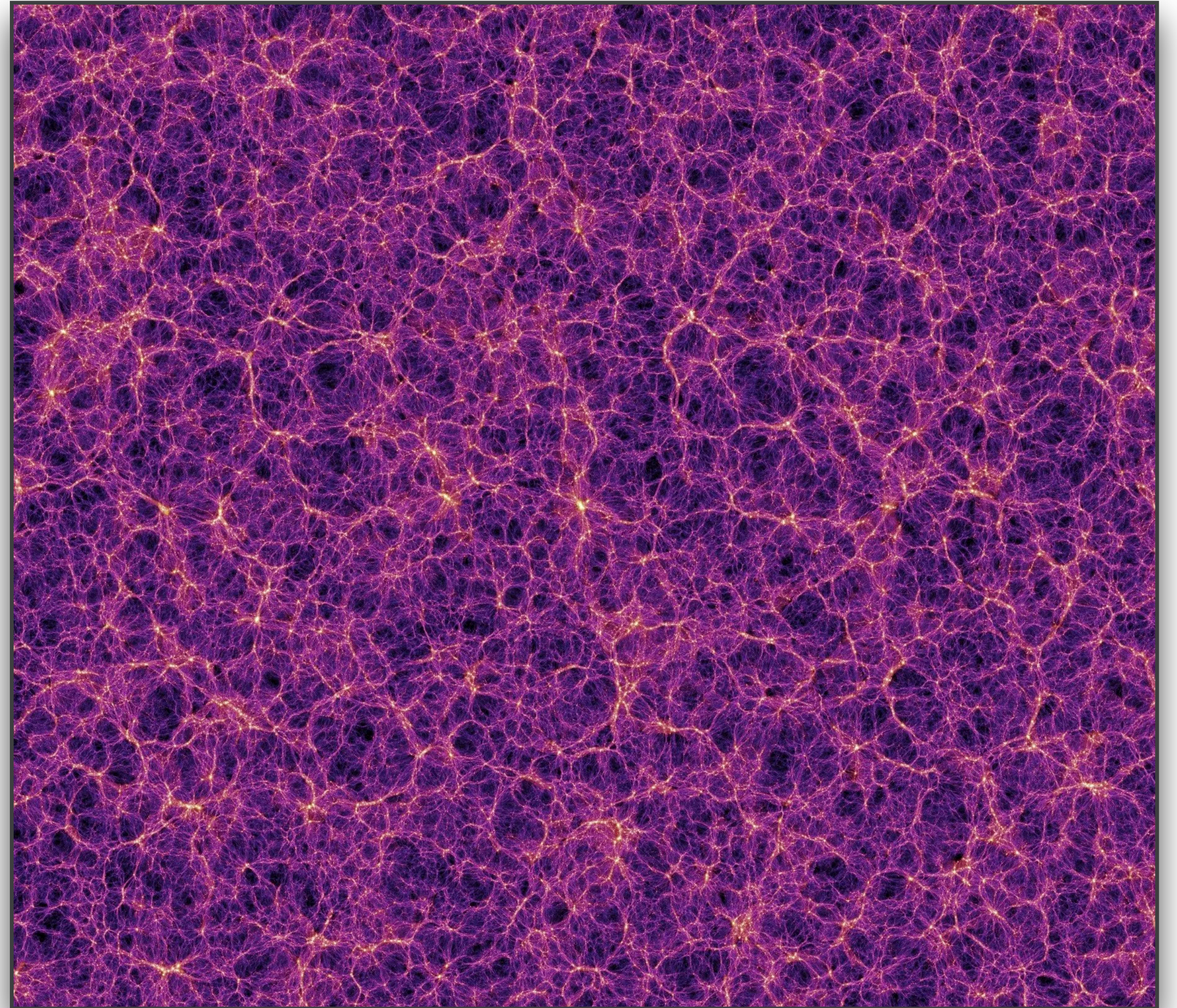
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→ 93 billion ly/28 Gpc



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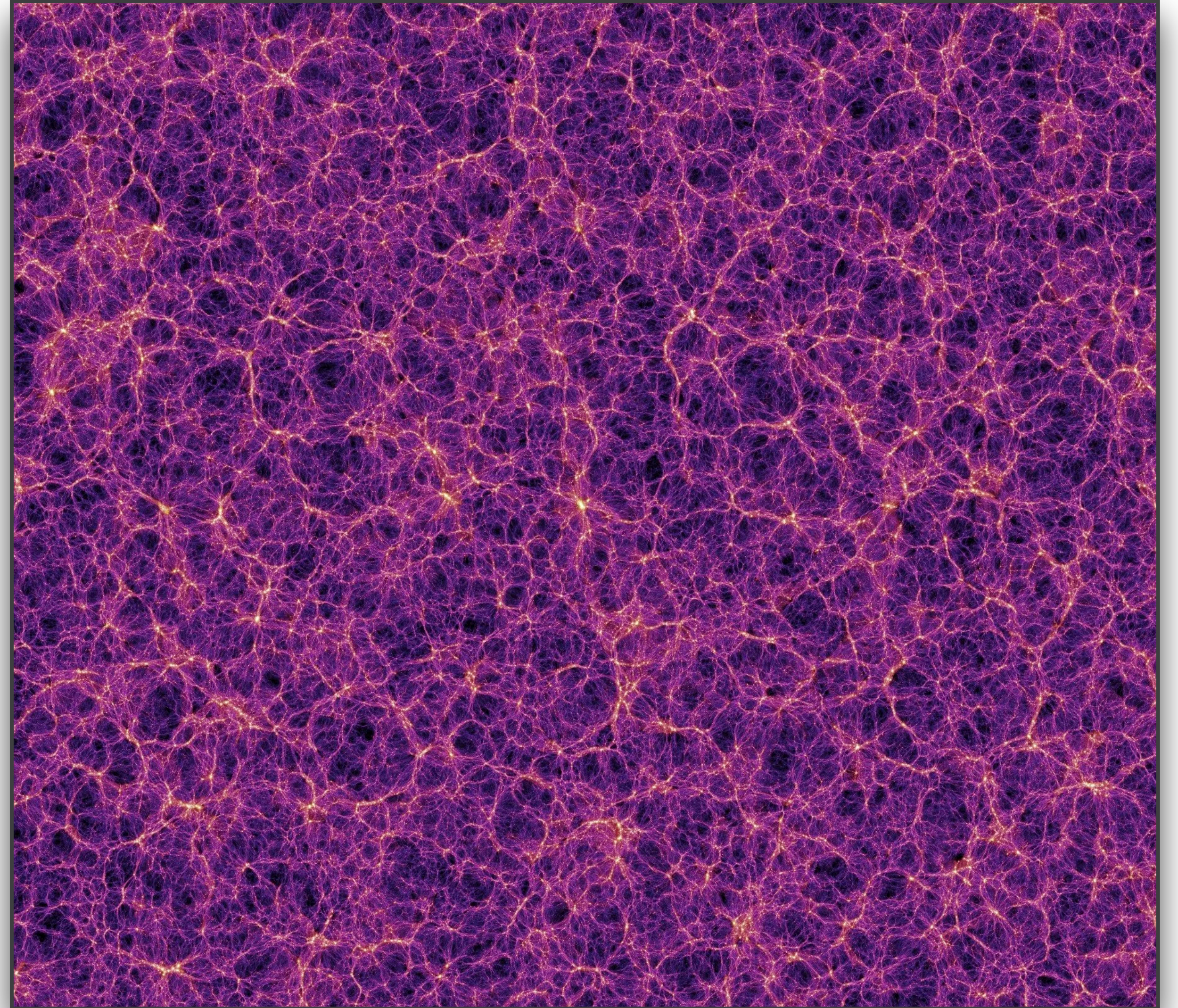
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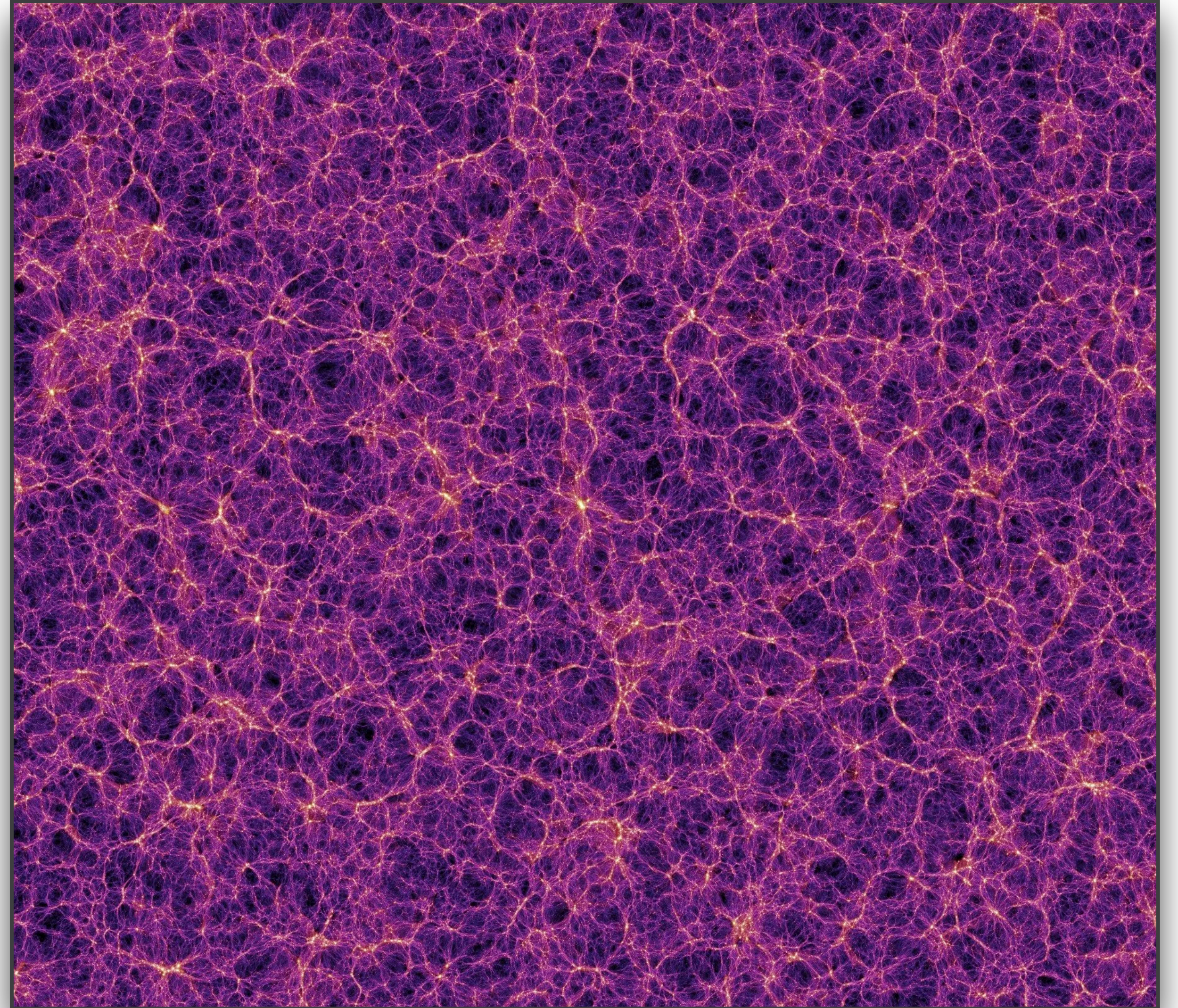
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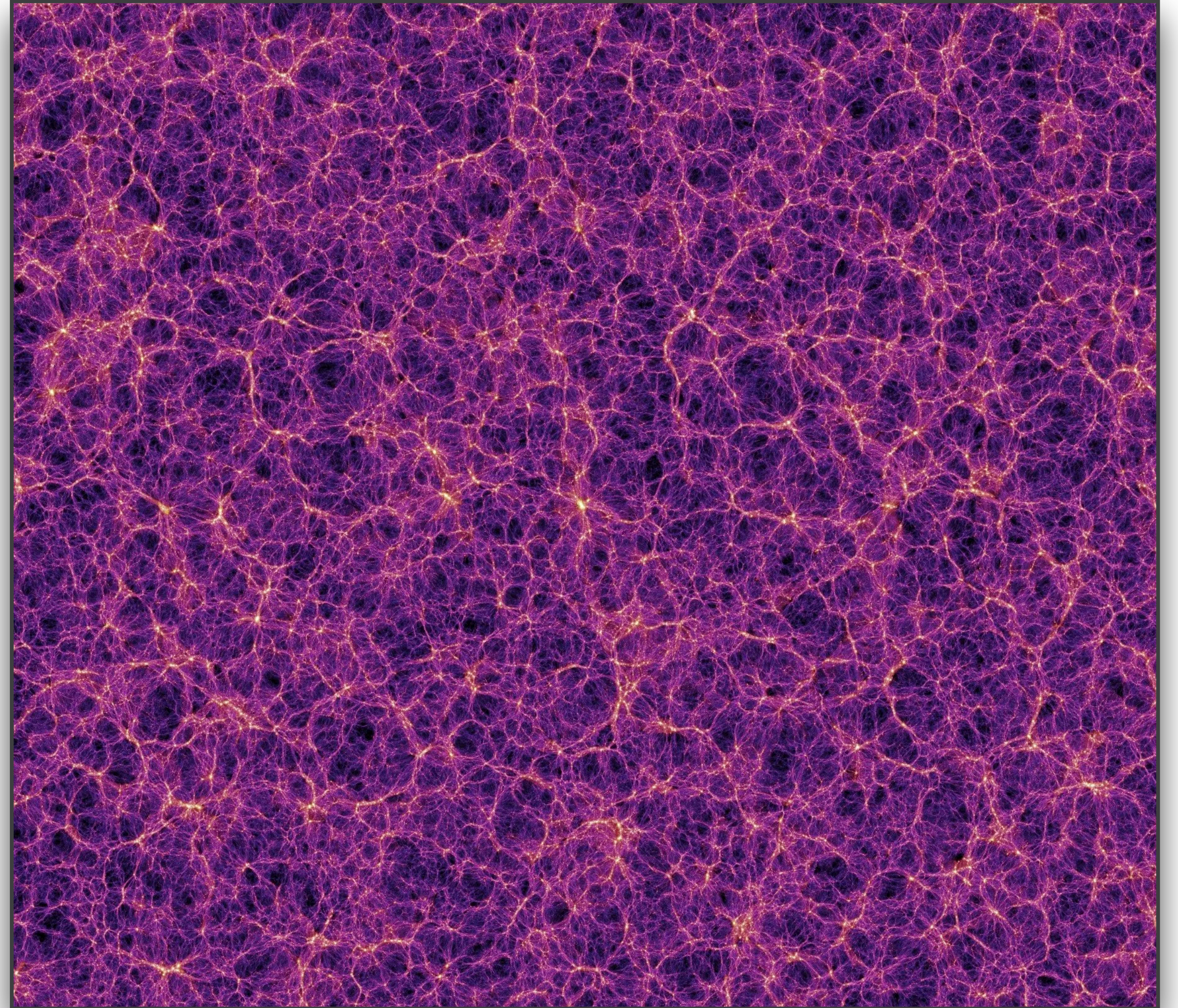
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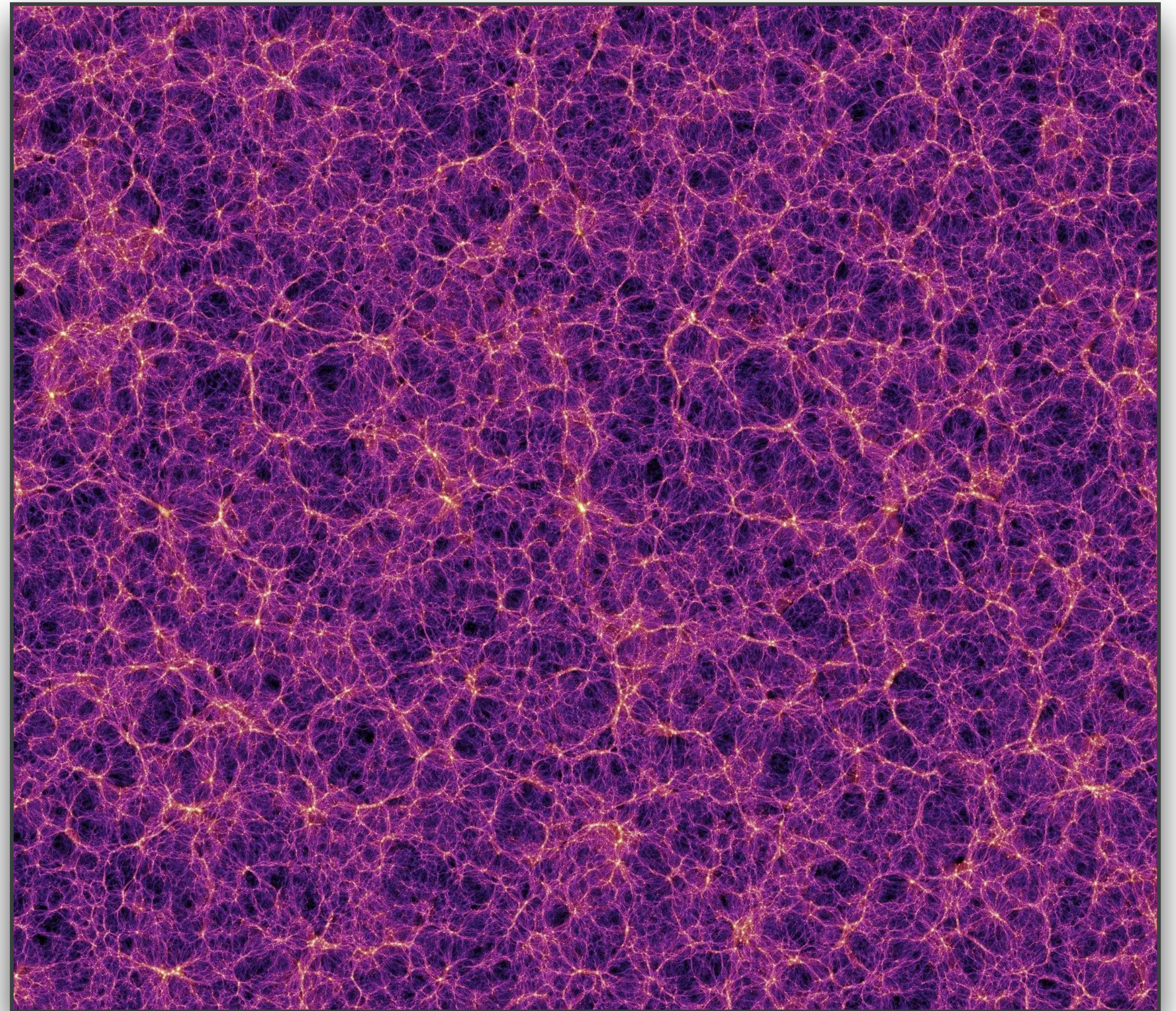
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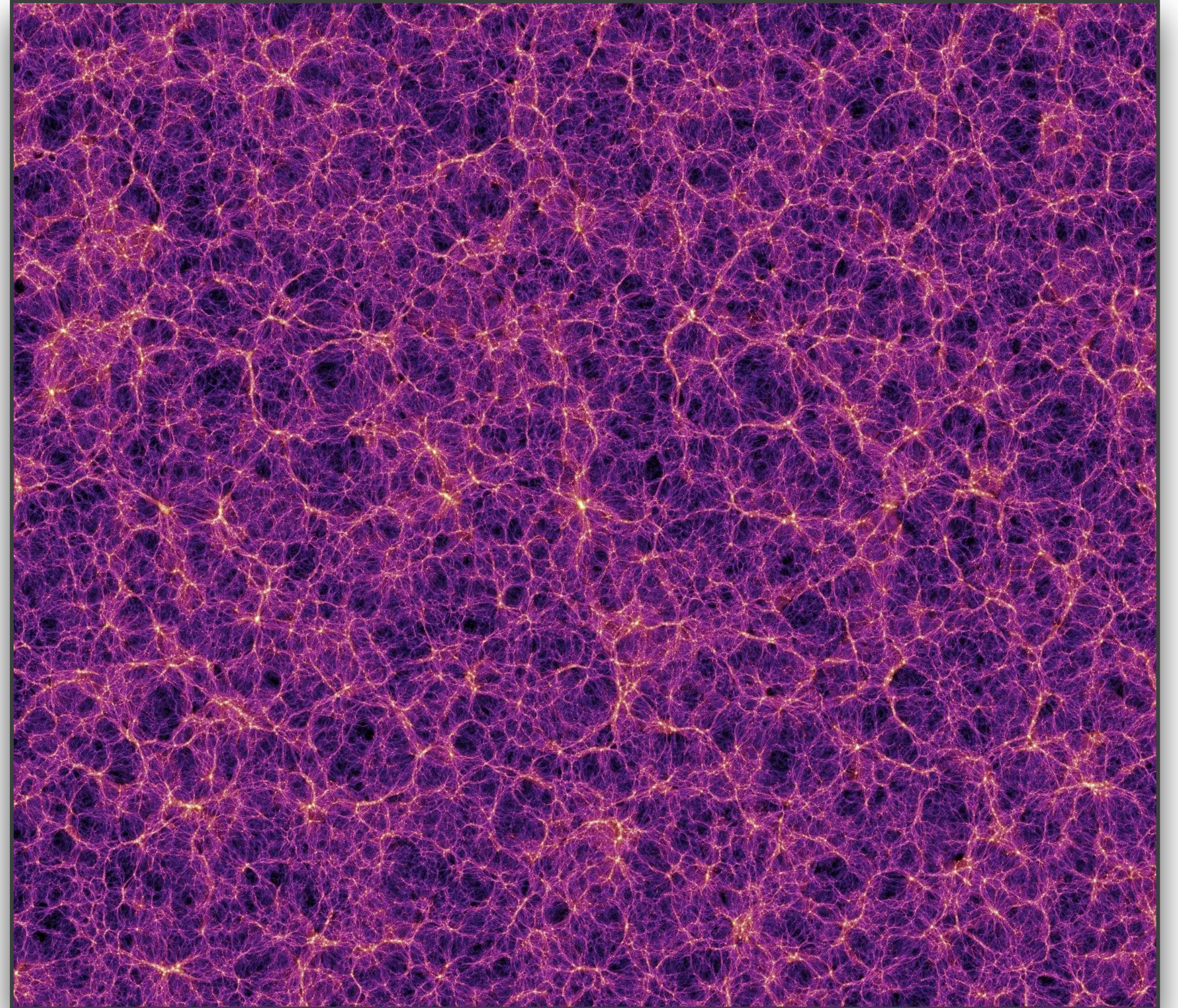
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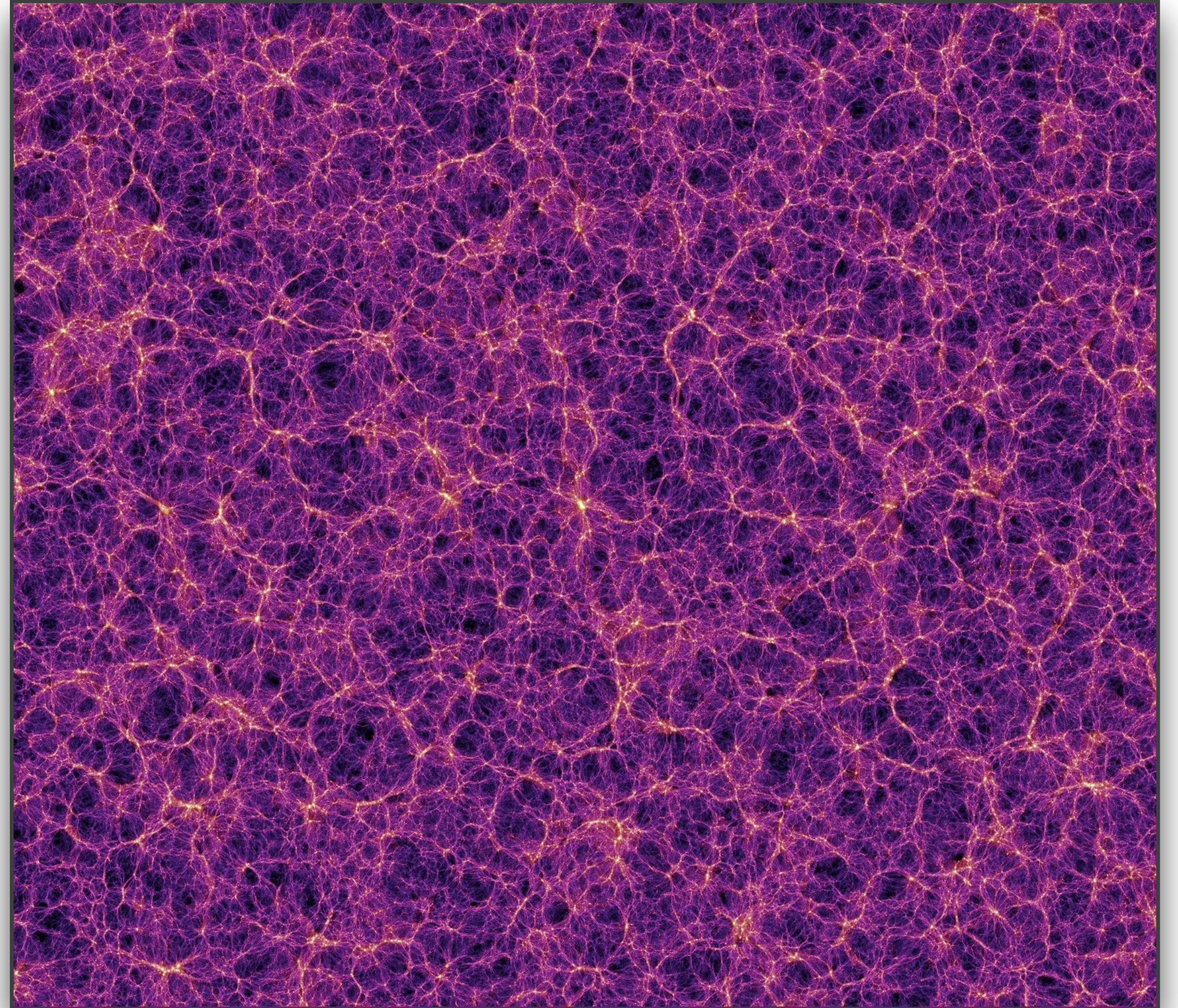
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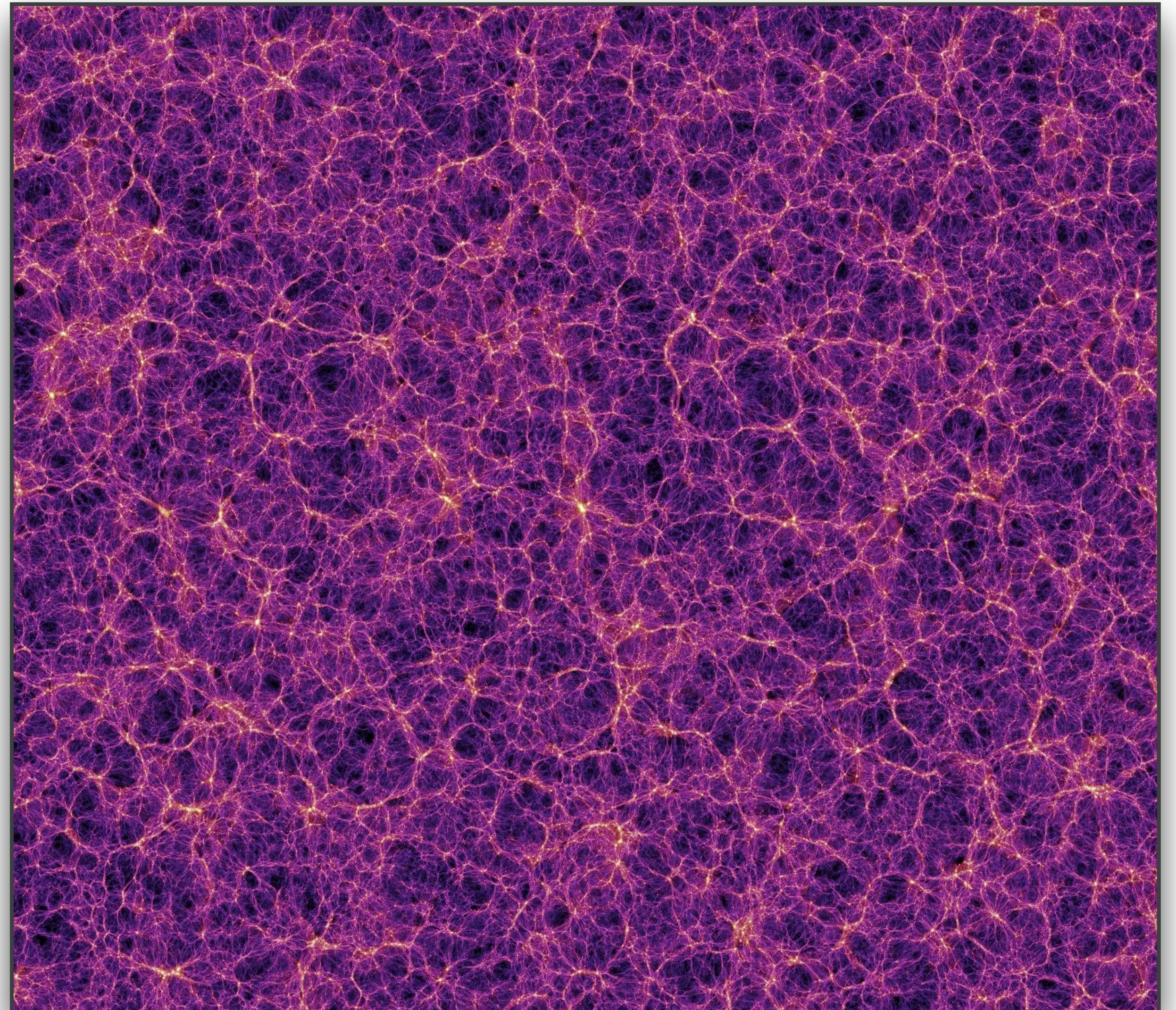
→ ~ 411

Average number of neutrinos [cm⁻³]

→ ~ 336

Energy density of components

$$\Omega = \frac{\rho}{\rho_c} \quad \rightarrow \quad \begin{array}{ll} \Omega_{\text{rad}} = & \Omega_b = \\ \Omega_{\text{DM}} = & \Omega_{\Lambda} = \end{array}$$



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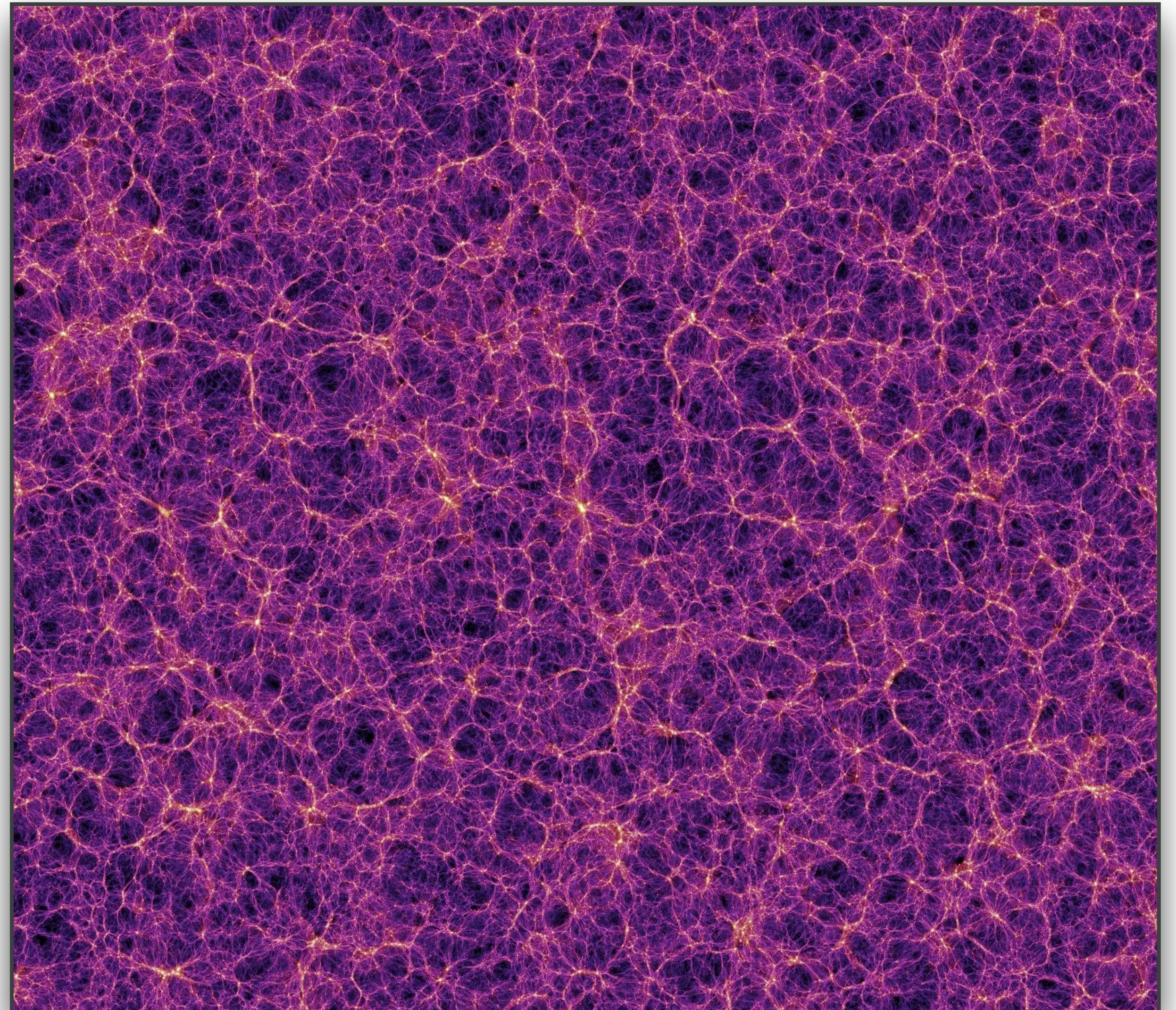
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Average number of neutrinos [cm⁻³]

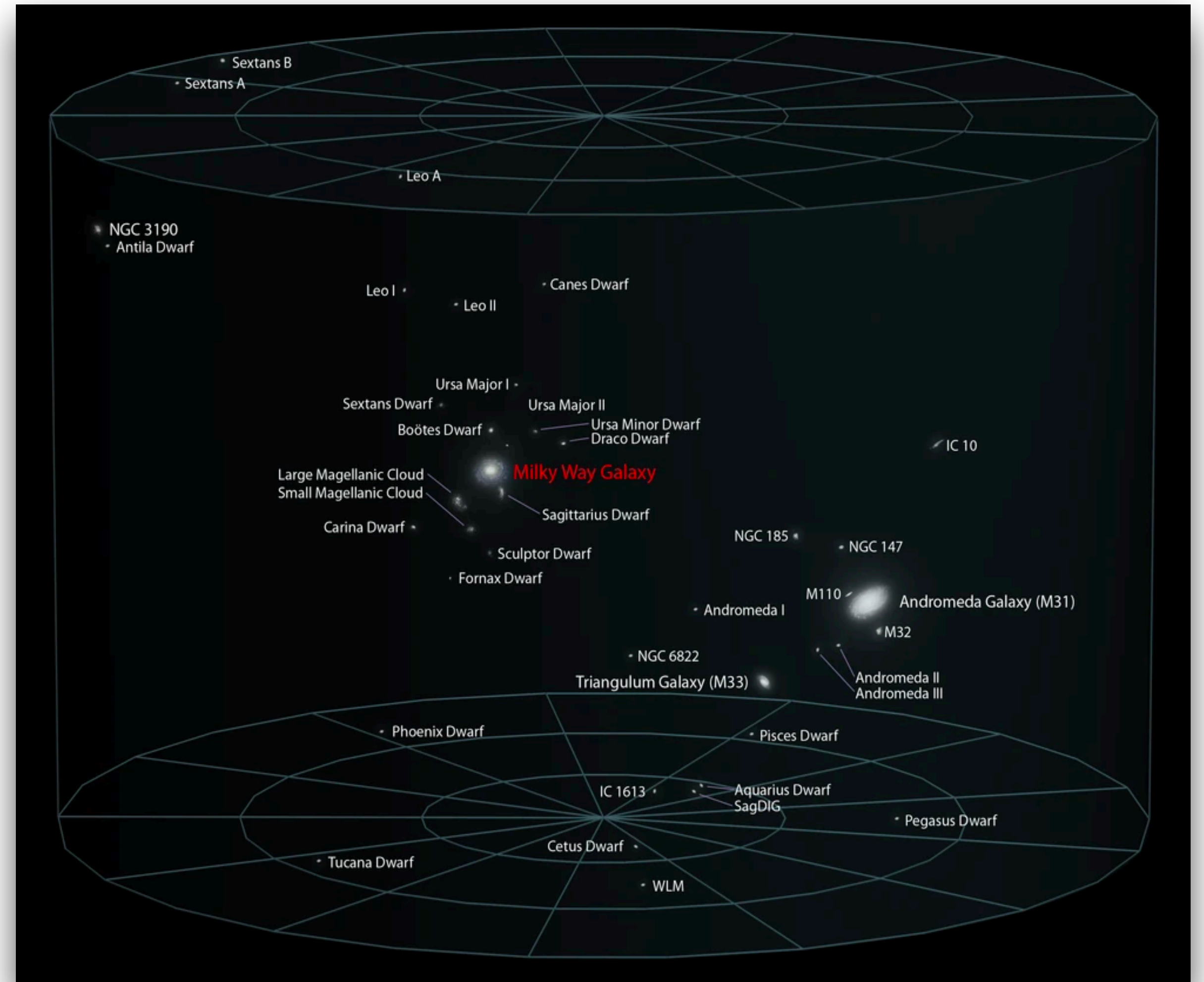
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Energy density of components

$$\Omega = \frac{\rho}{\rho_c} \quad \rightarrow \quad \begin{array}{ll} \Omega_{\text{rad}} = 0.0015 & \Omega_b = 0.05 \\ \Omega_{\text{DM}} = 0.26 & \Omega_{\Lambda} = 0.69 \end{array}$$

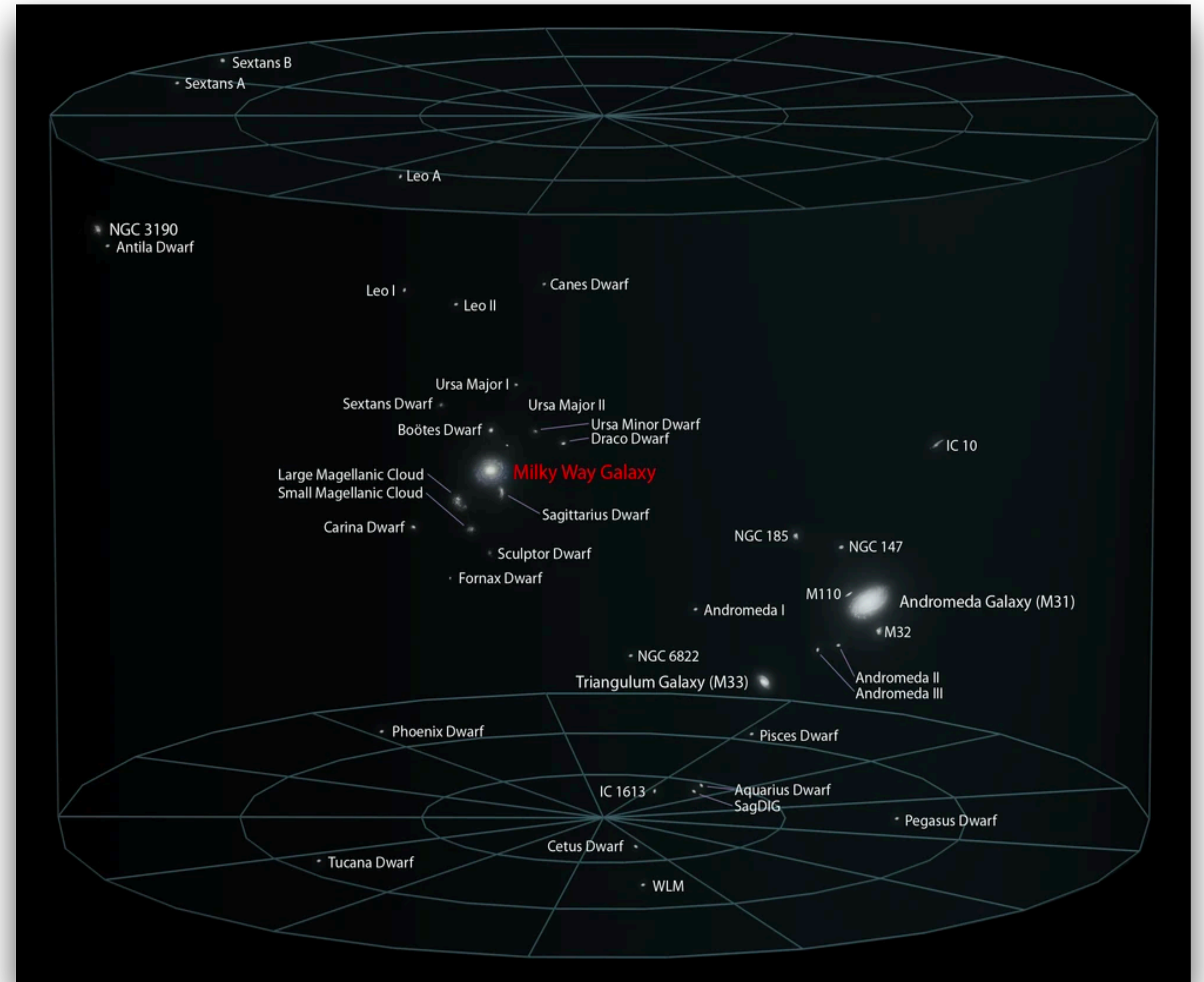


How well do you know your Local group?



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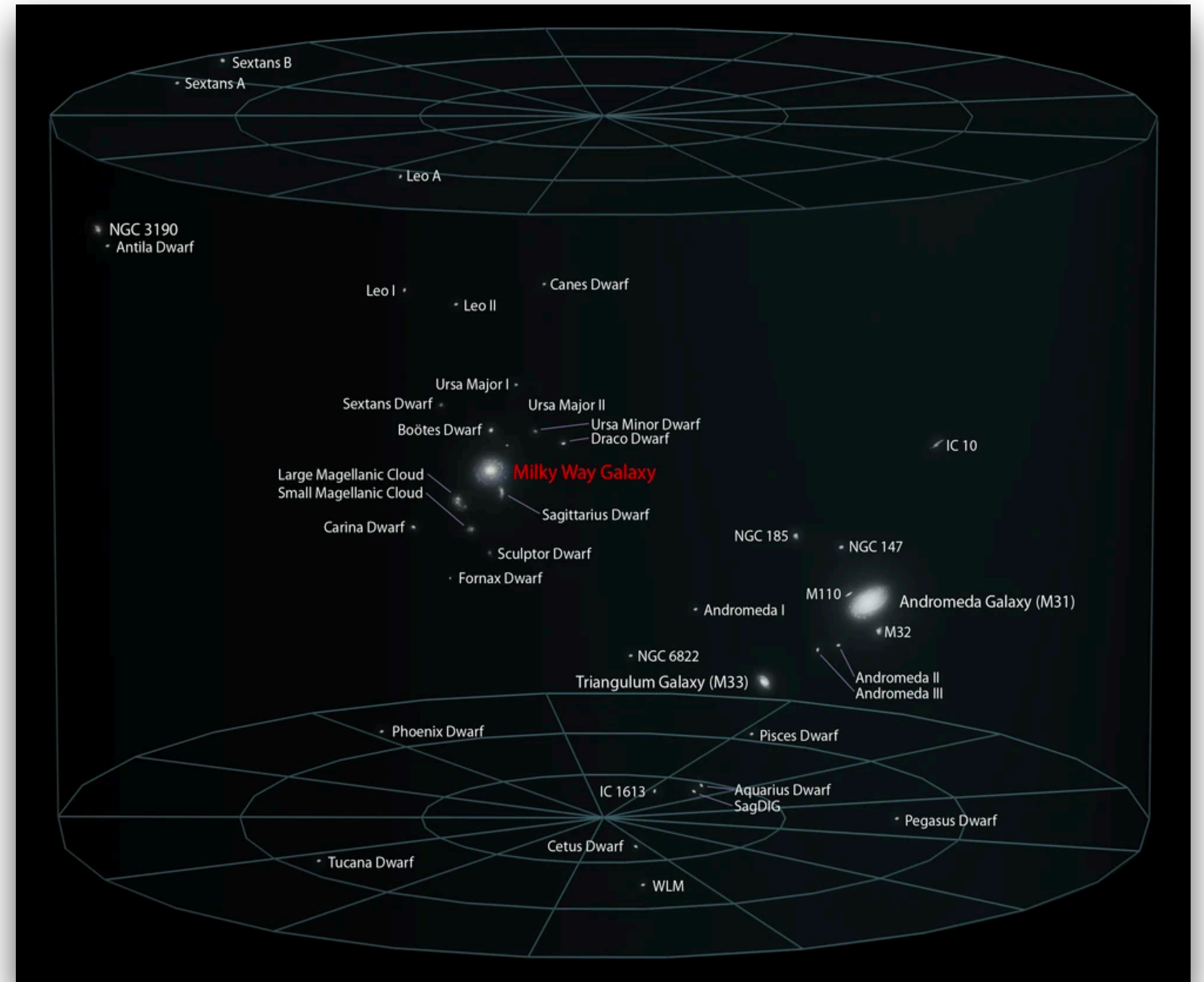
Distance to nearest major galaxy



How well do you know your Local group?

Distance to nearest major galaxy

→ 765 kpc / 250 million ly (Andromeda/M31)

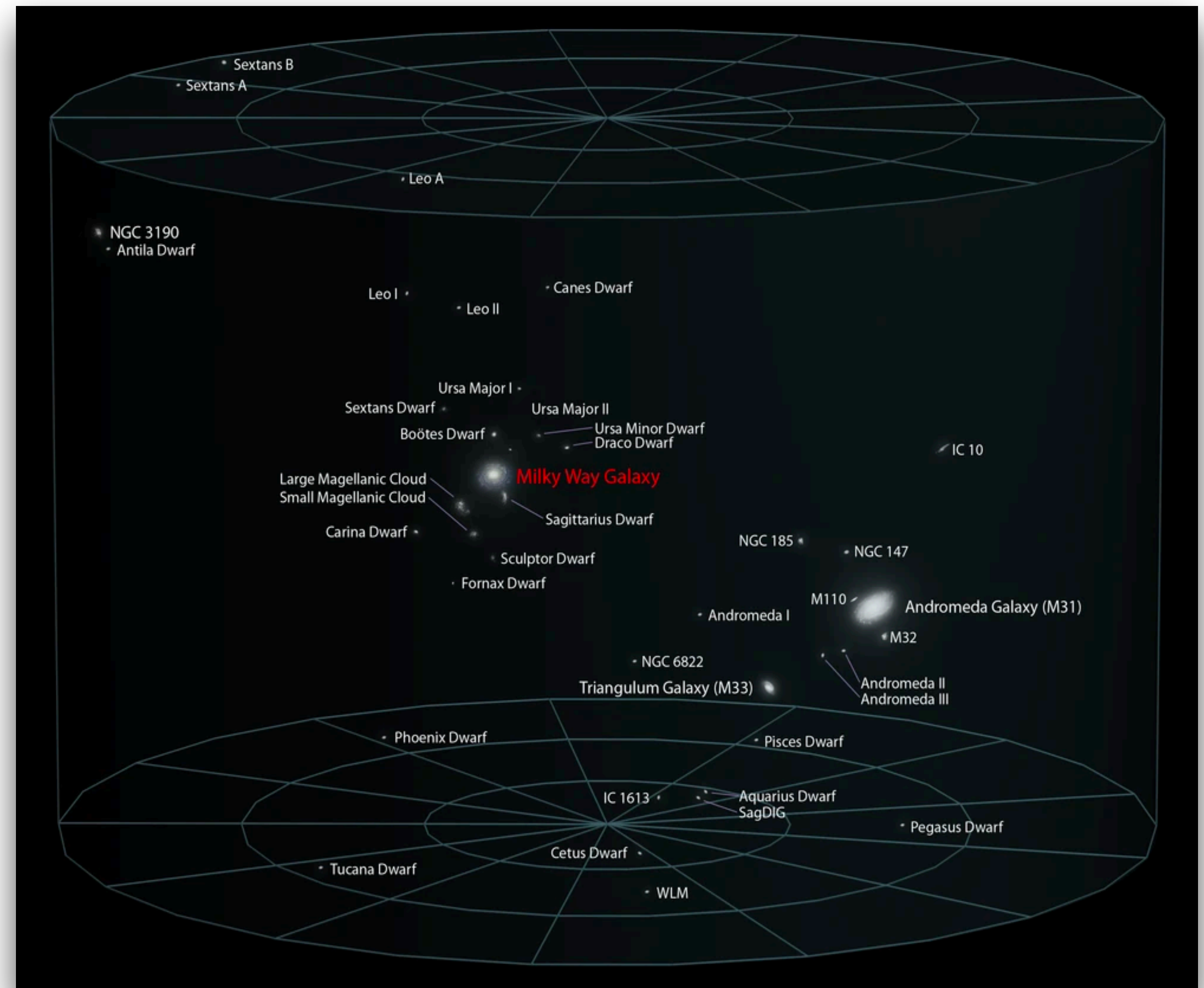


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Radius of Milky Way's DM halo



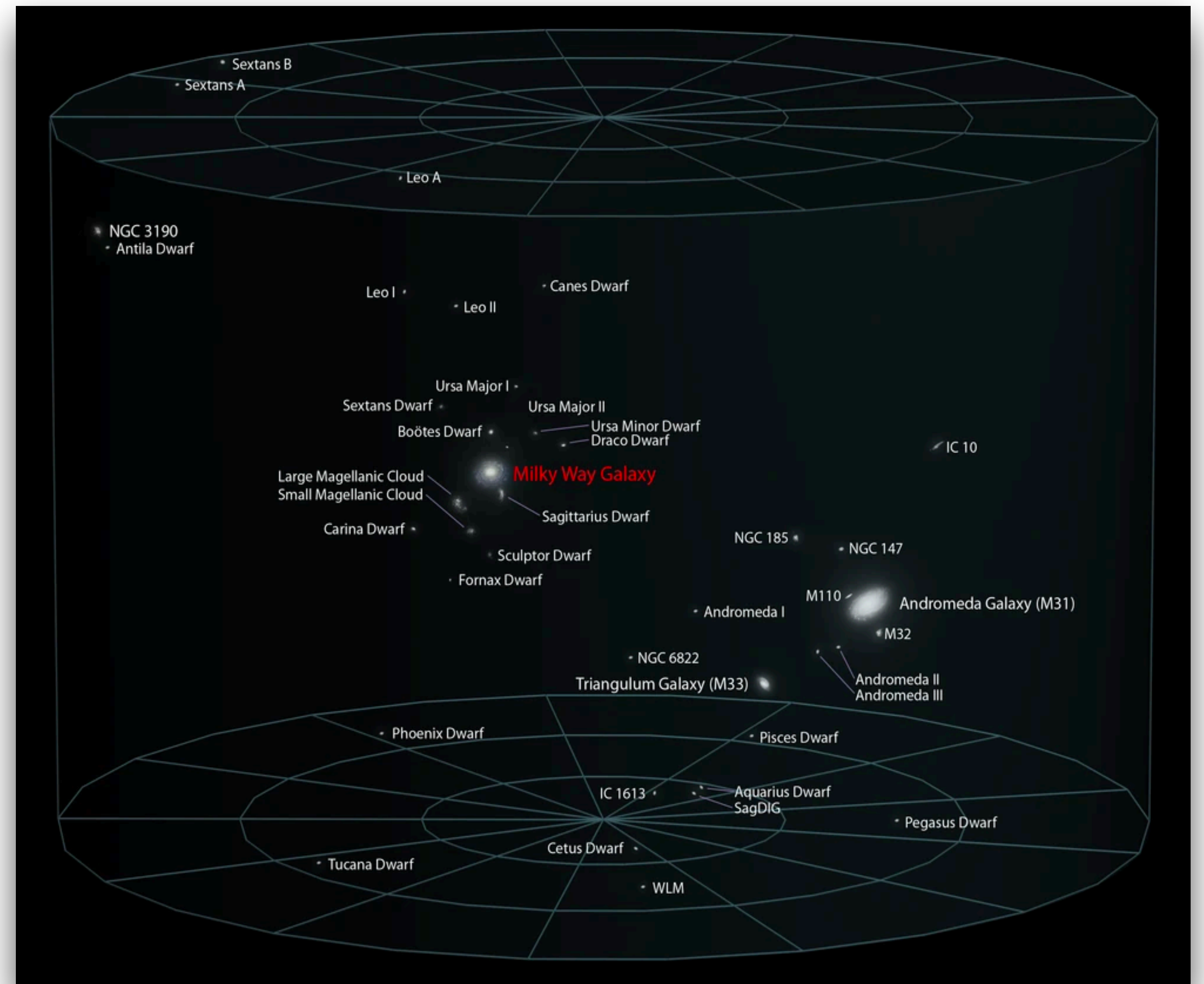
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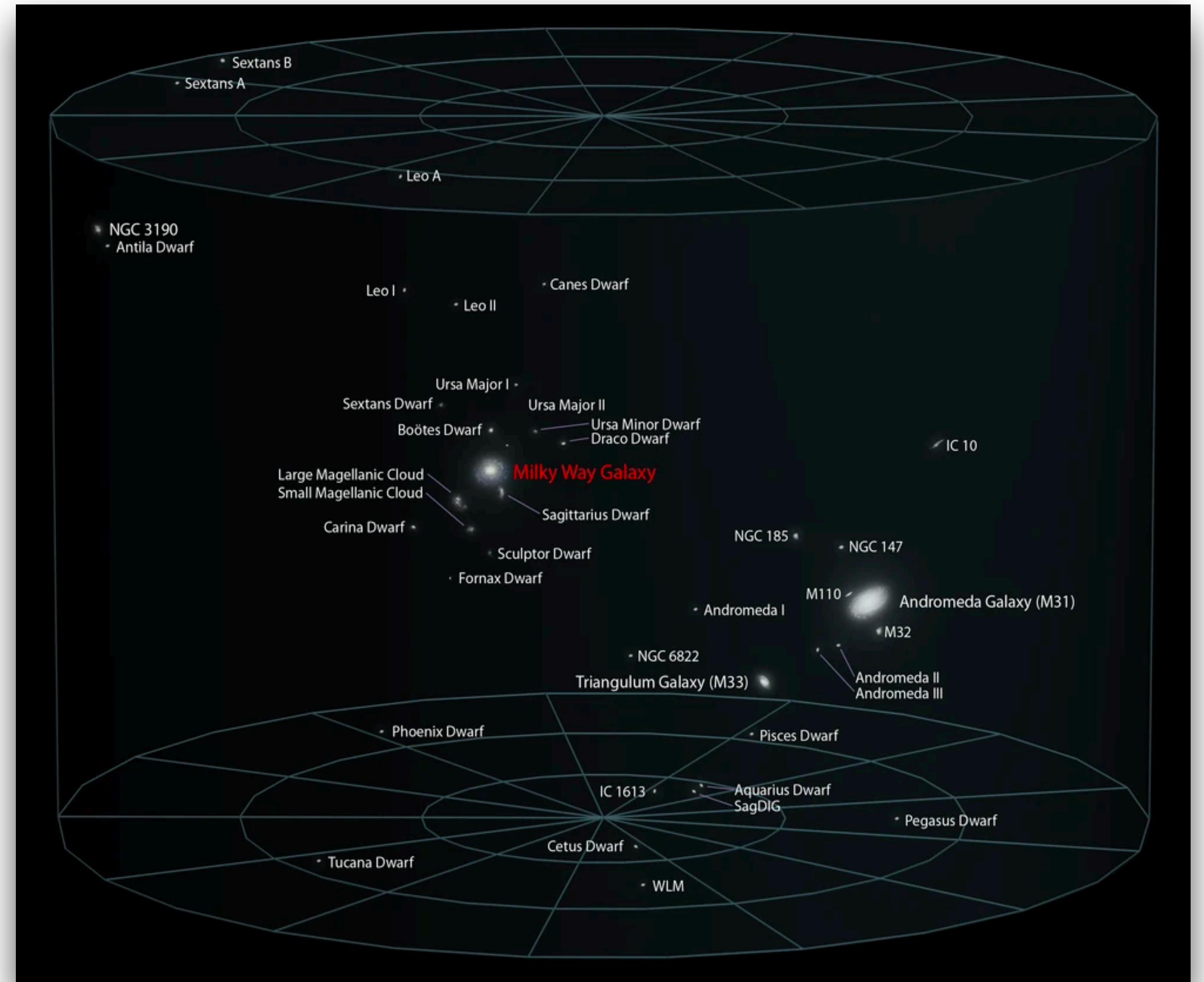
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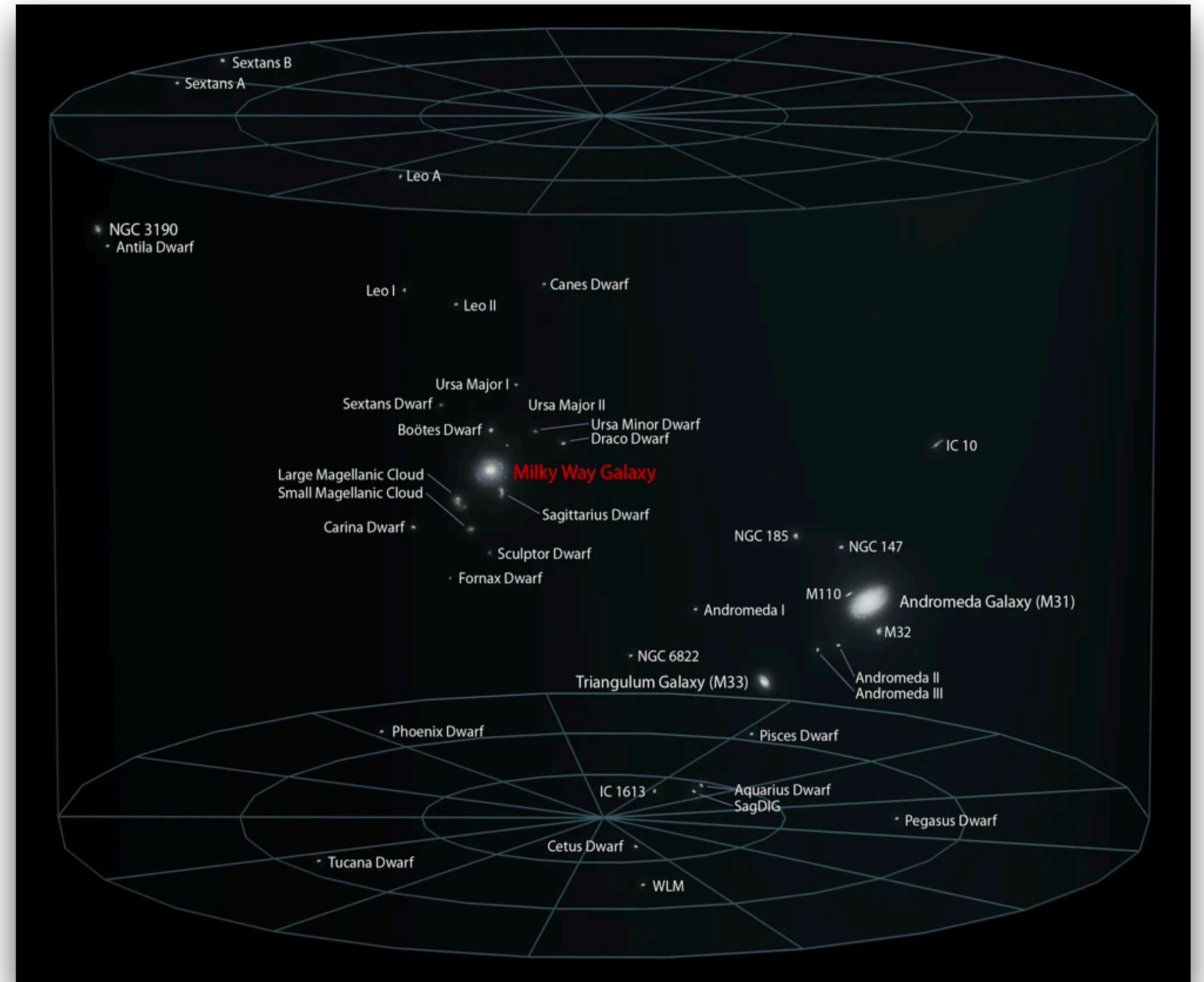
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→ $\sim 10^{11-12} M_{\odot}$



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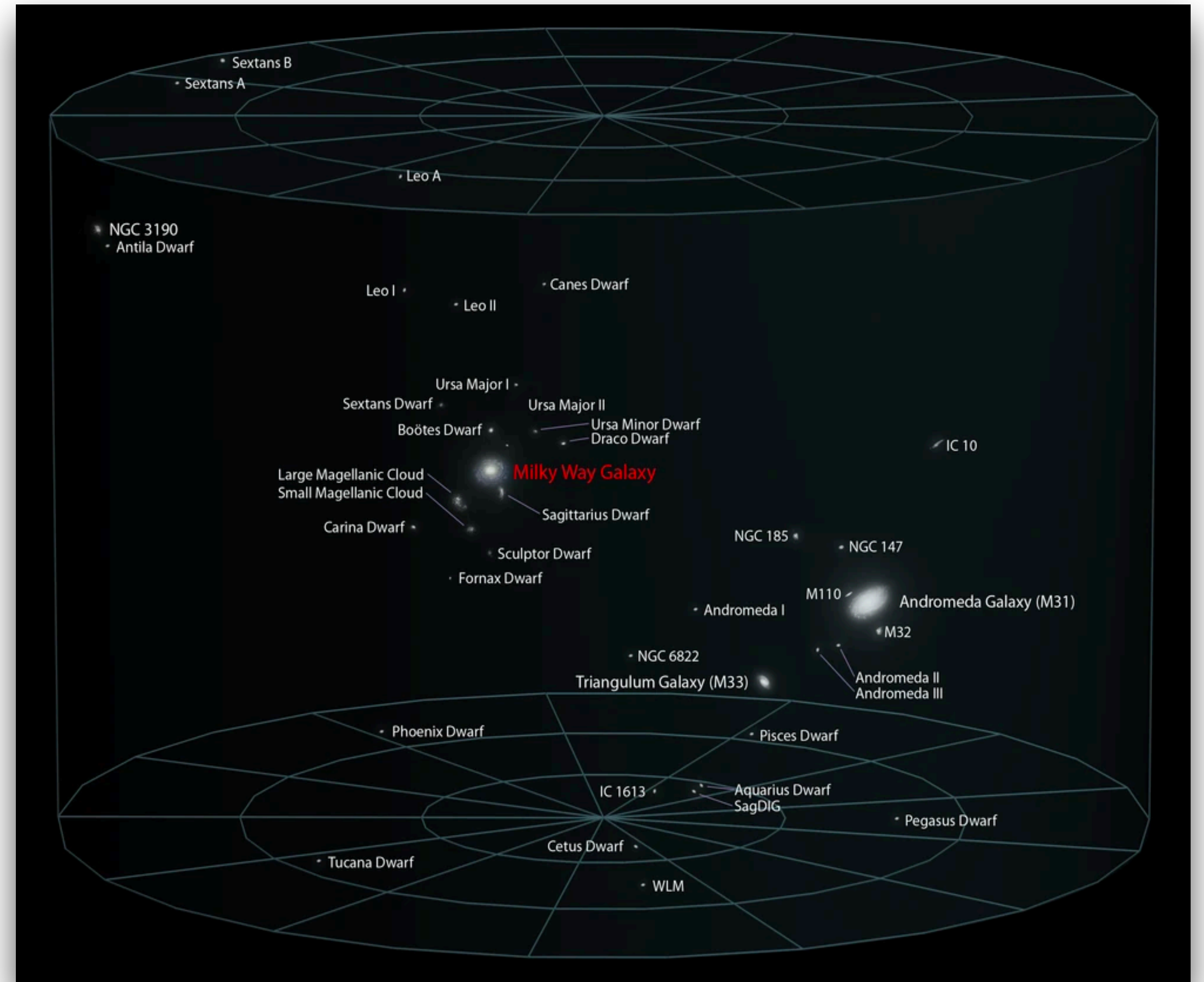
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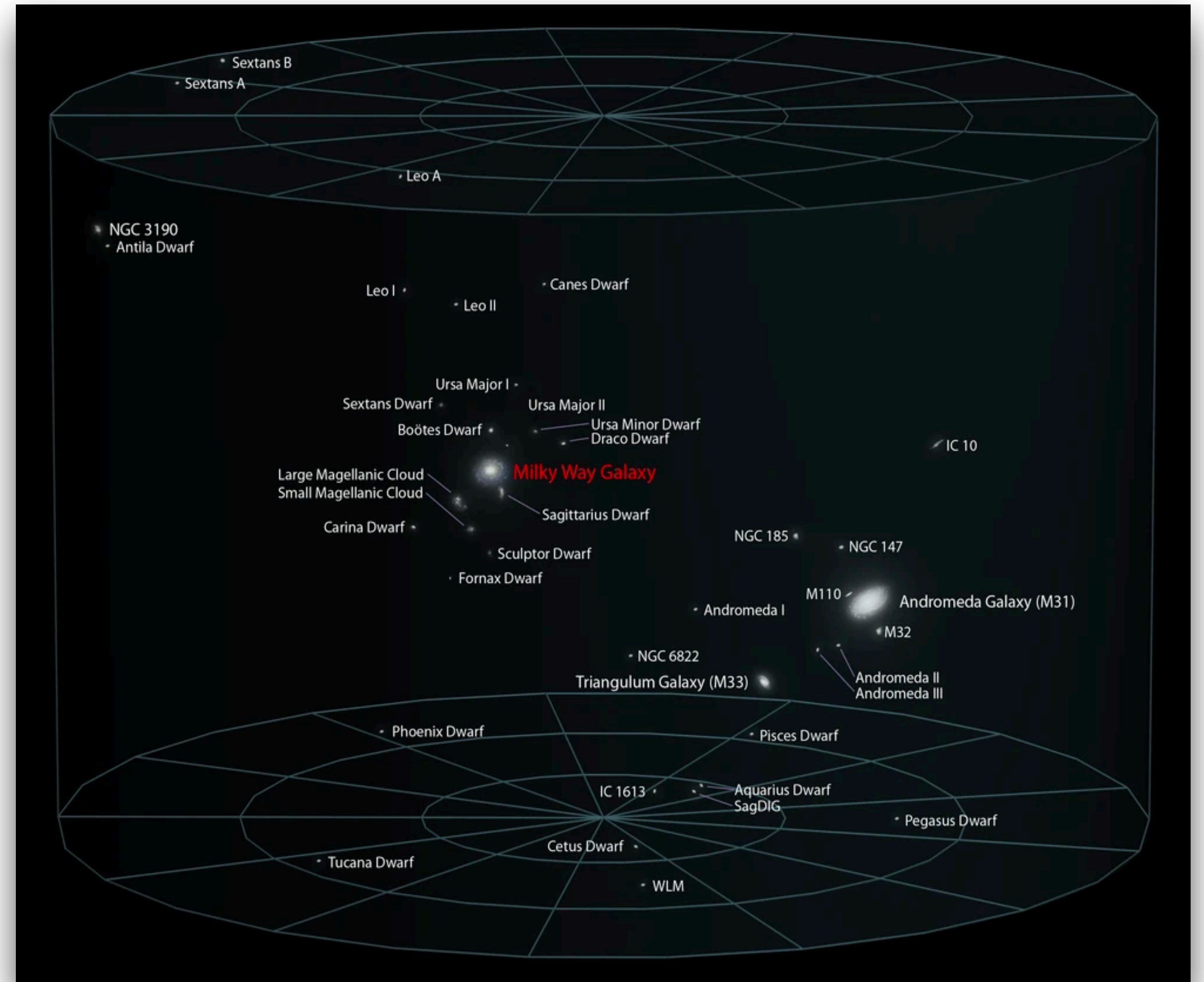
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Velocity of Milky Way

→ ~ 600 km/s



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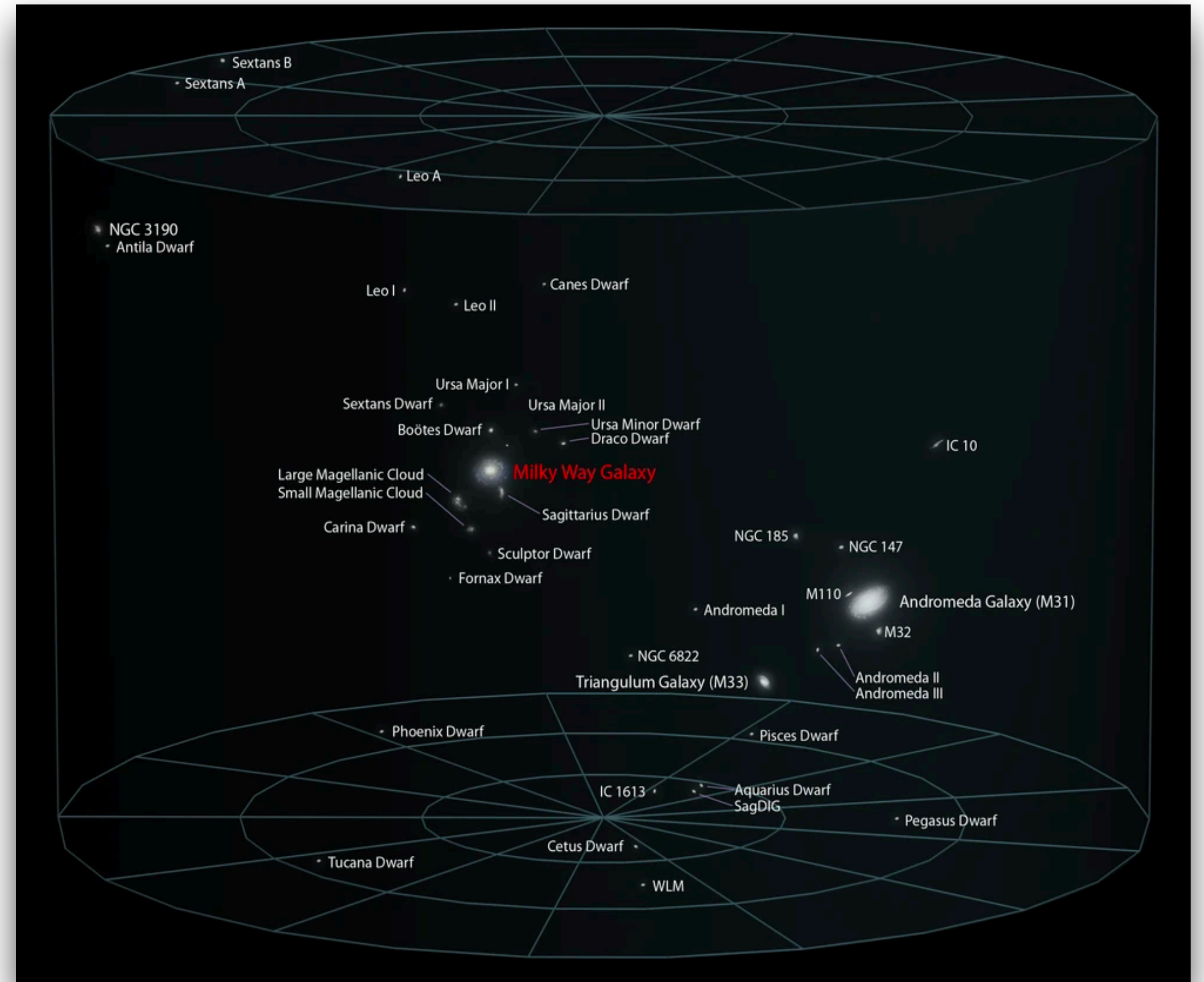
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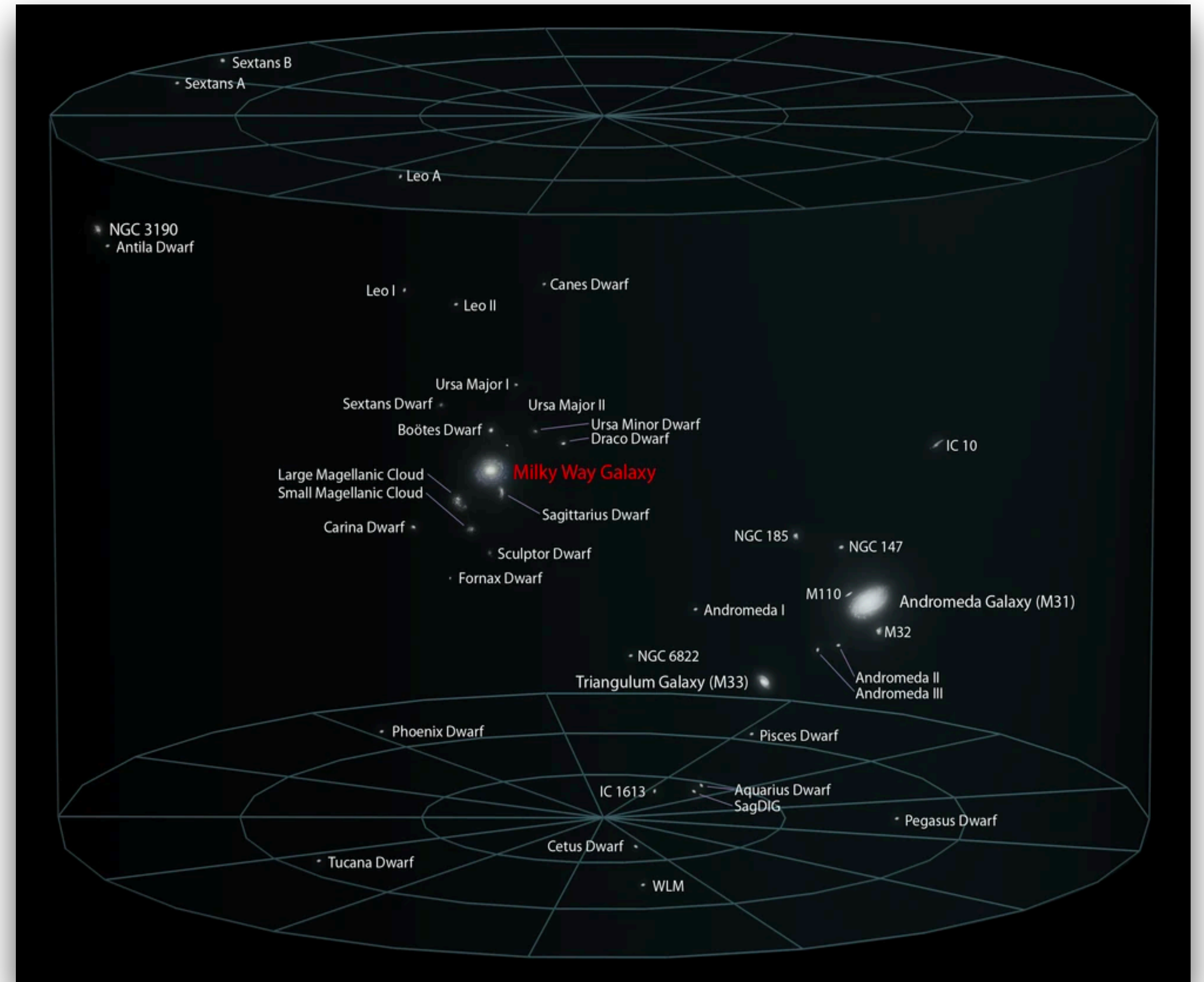
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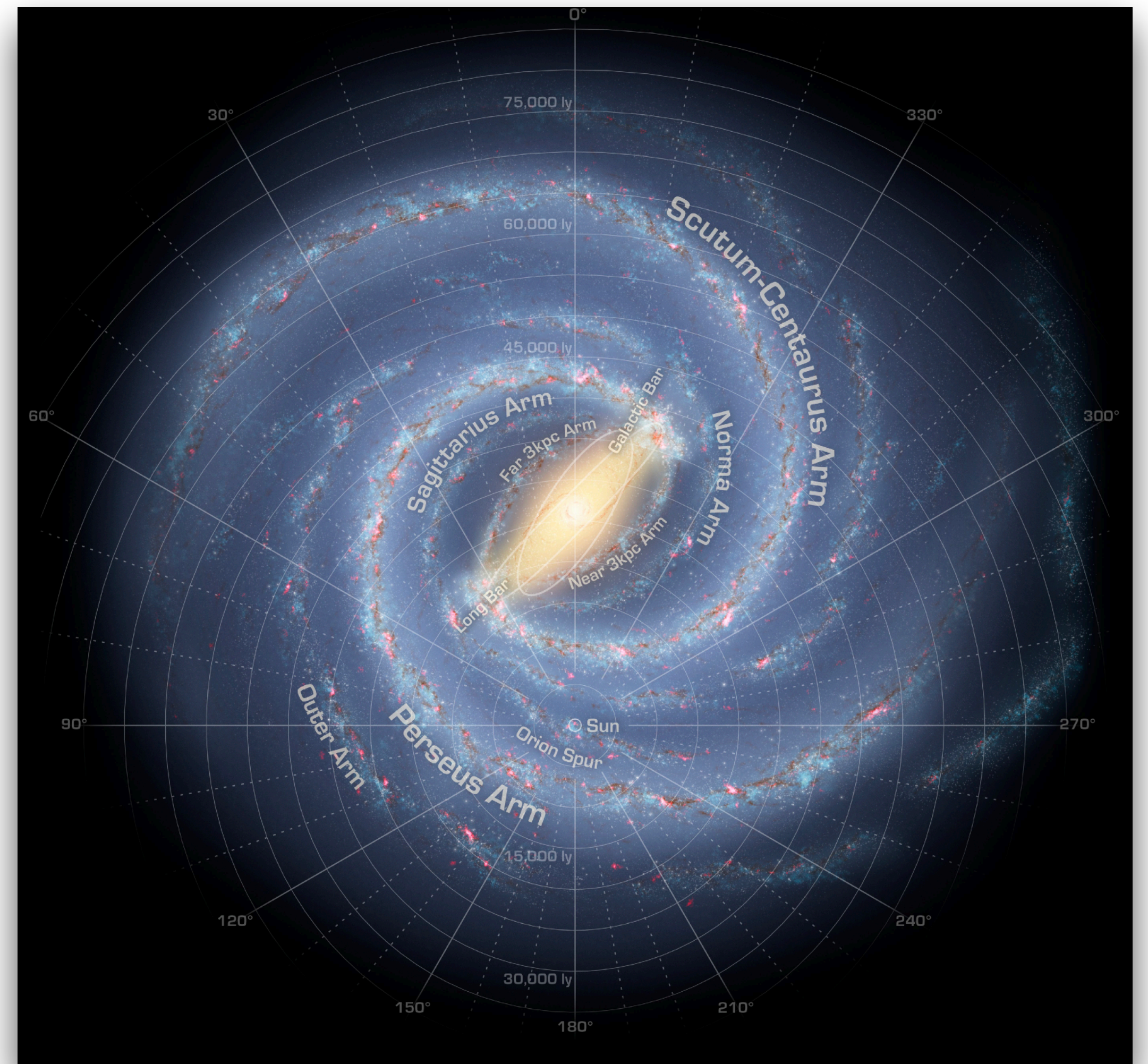
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What our galaxy will be called in 4.5 bn years

→ Milkdromeda

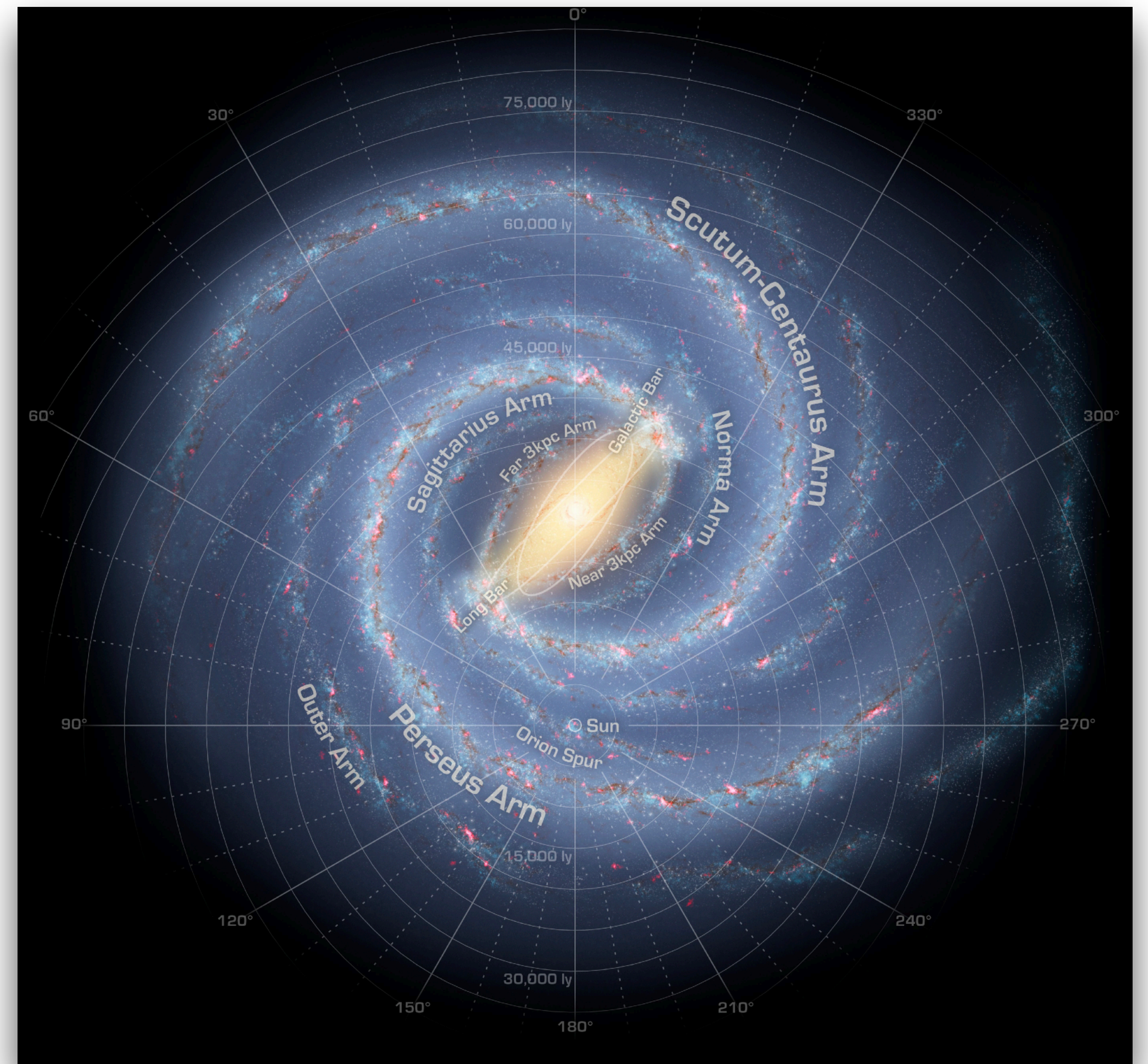


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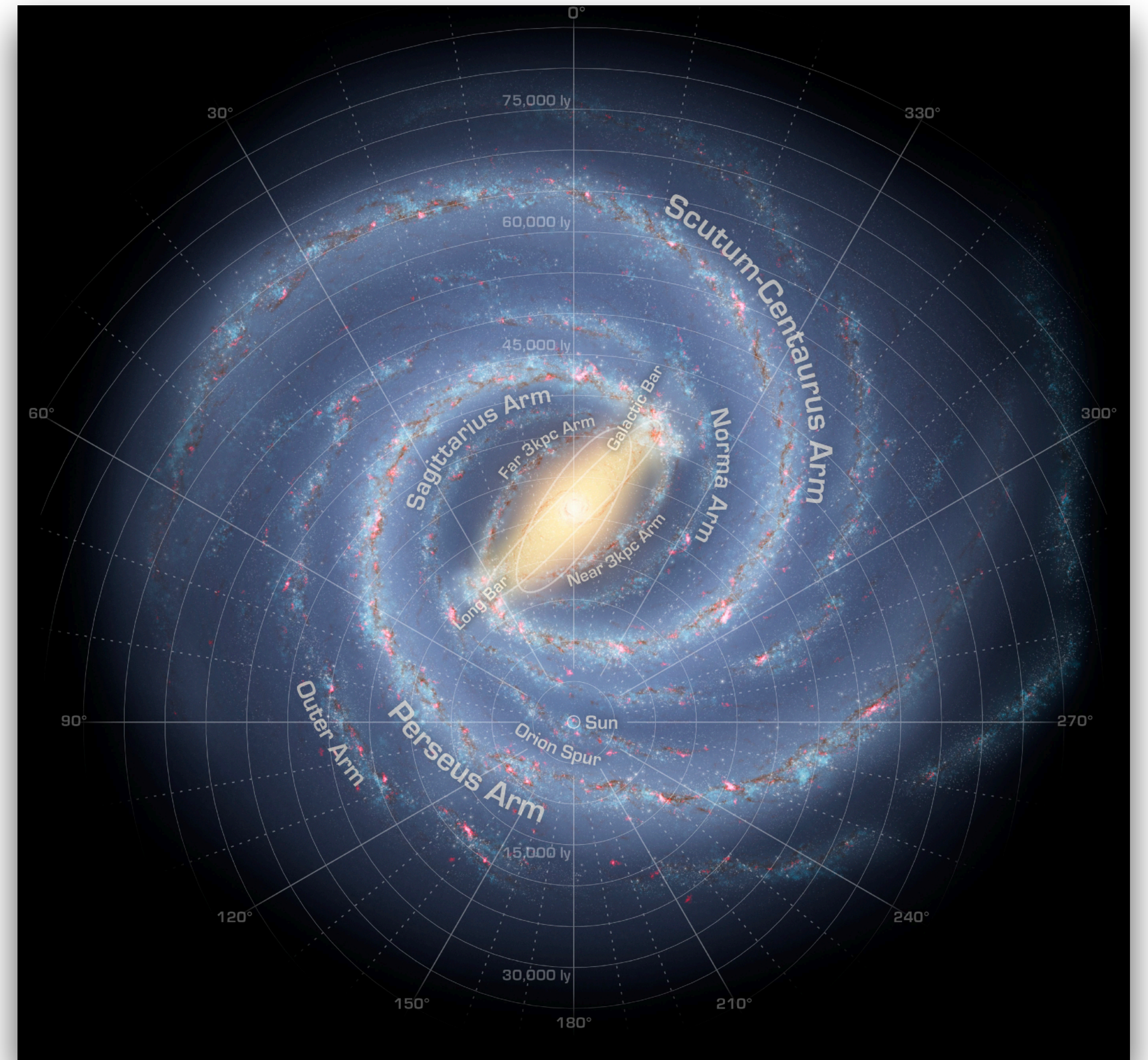
Diameter of Milky Way disk



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Diameter of Milky Way disk

→ ~30 kpc

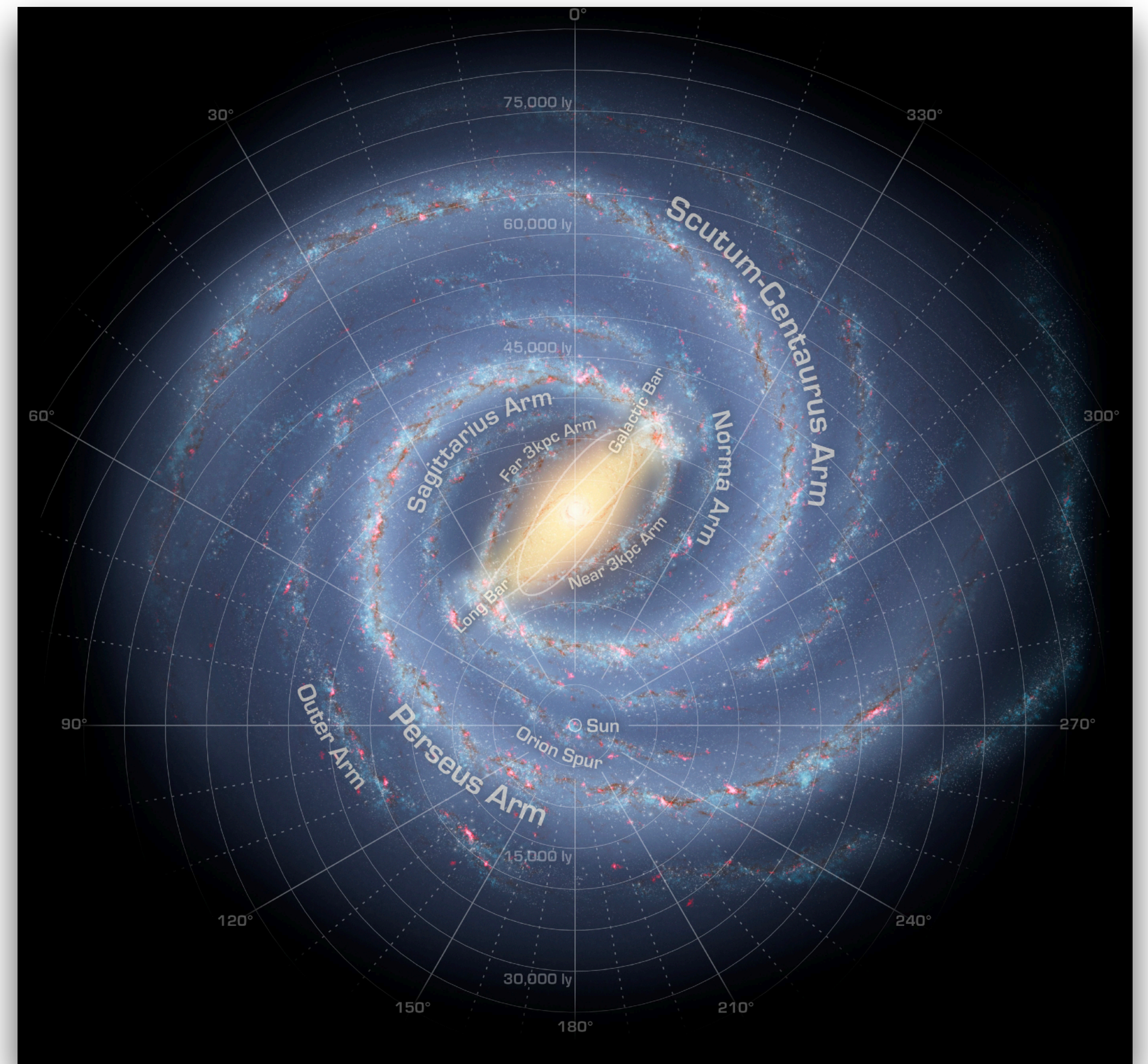


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Diameter of Milky Way disk

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Radius of solar orbit



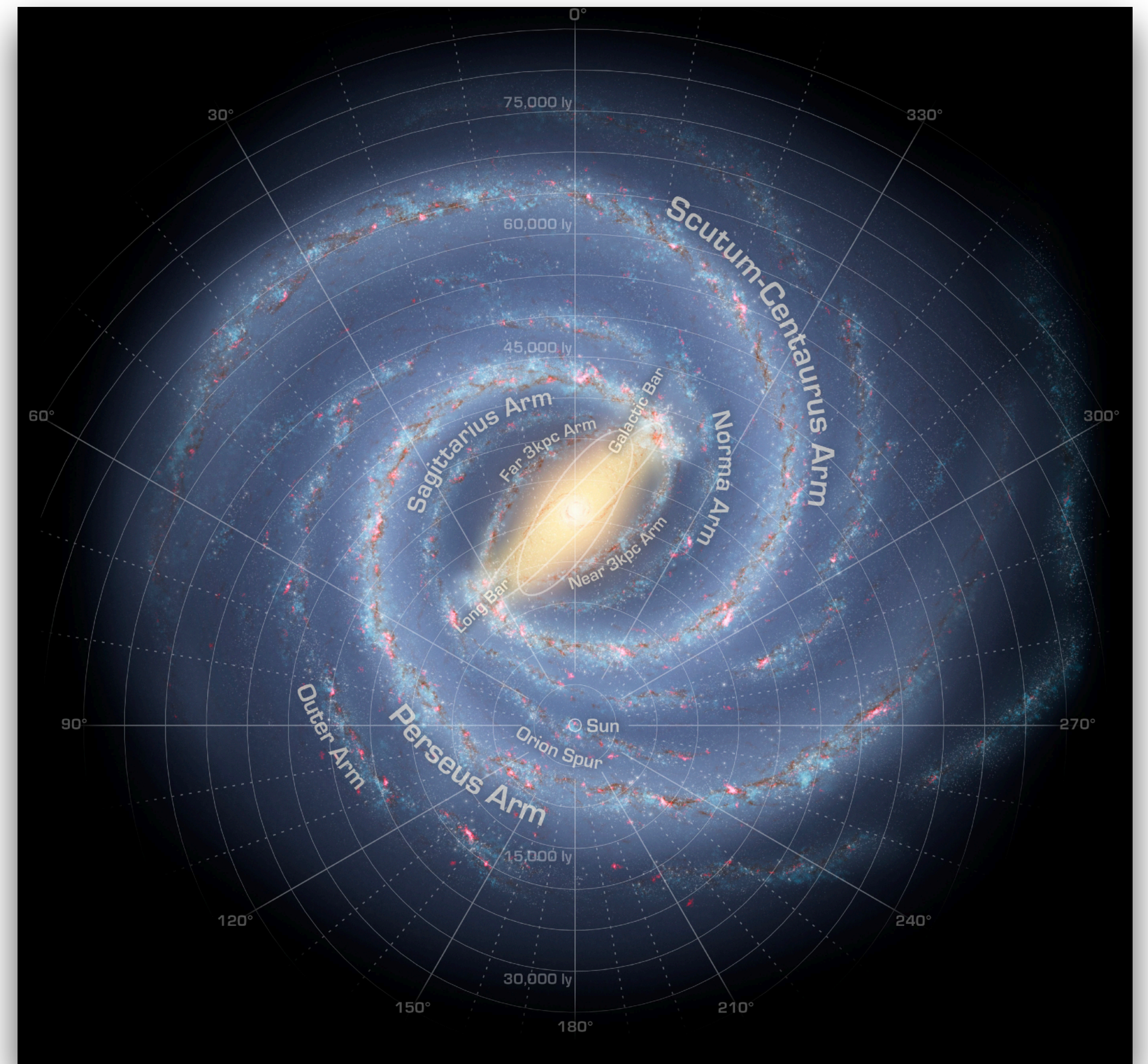
How well do you know your galaxy?

Diameter of Milky Way disk

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Radius of solar orbit

→ 8 kpc



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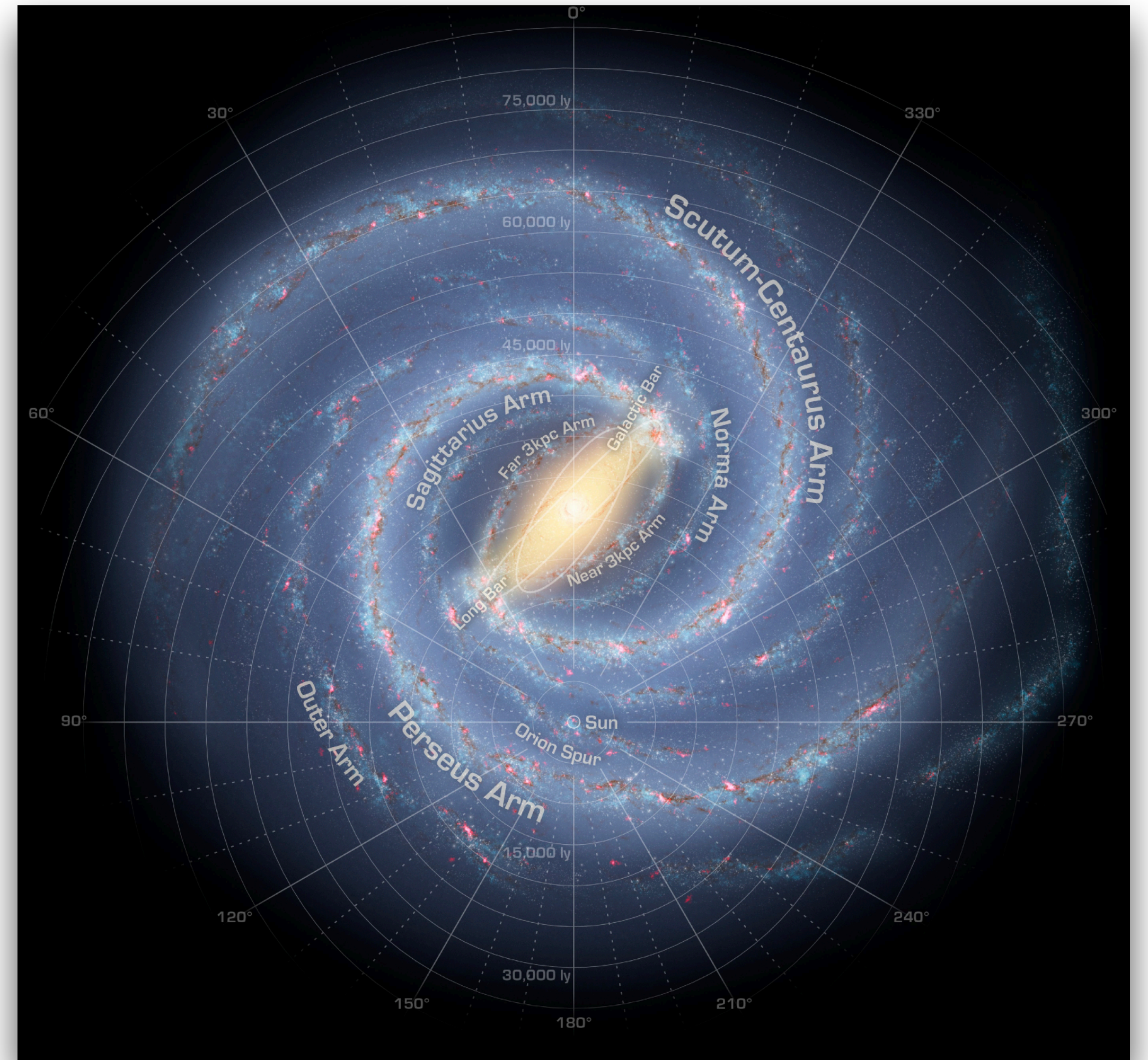
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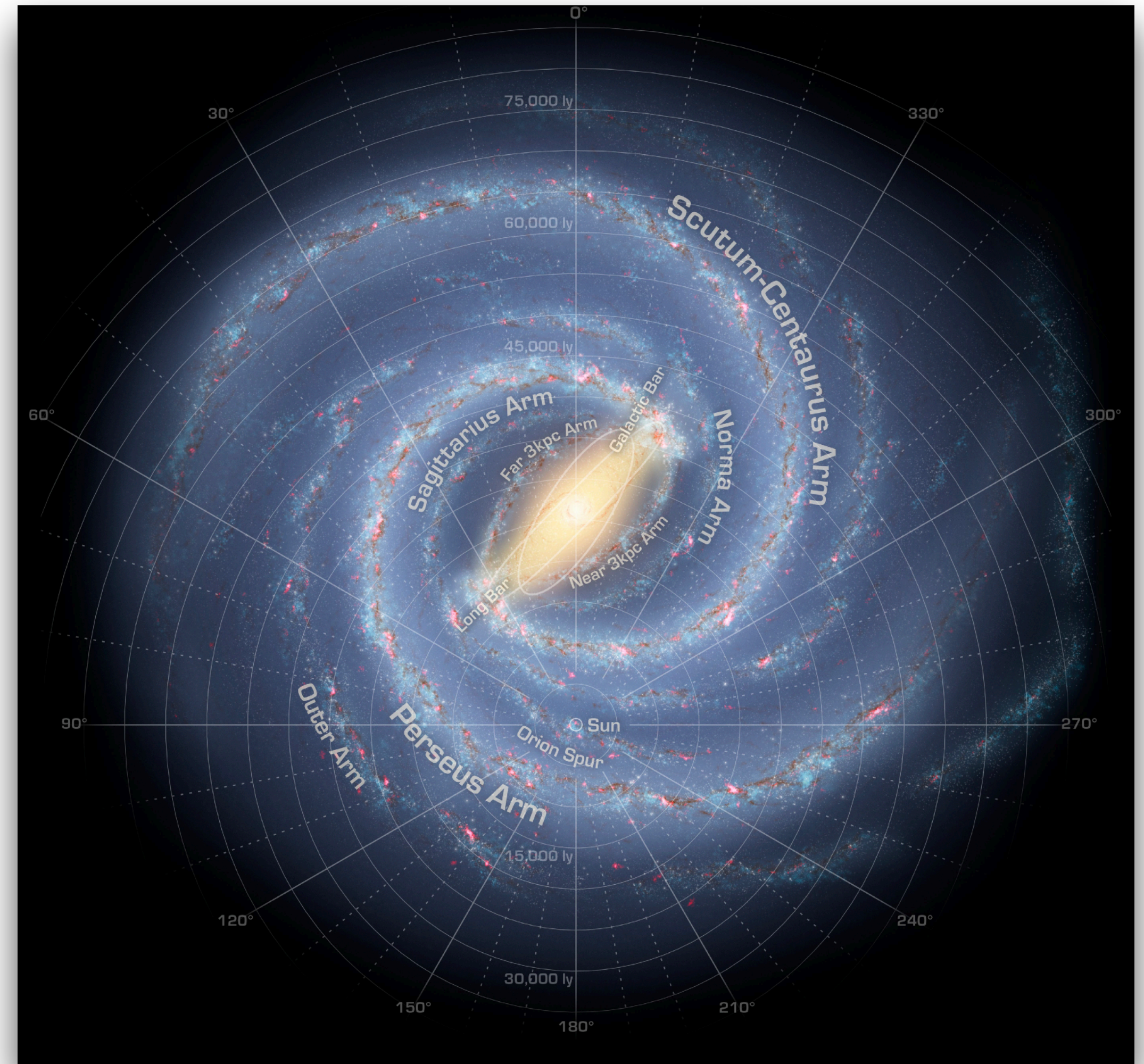
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Radius of solar orbit

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How many birthdays has the Sun had?

→ Solar age/Solar orbit = 4.6 Gyr/230 Myr = 20



How well do you know your galaxy?

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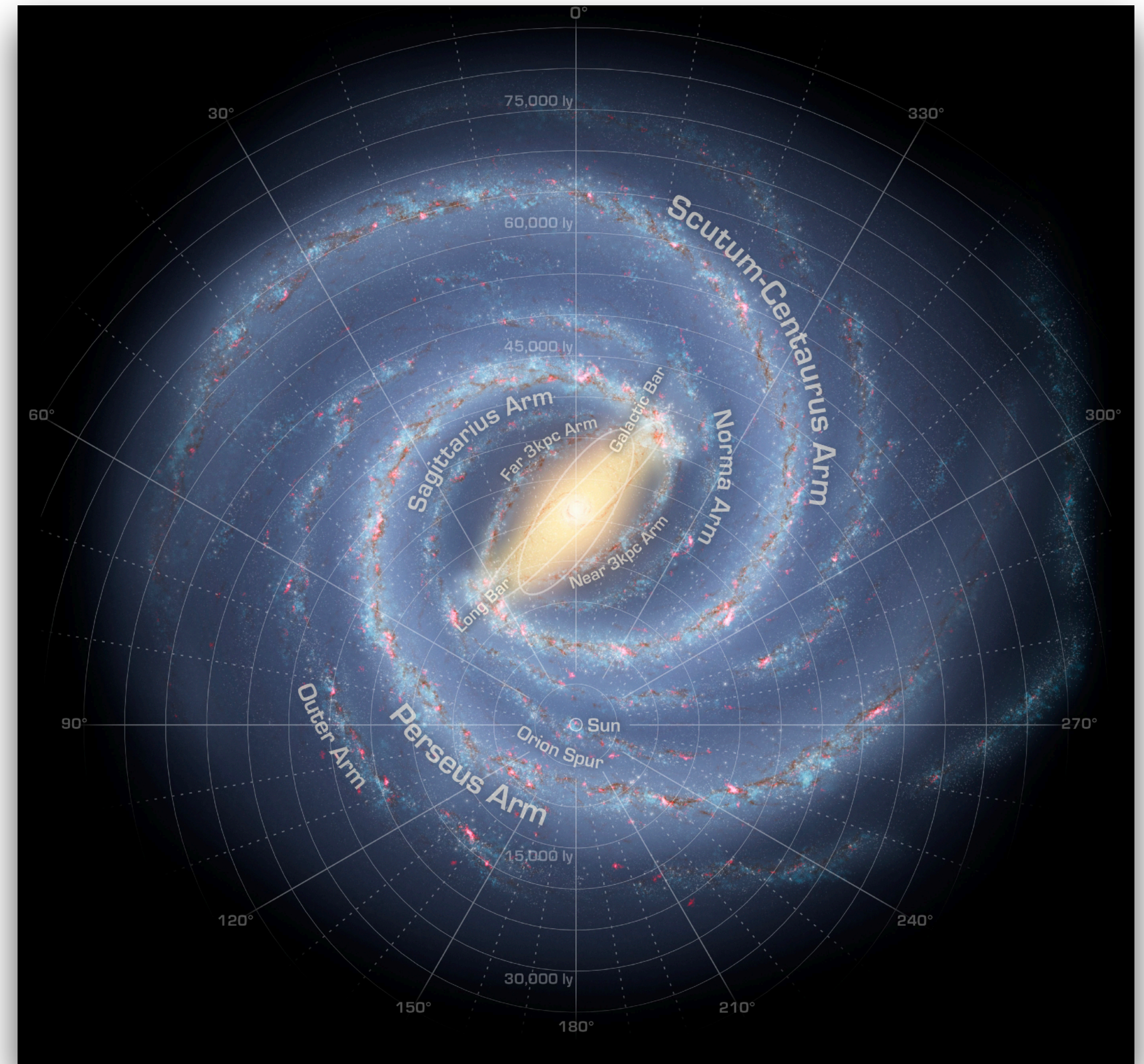
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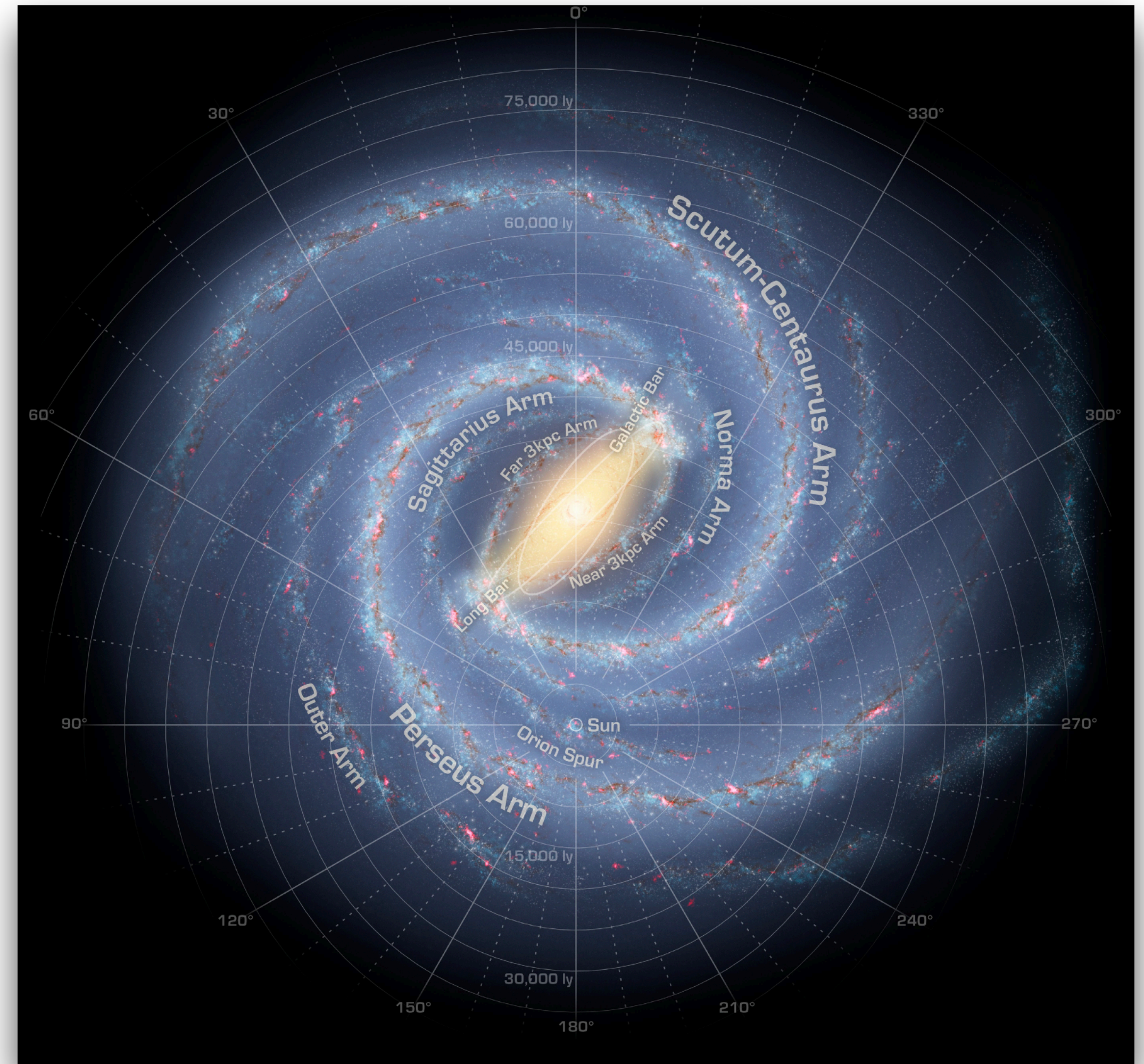
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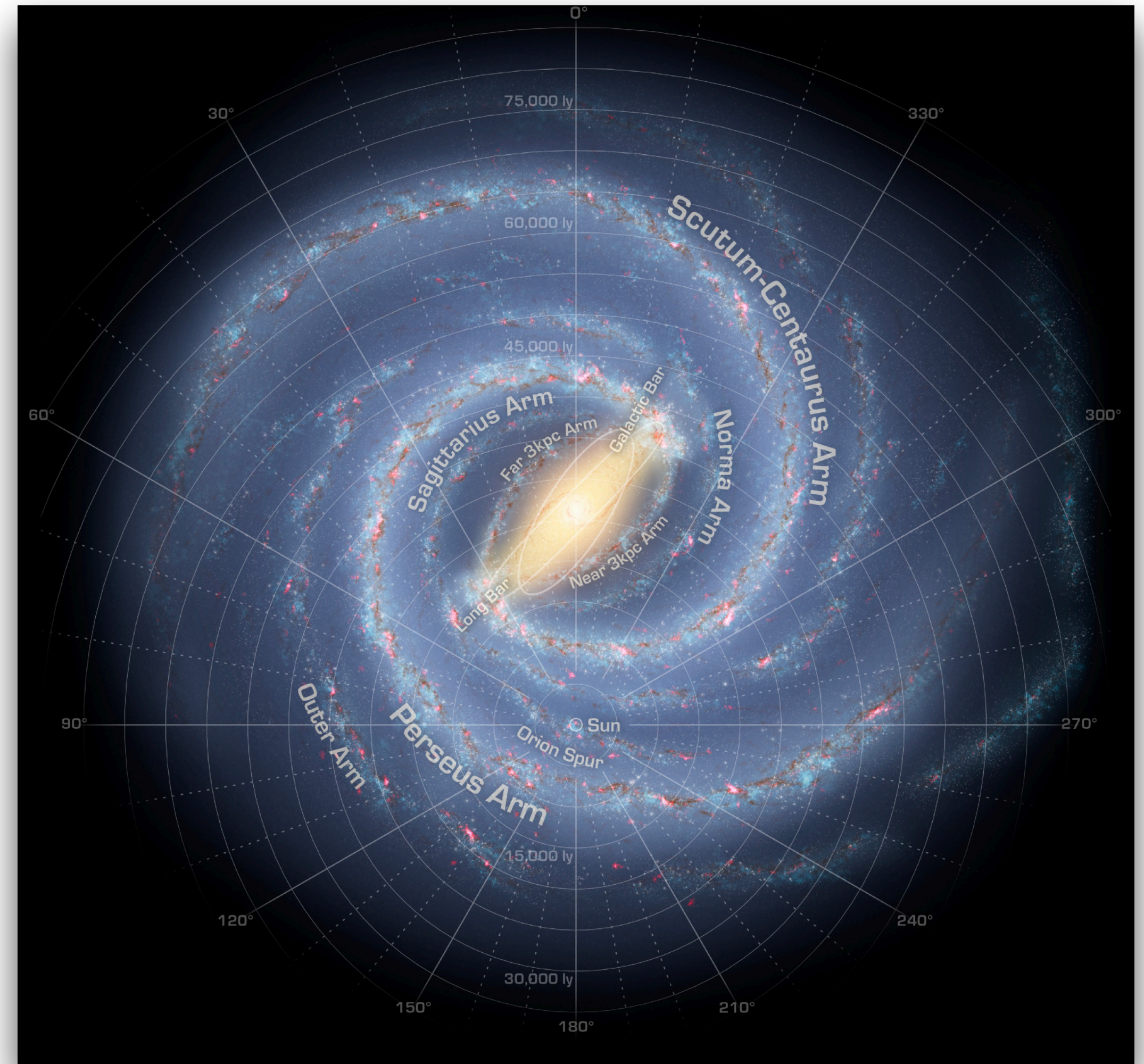
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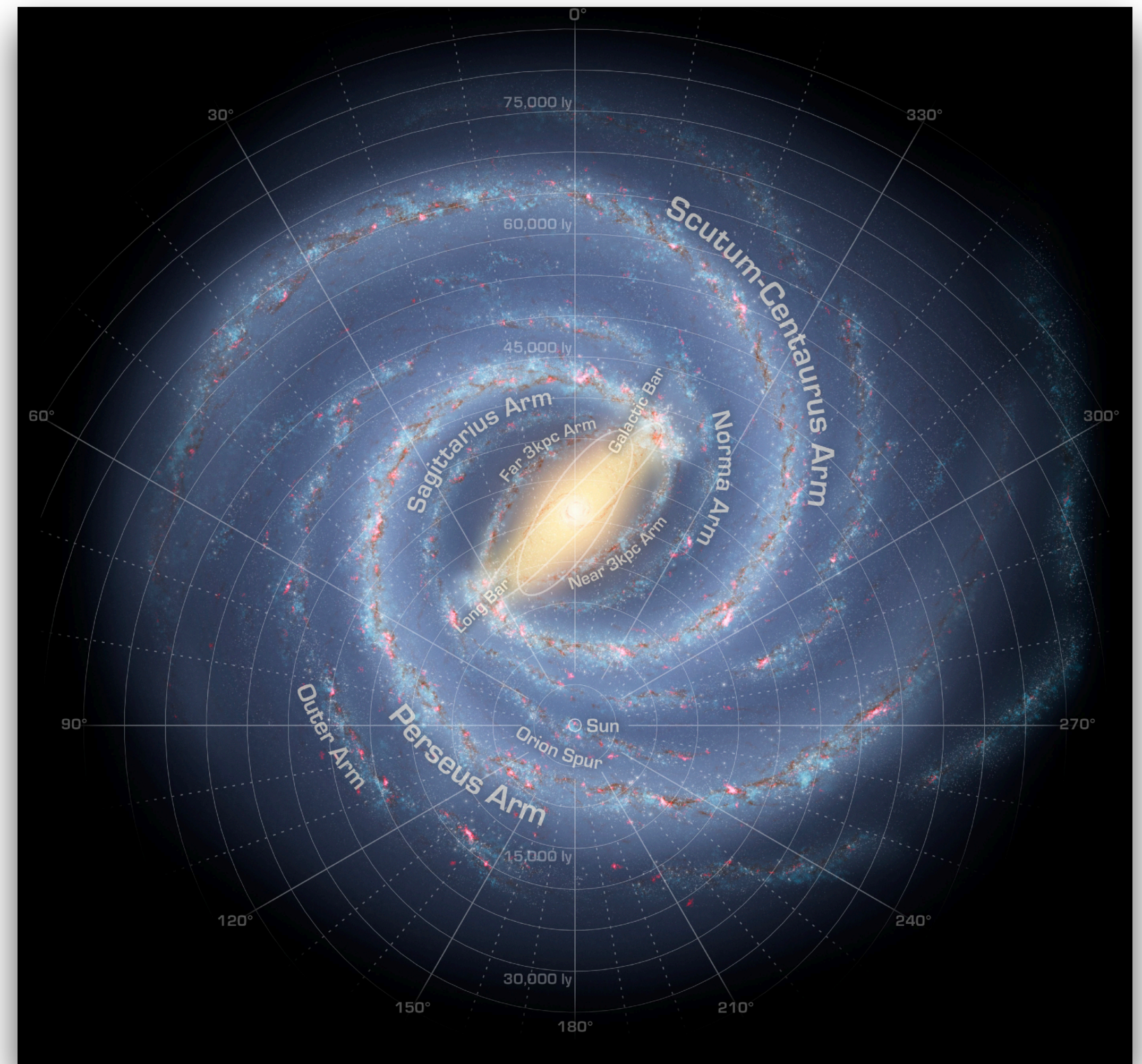
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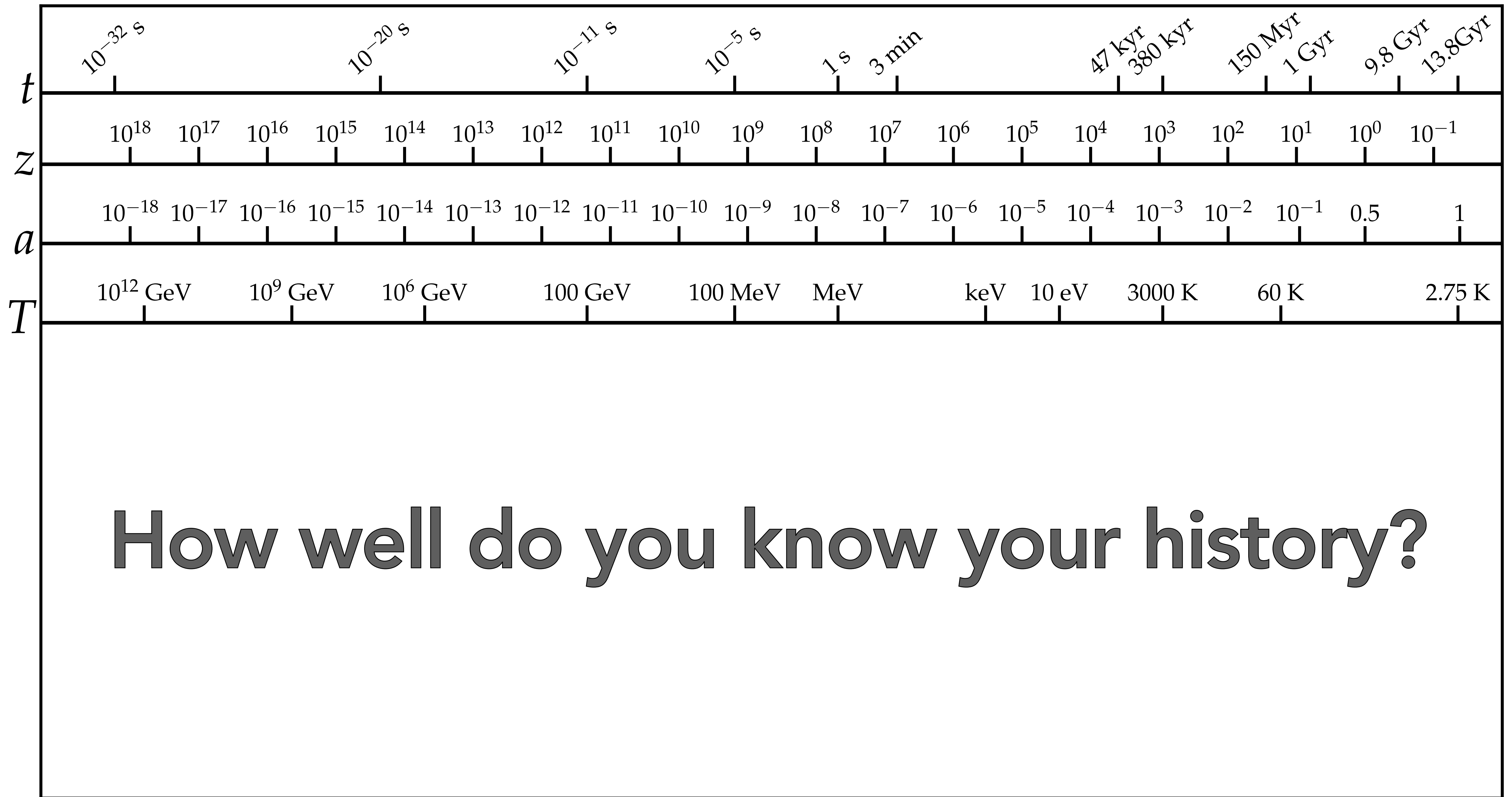
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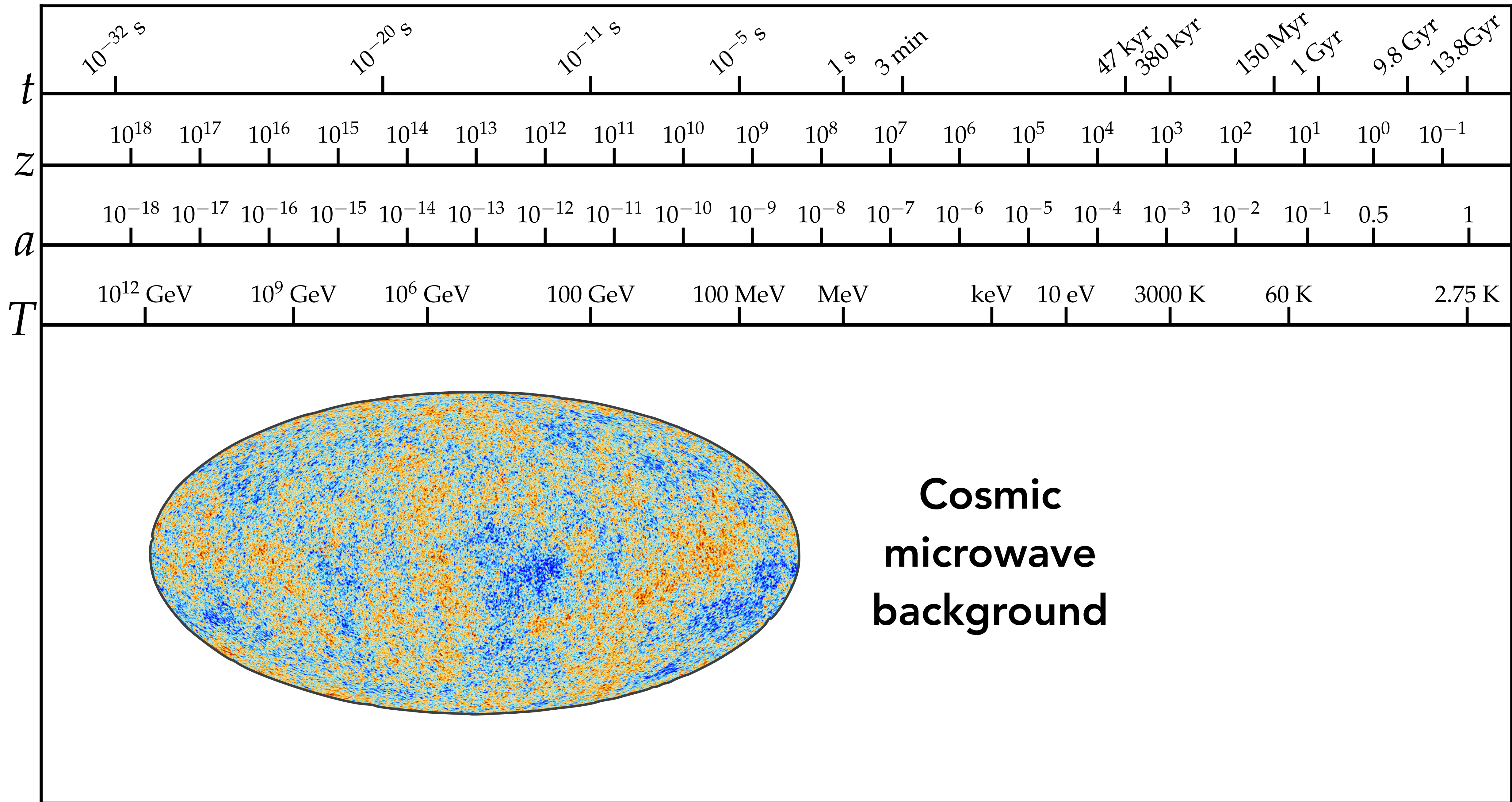
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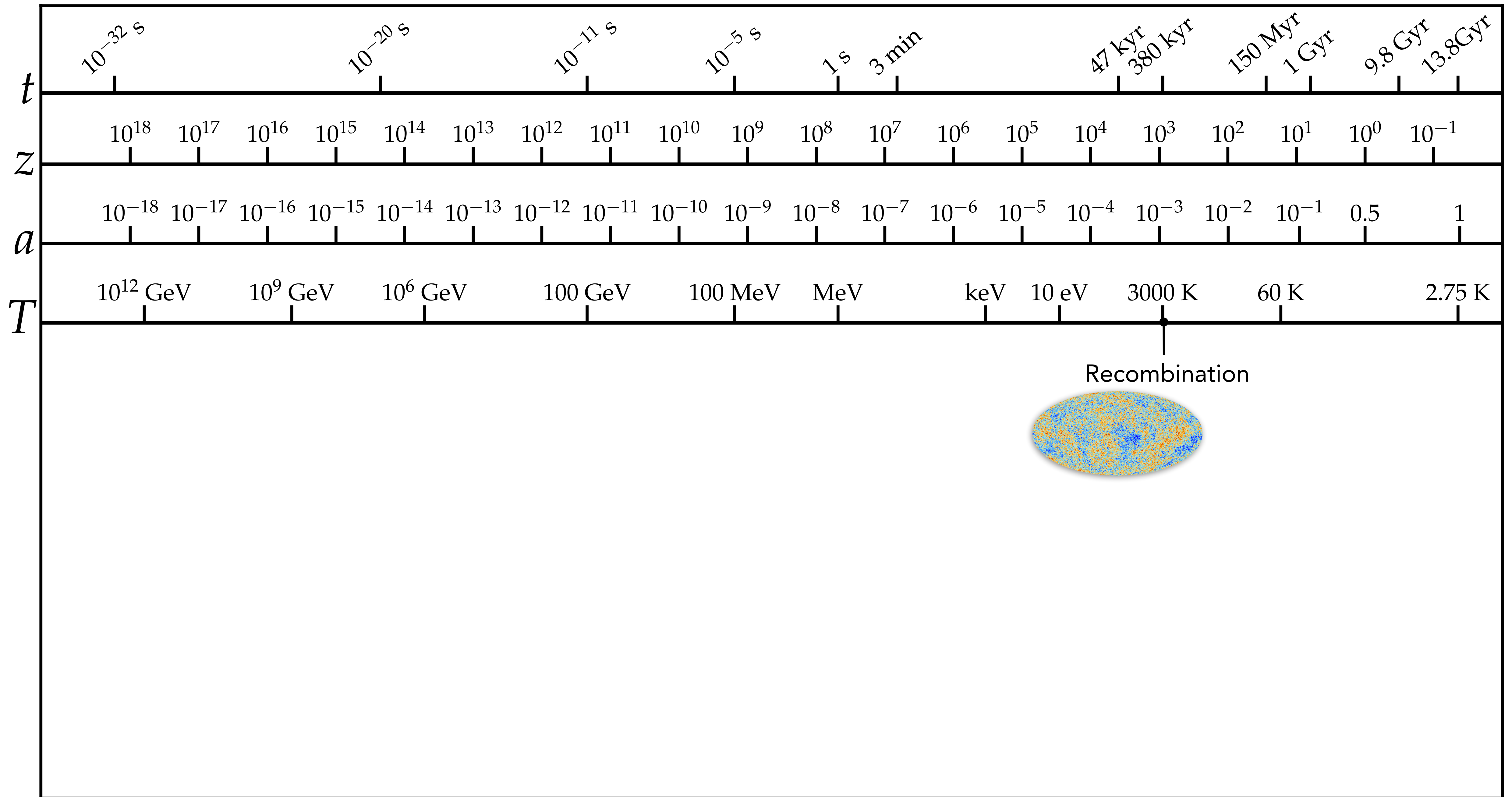
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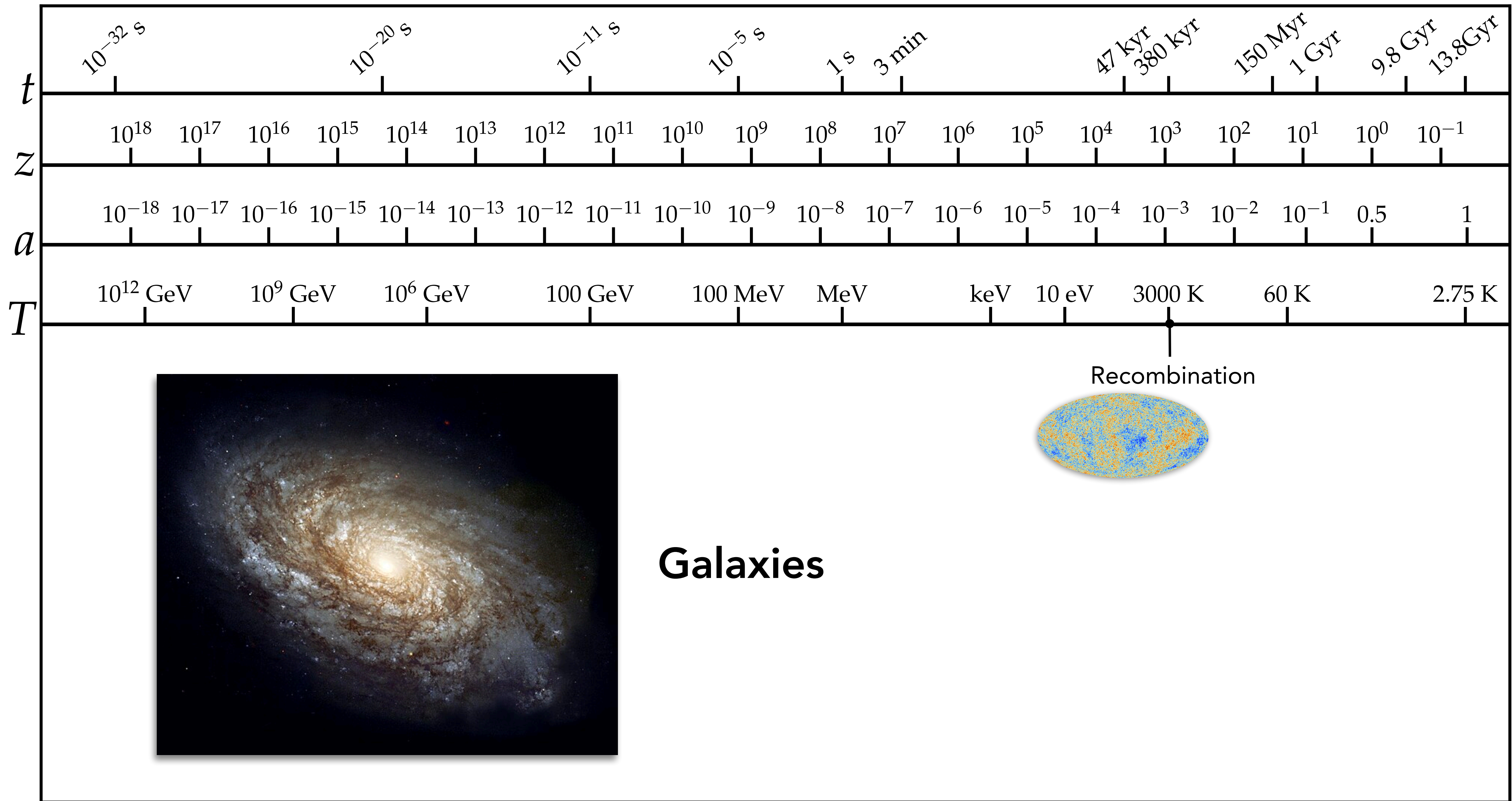
→ ?

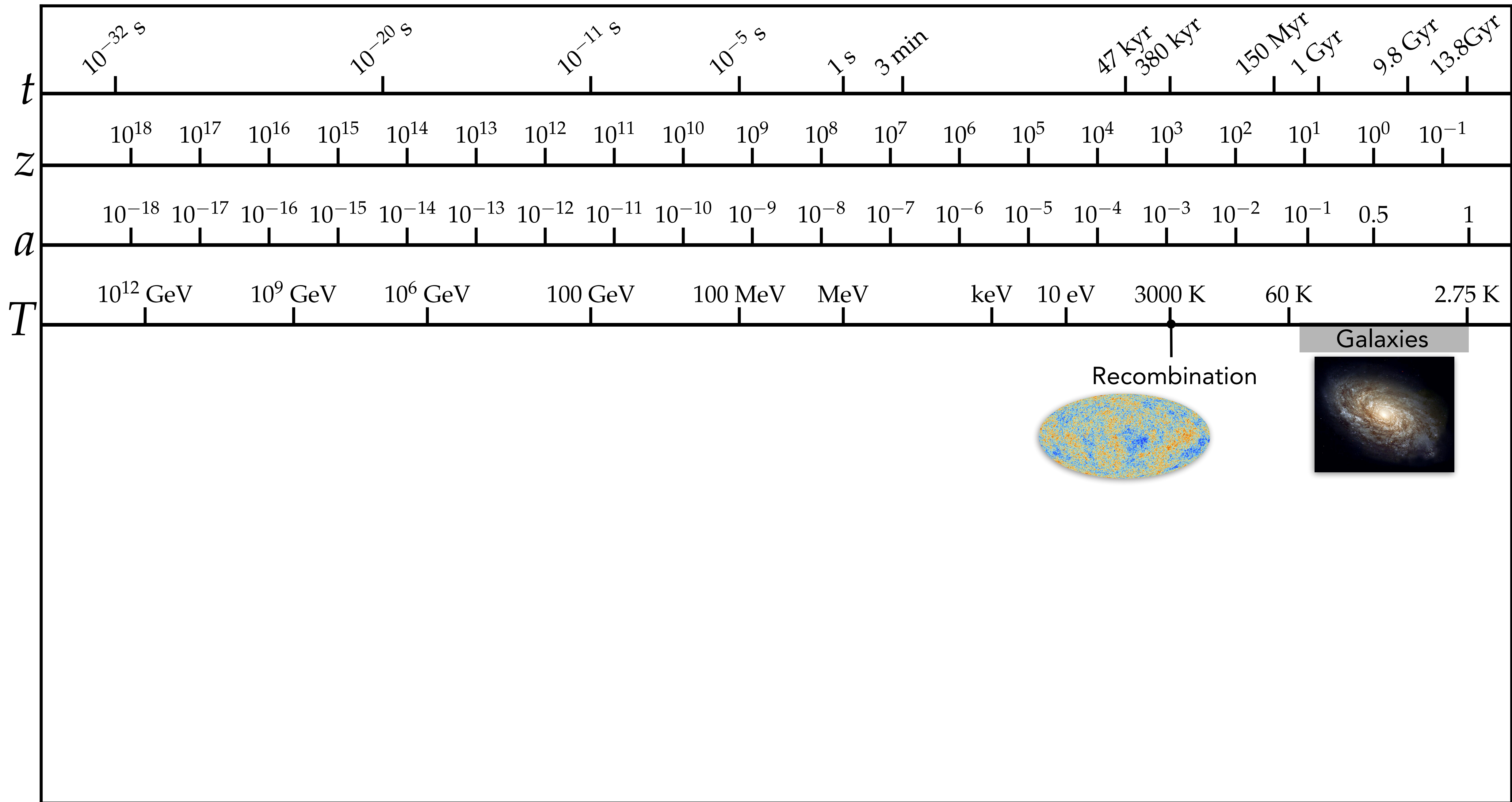


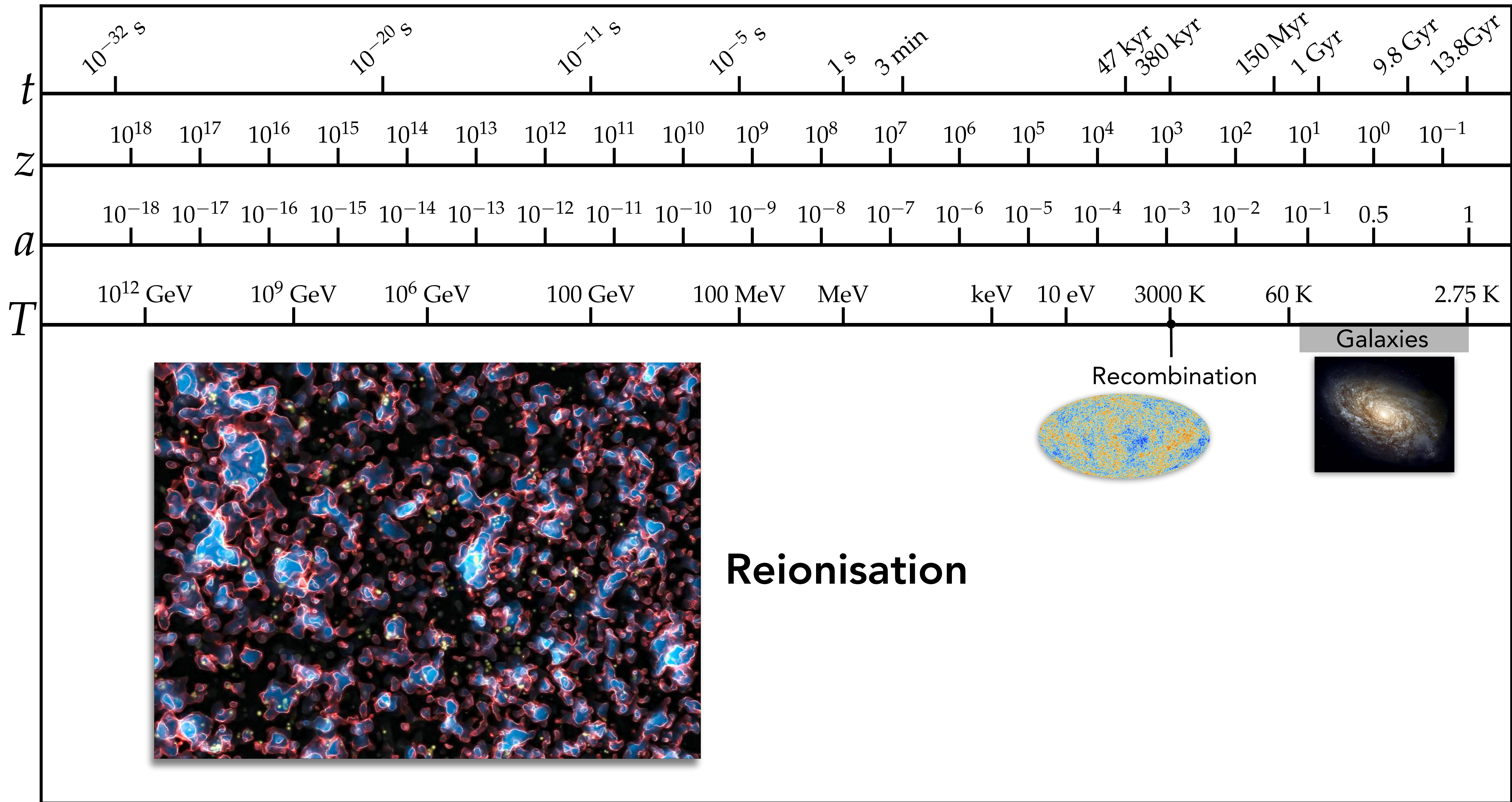


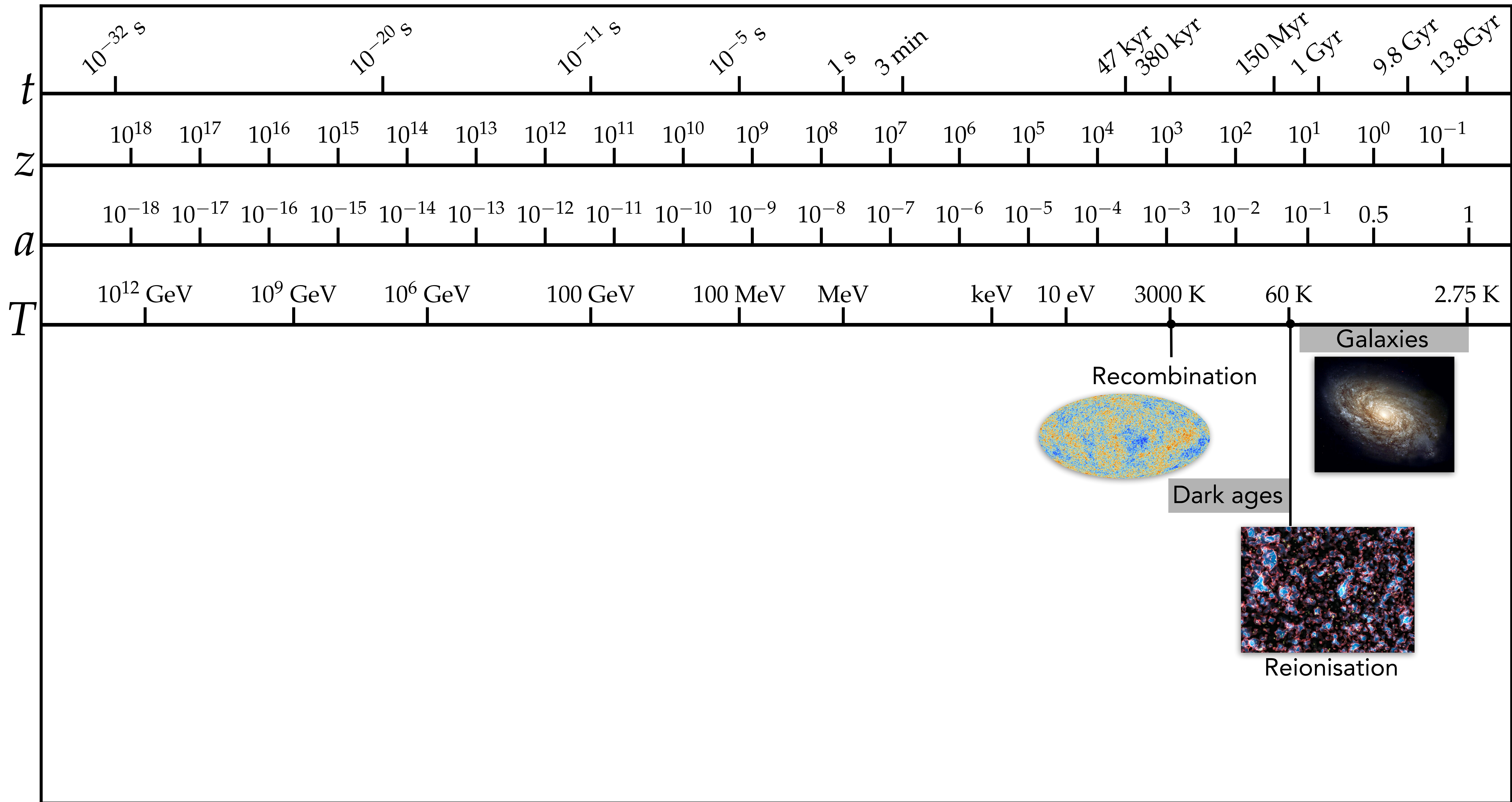


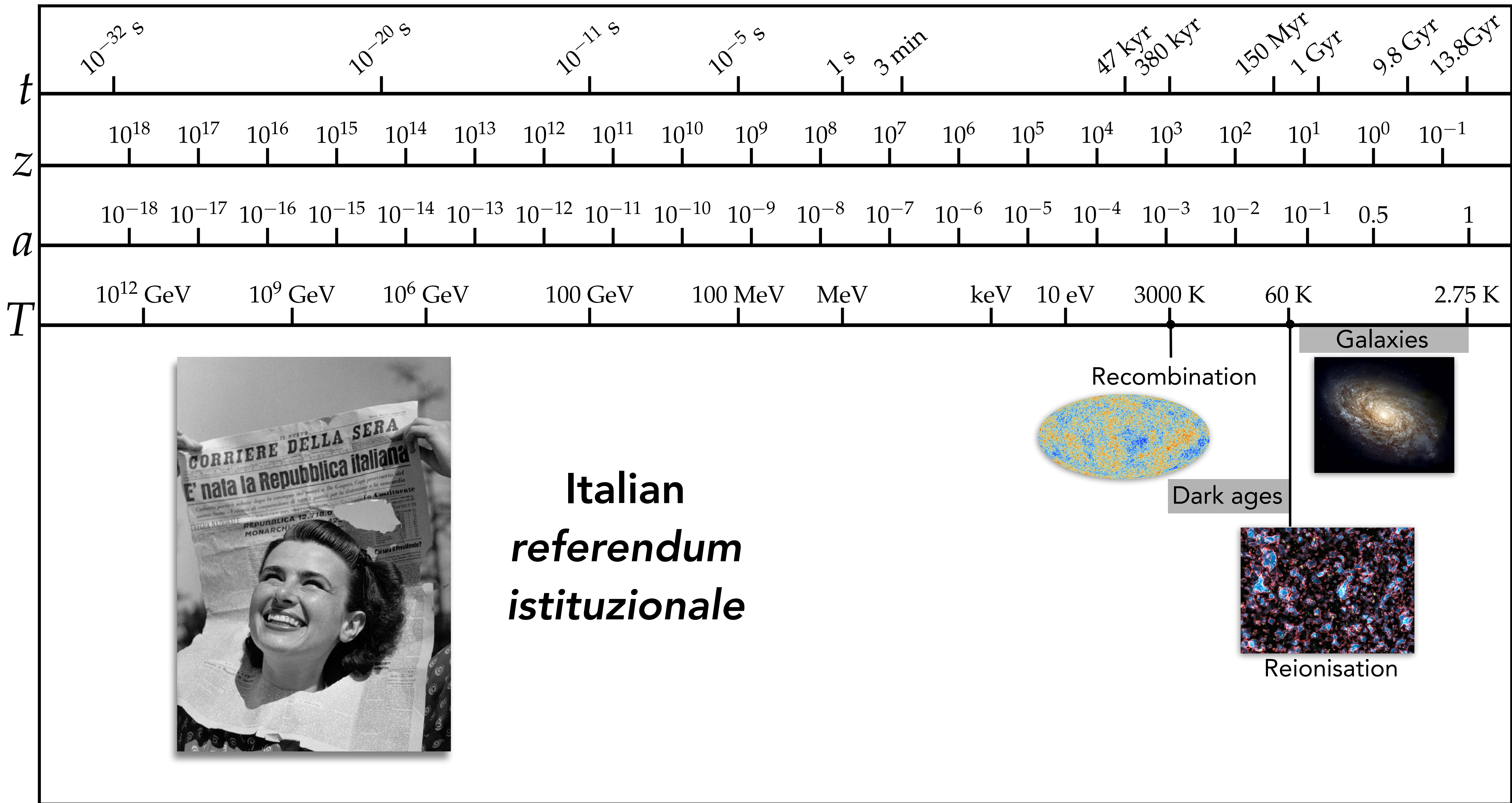


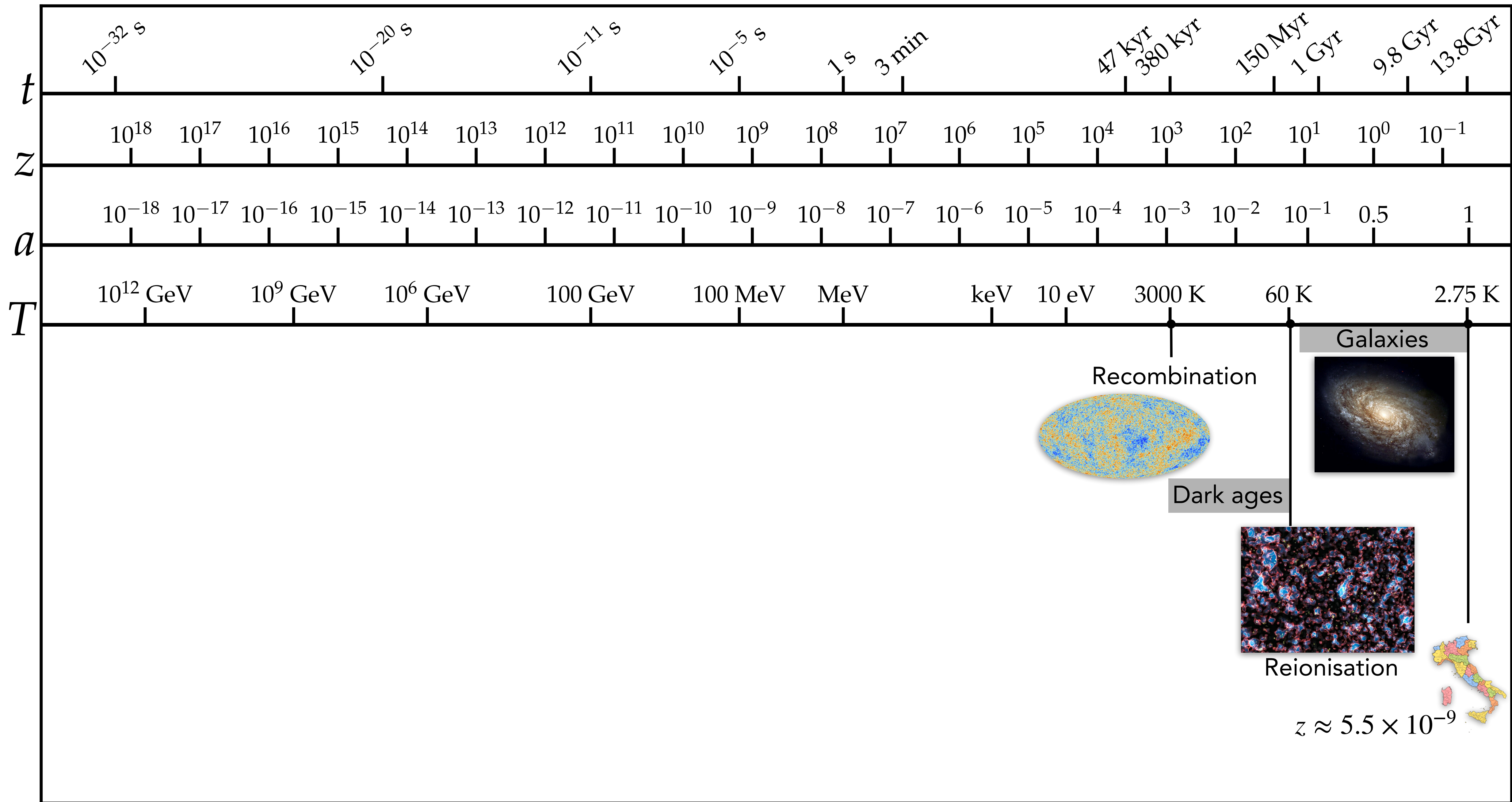


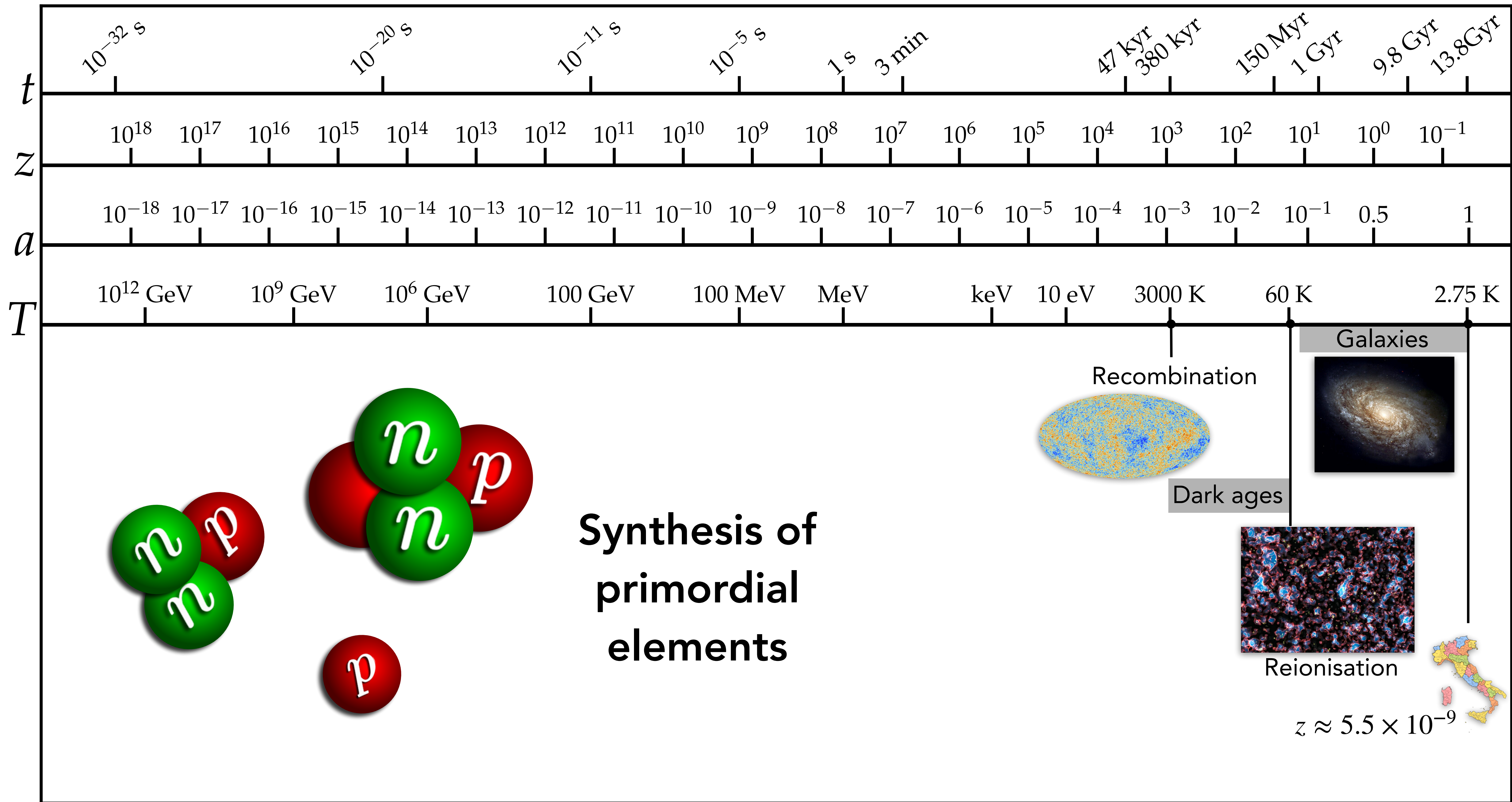


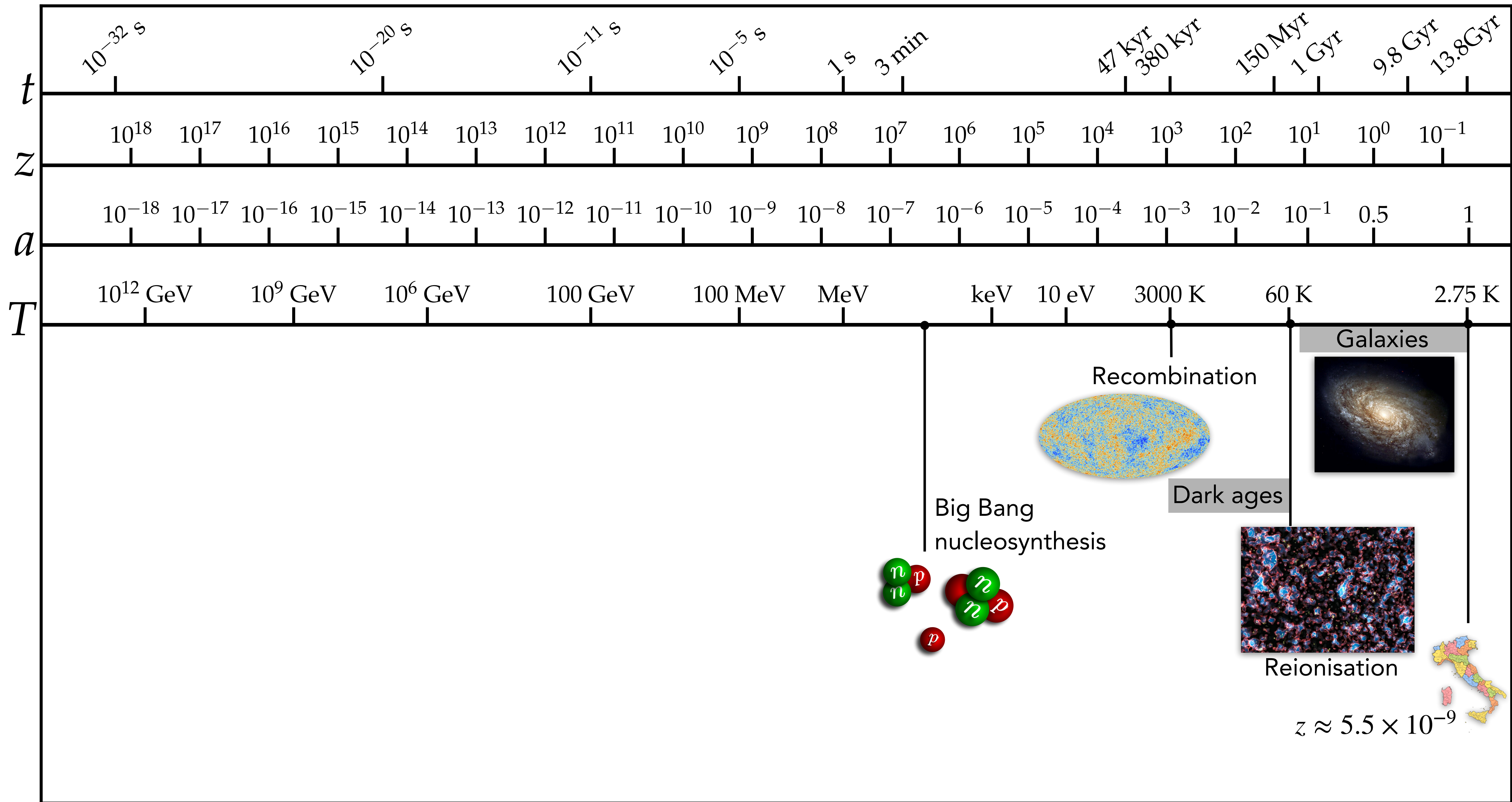


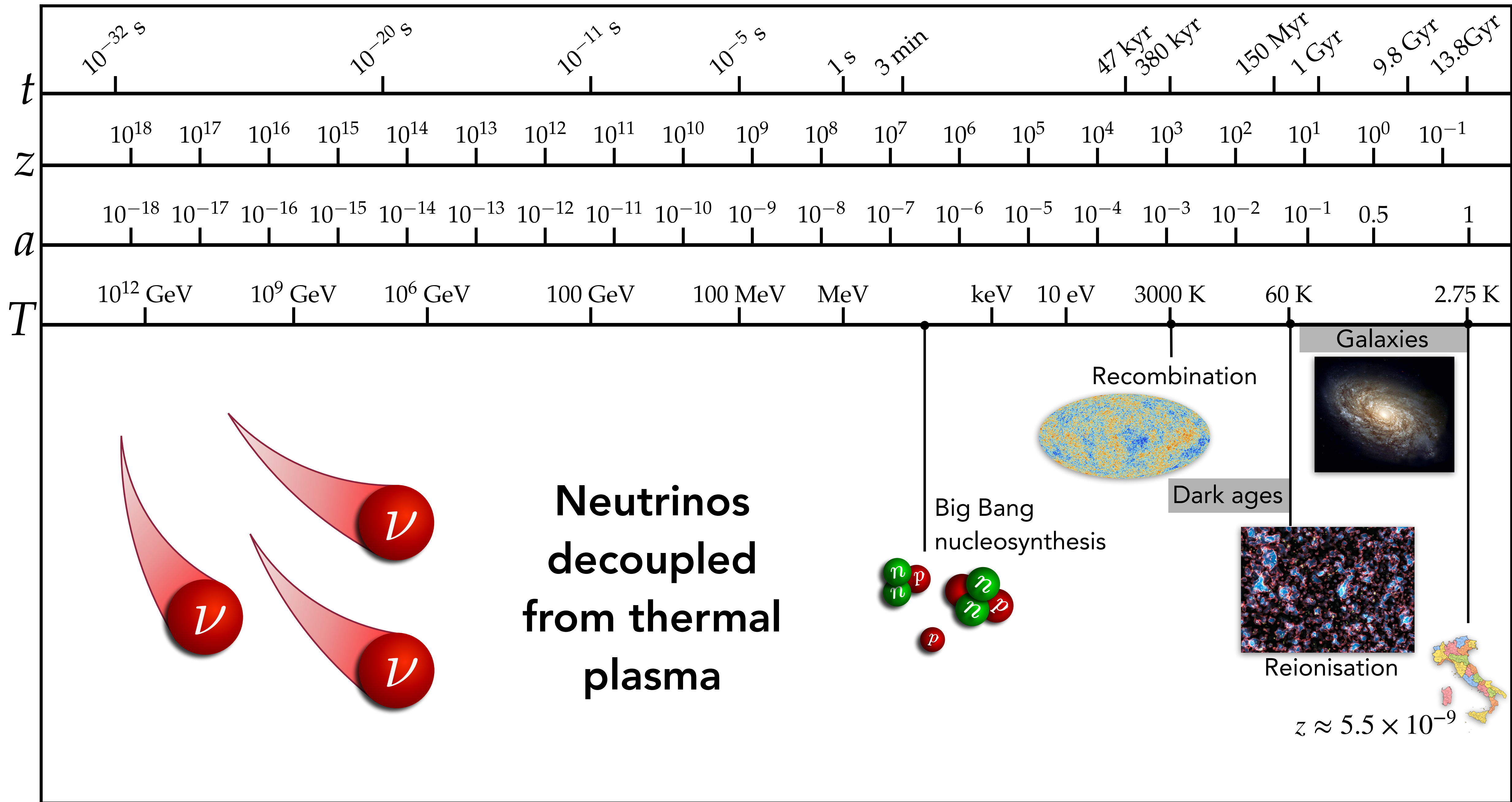


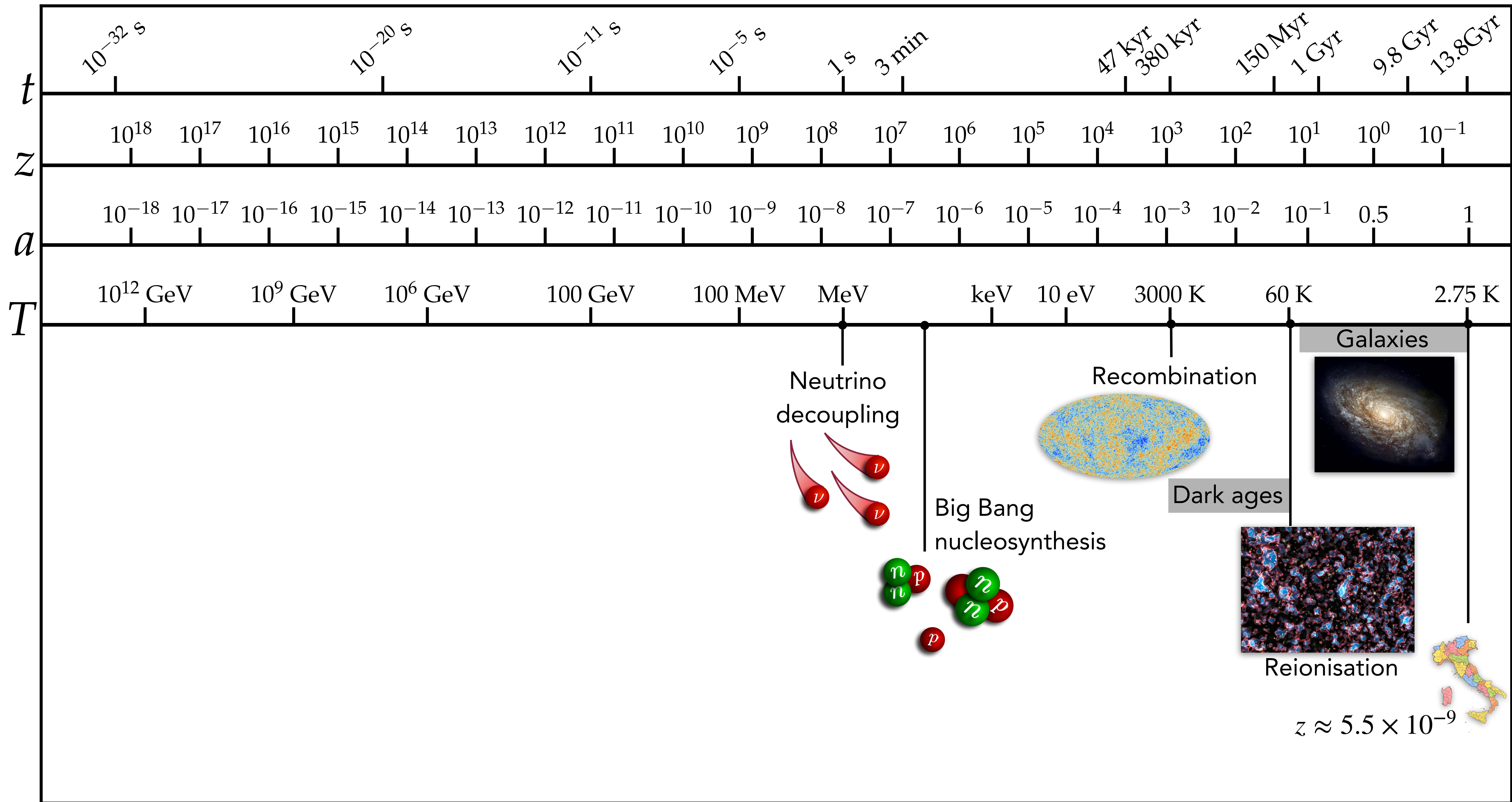


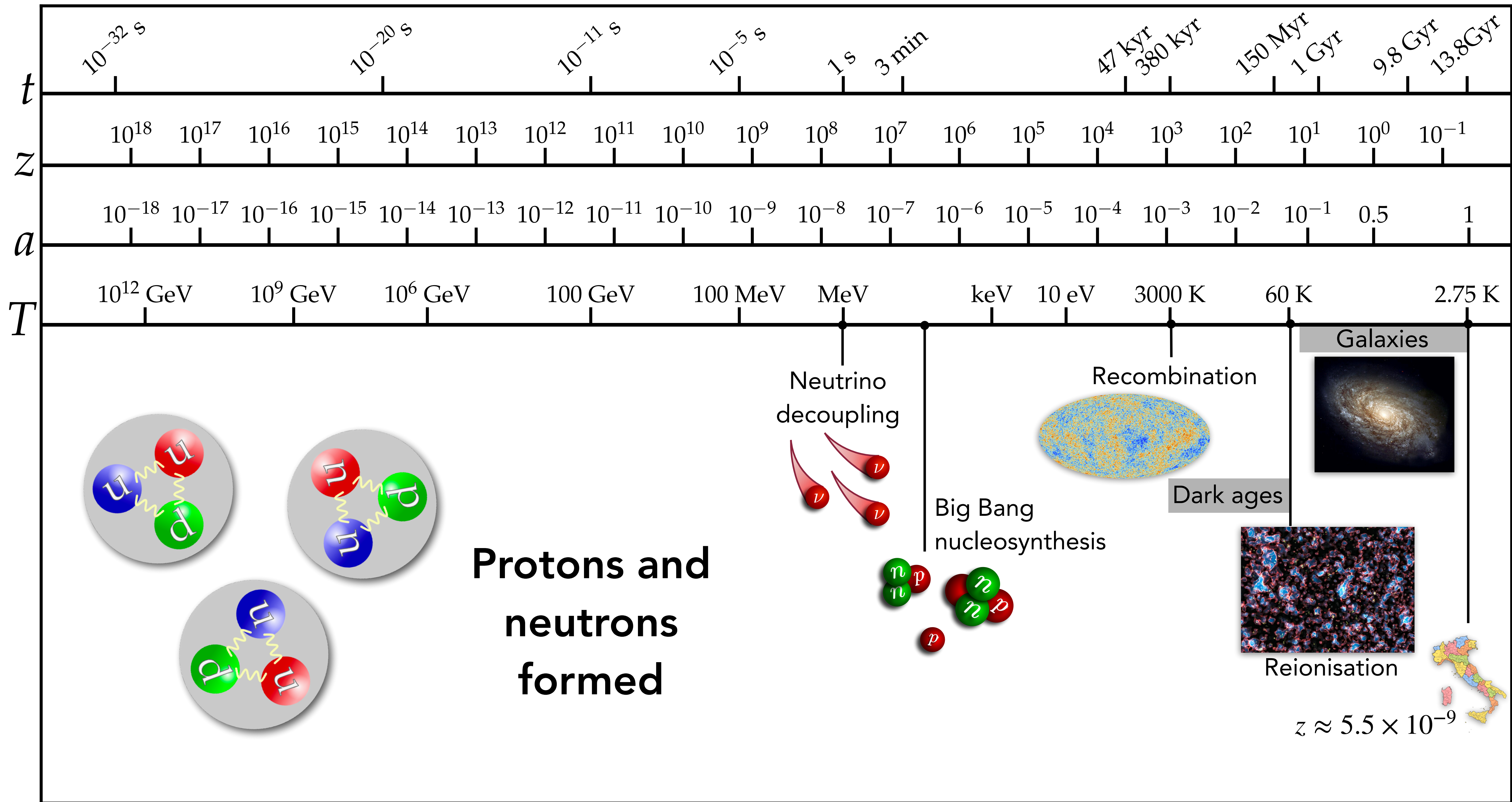


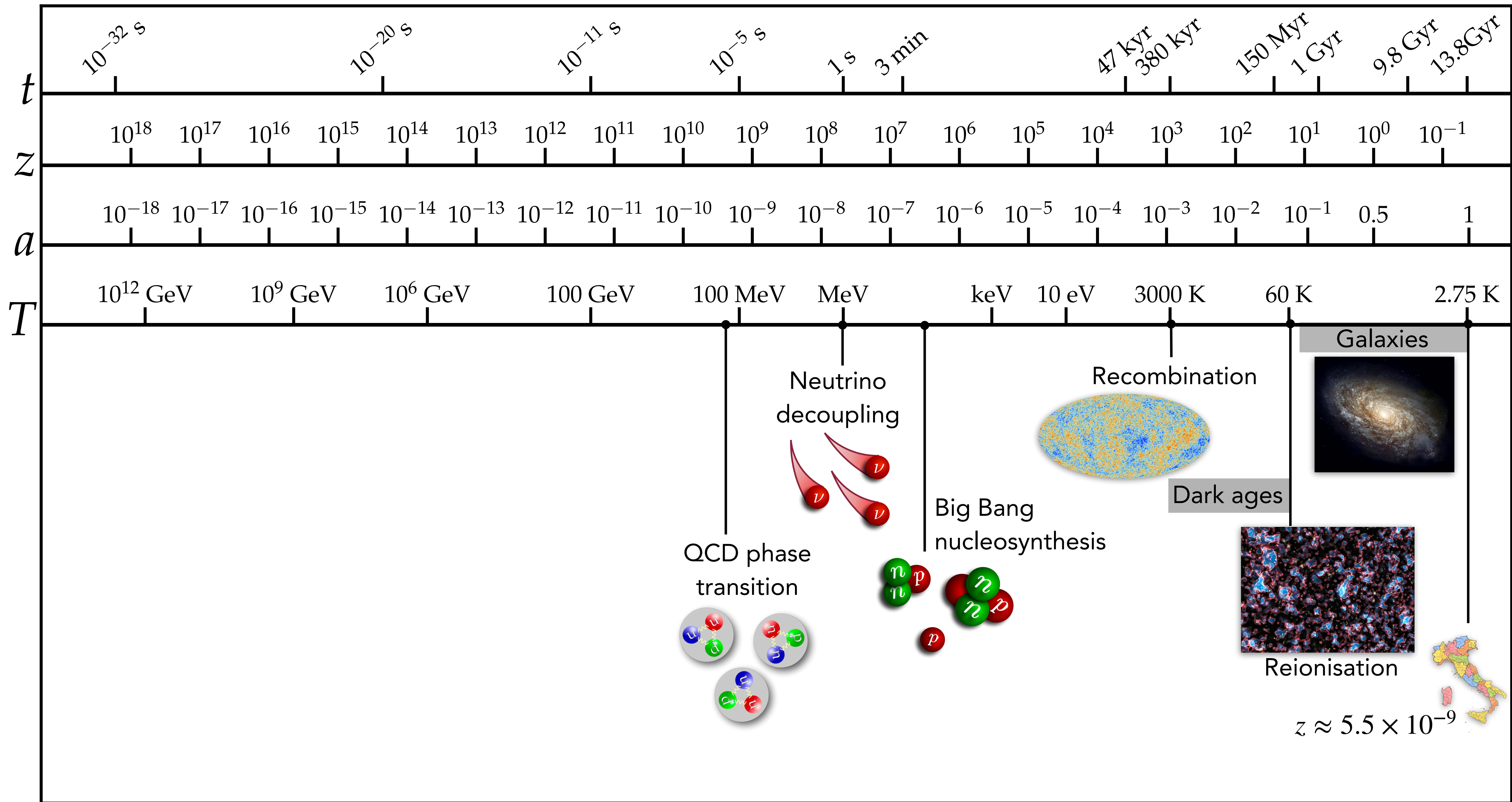


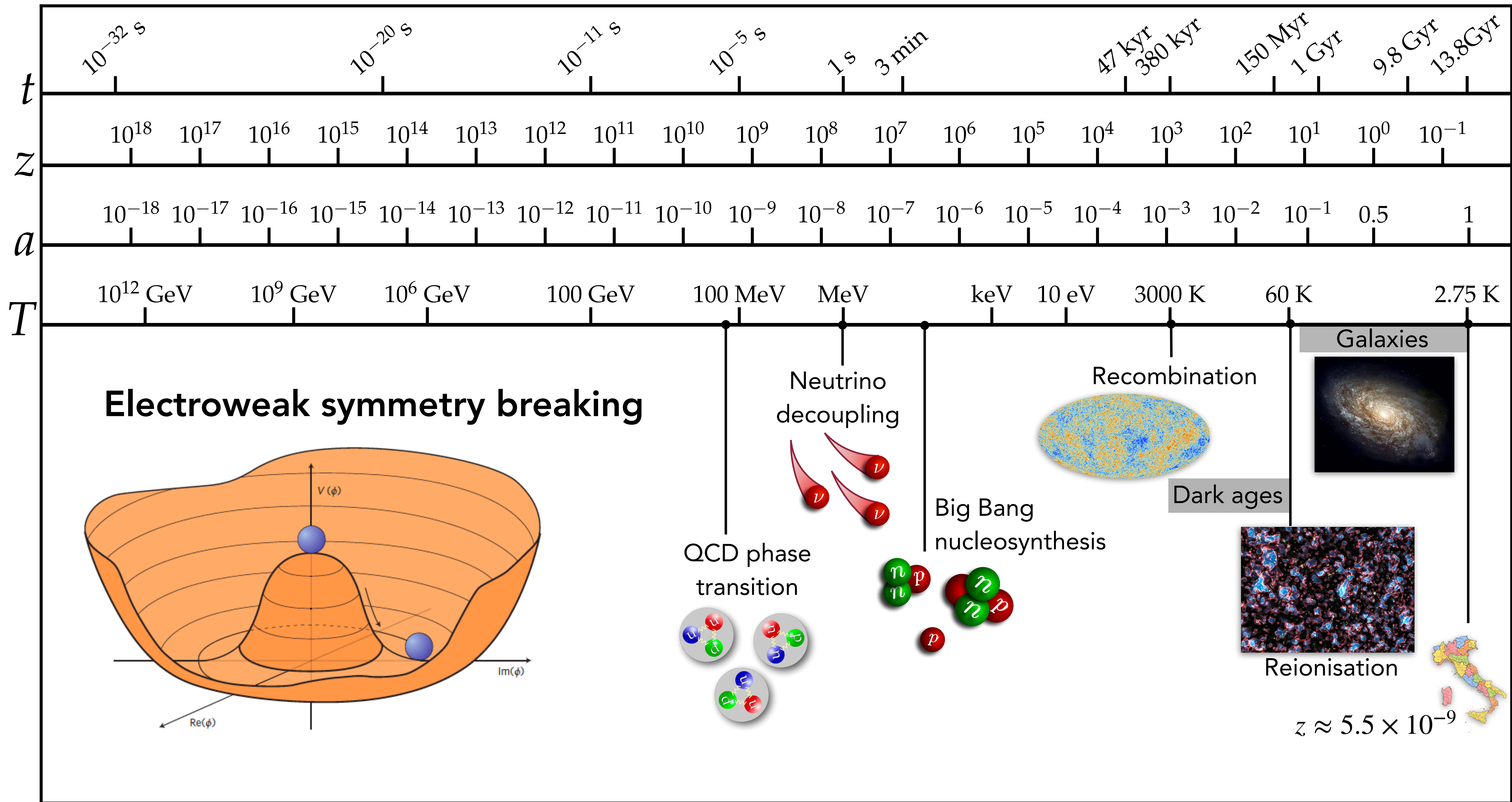


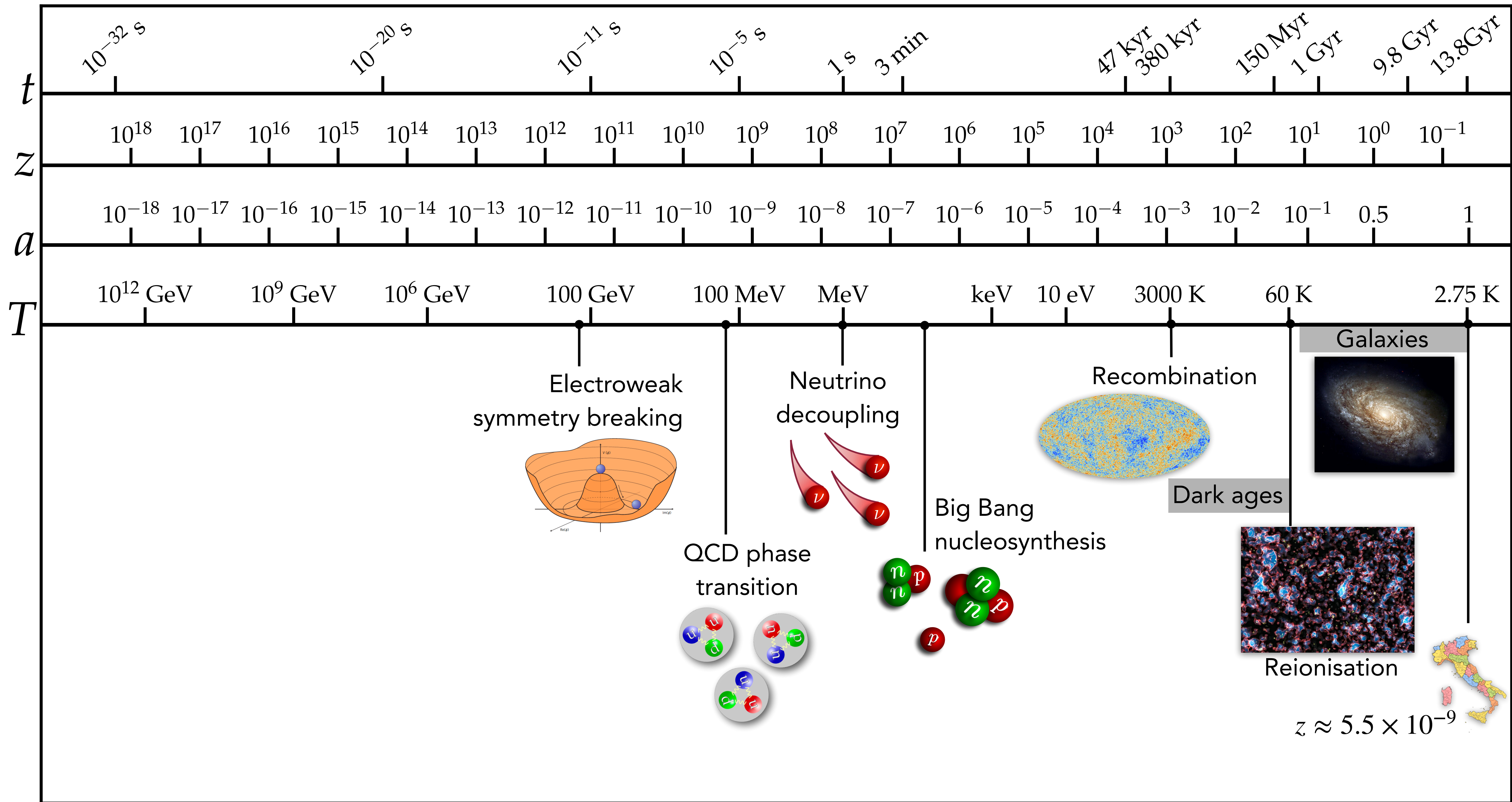


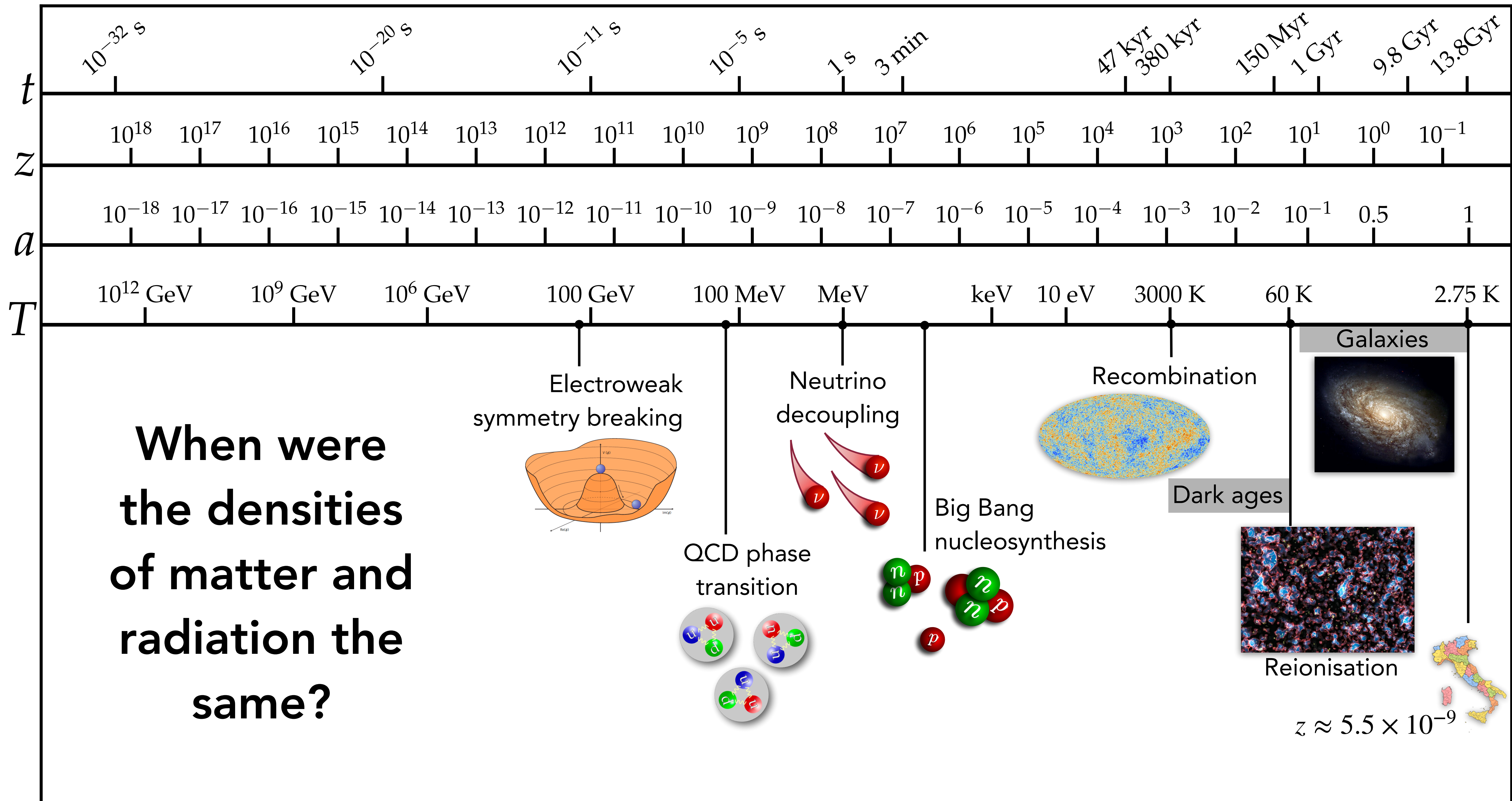


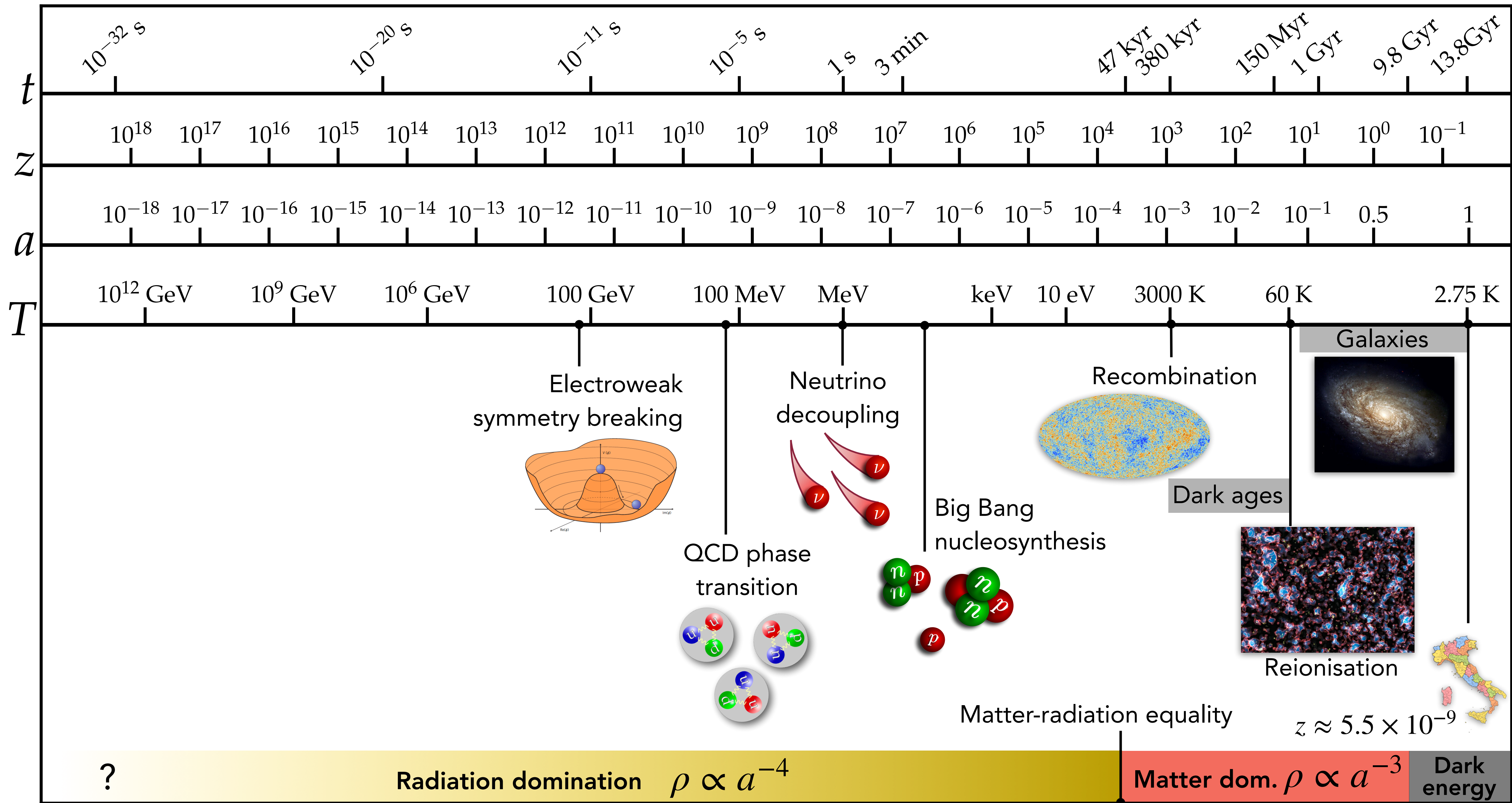


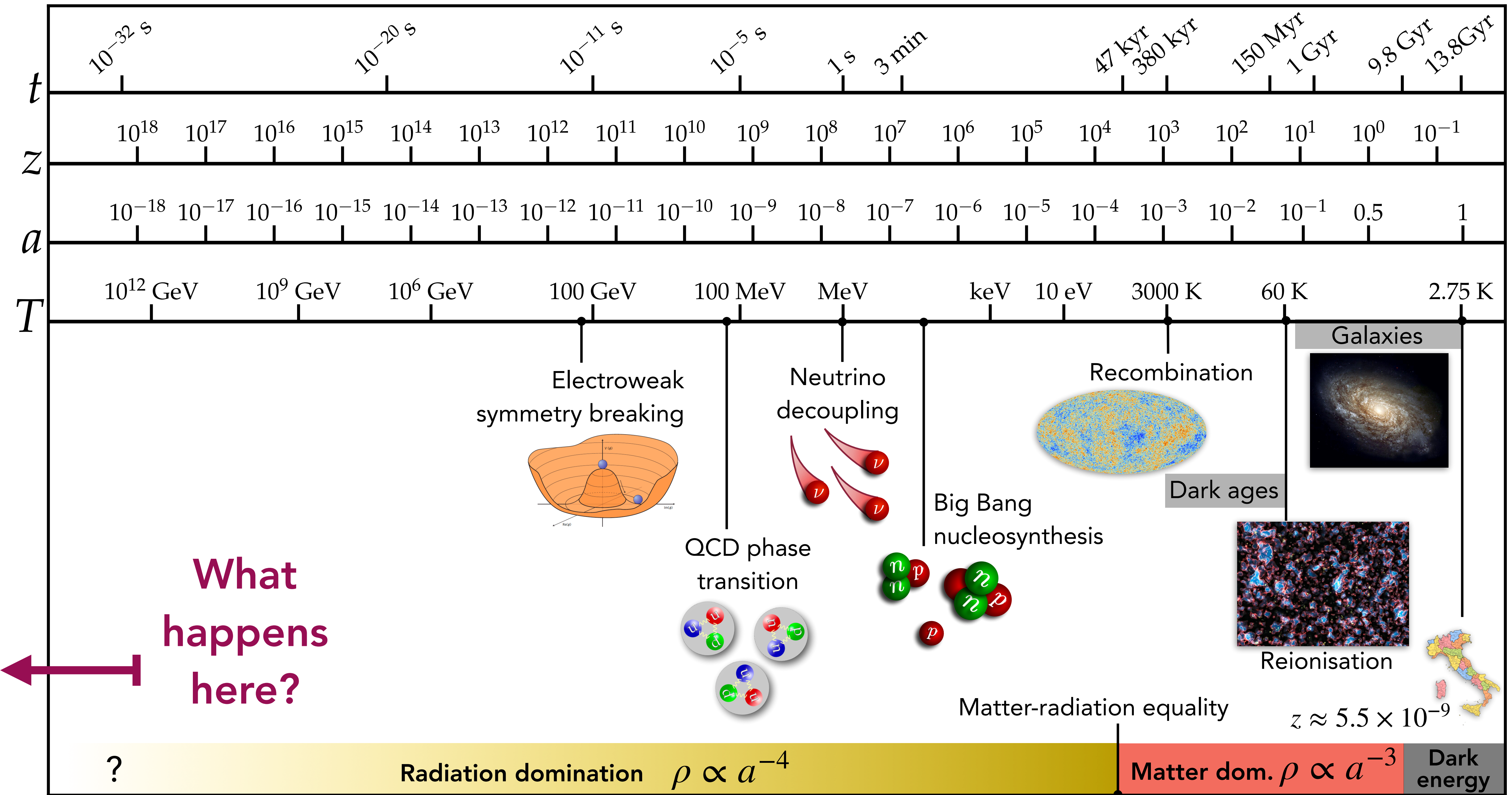


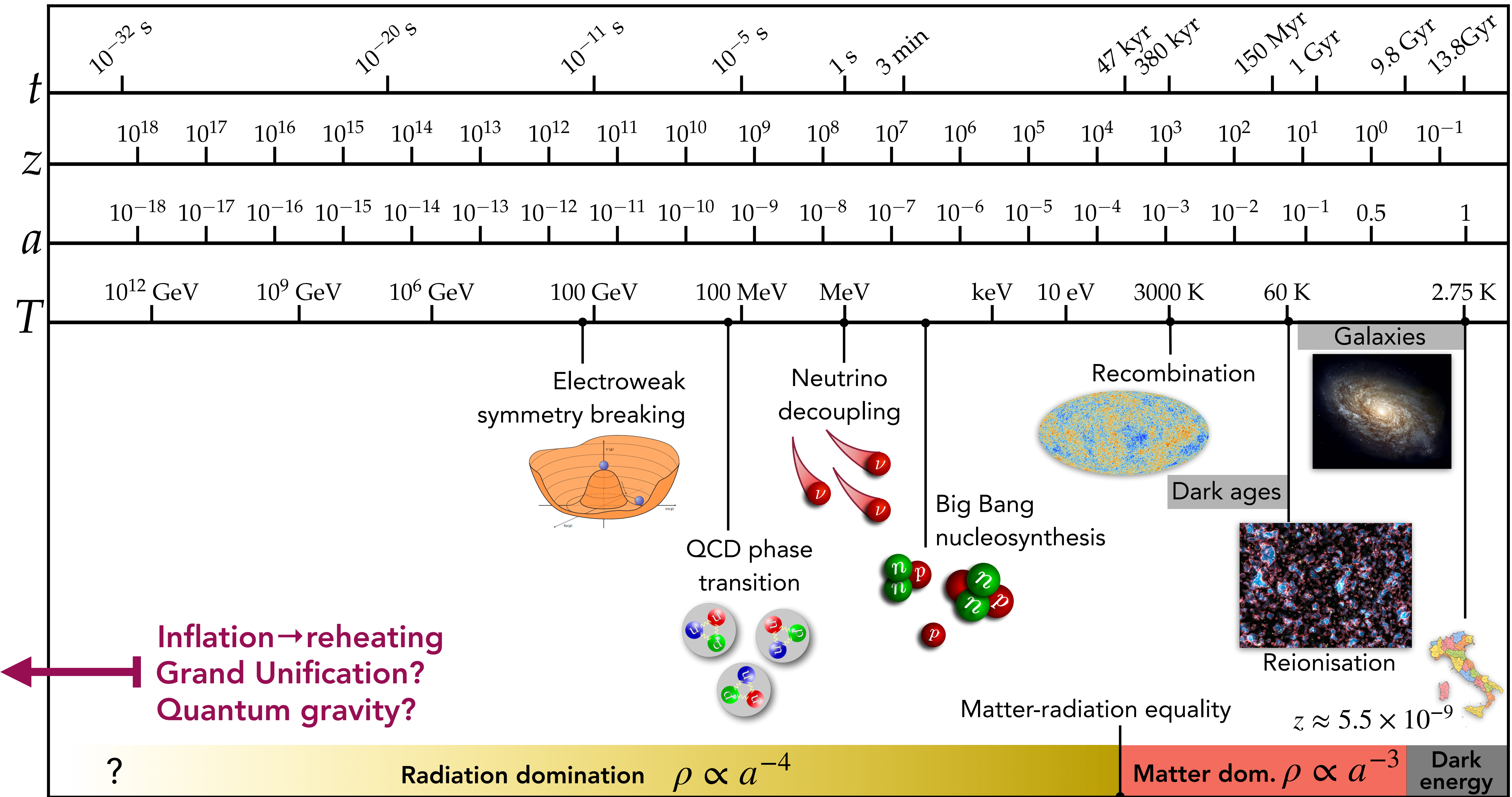


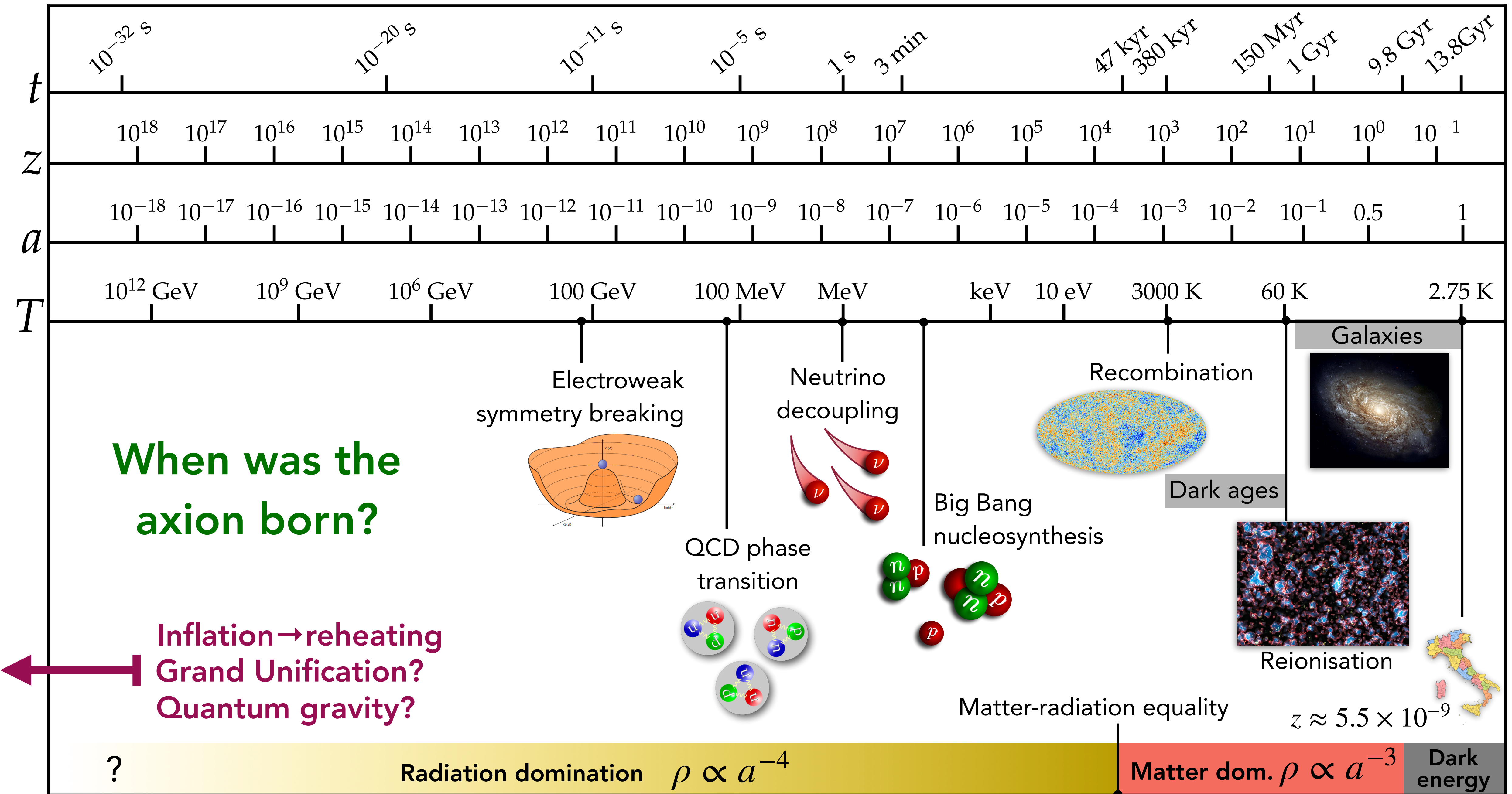


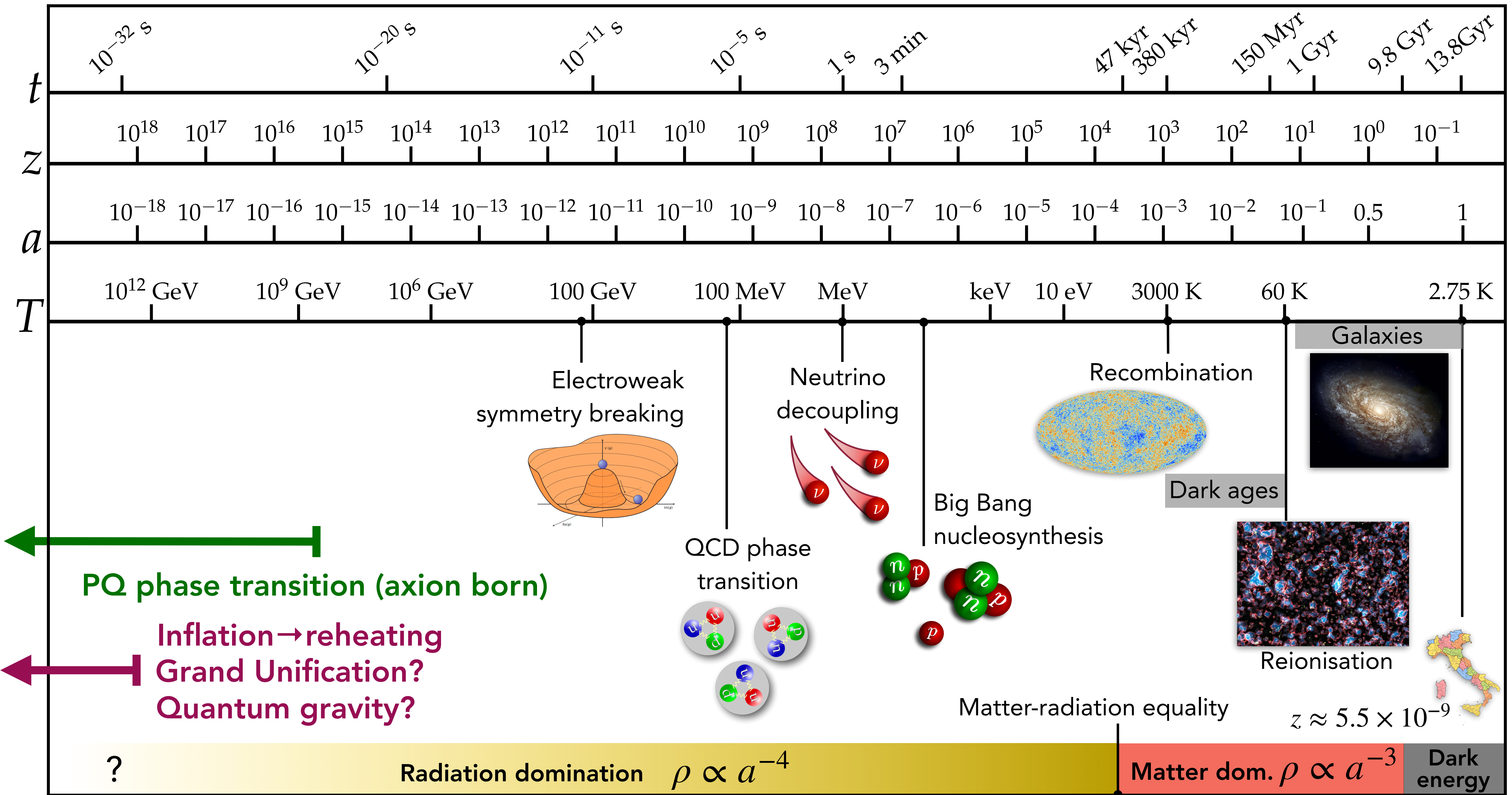




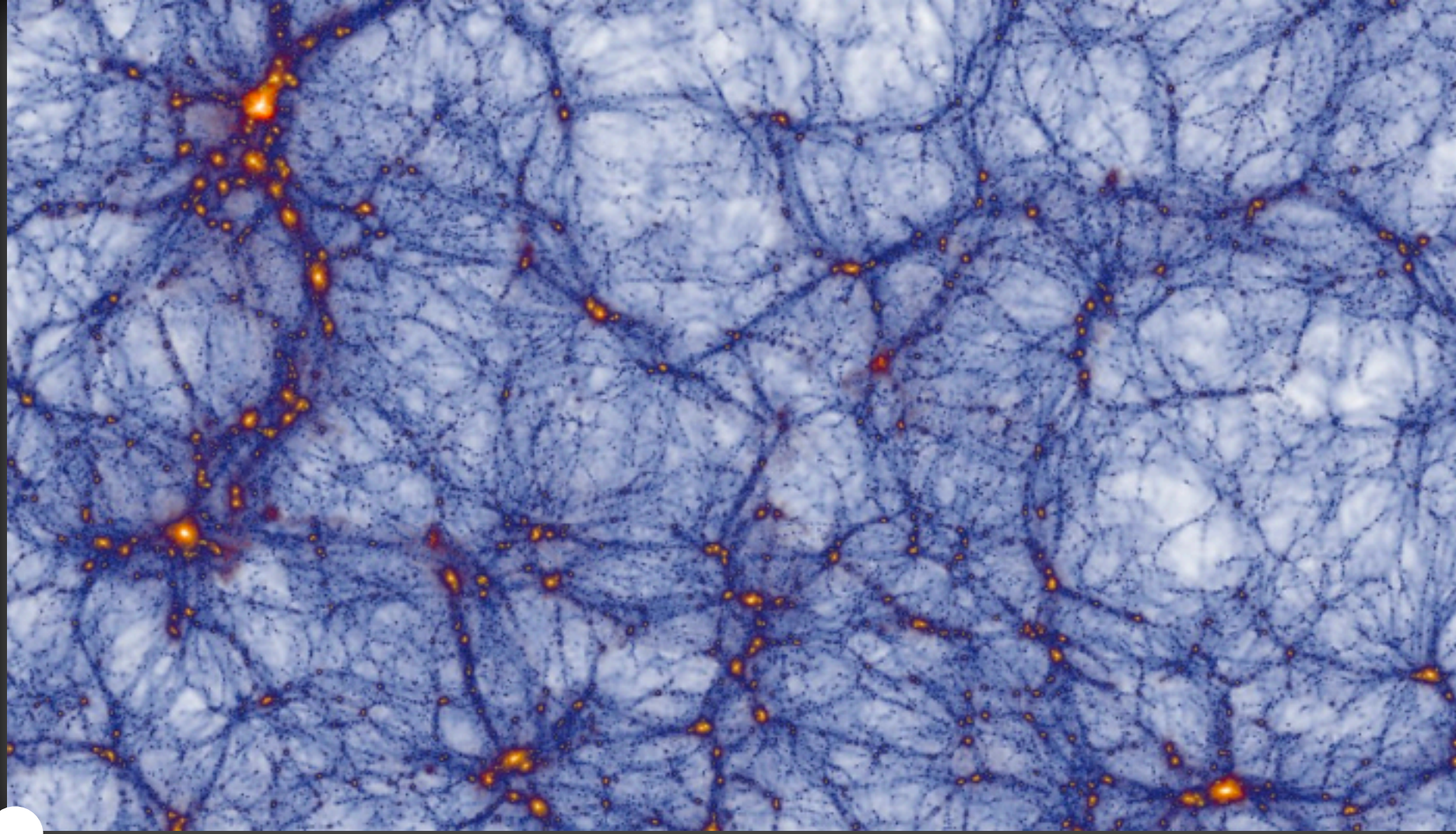






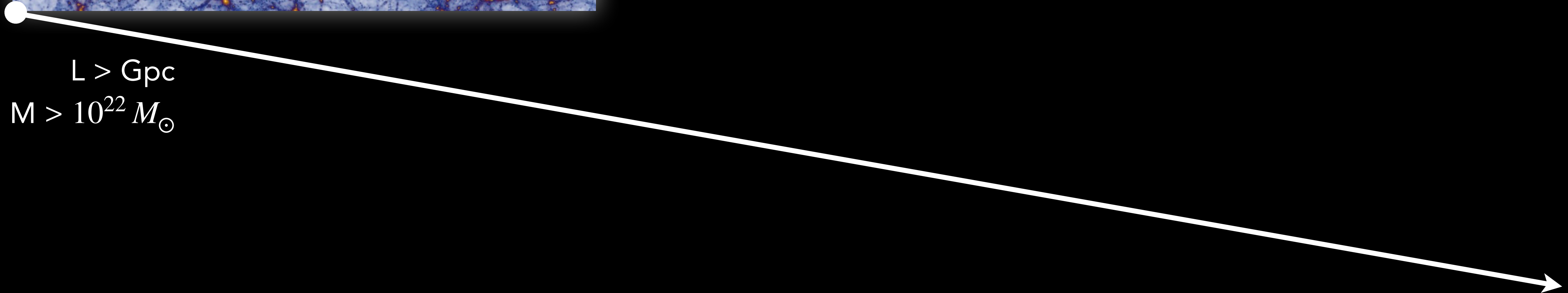


Large-scale structure

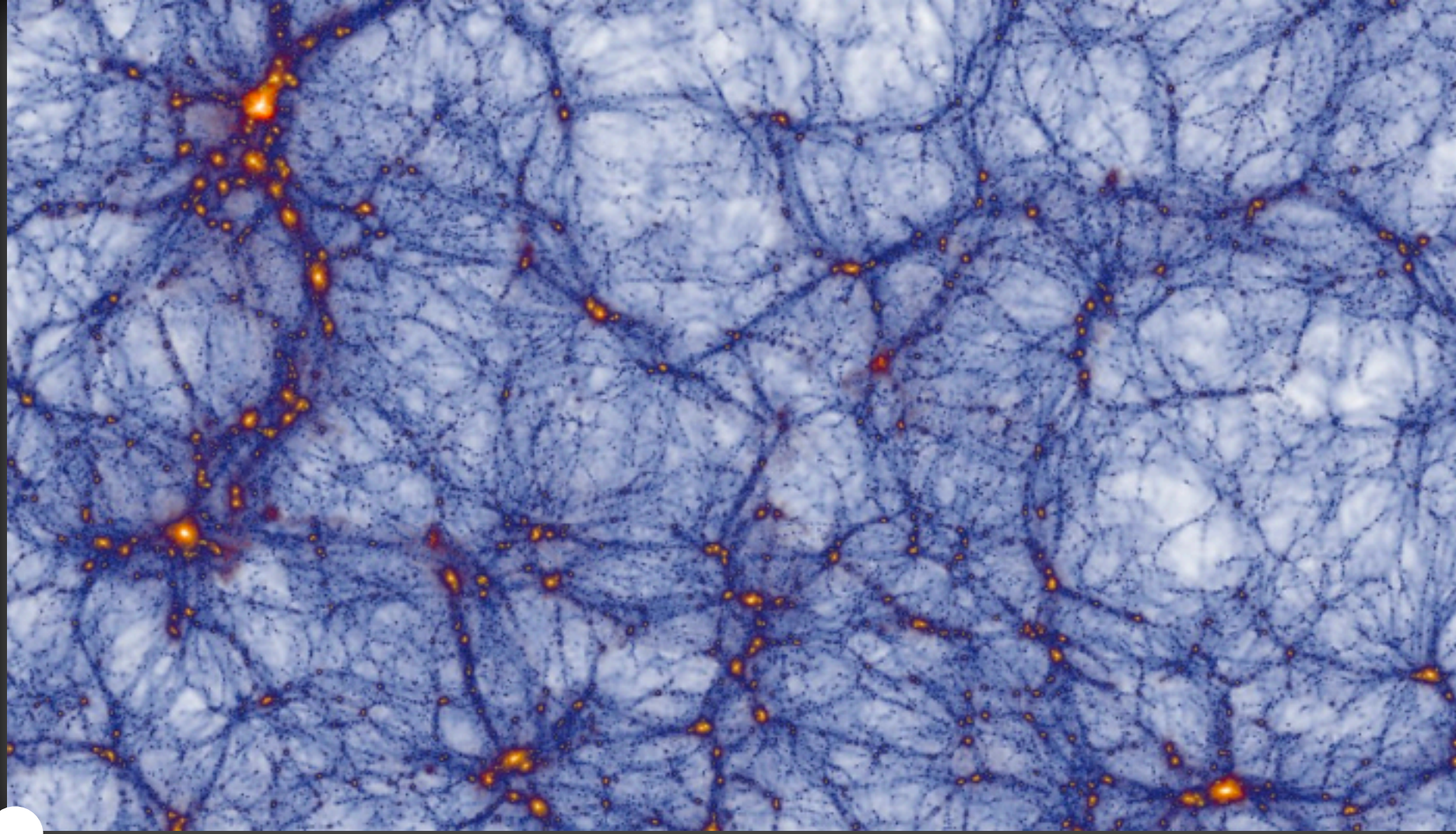


Evidence for dark matter across scales

$L > \text{Gpc}$
 $M > 10^{22} M_{\odot}$



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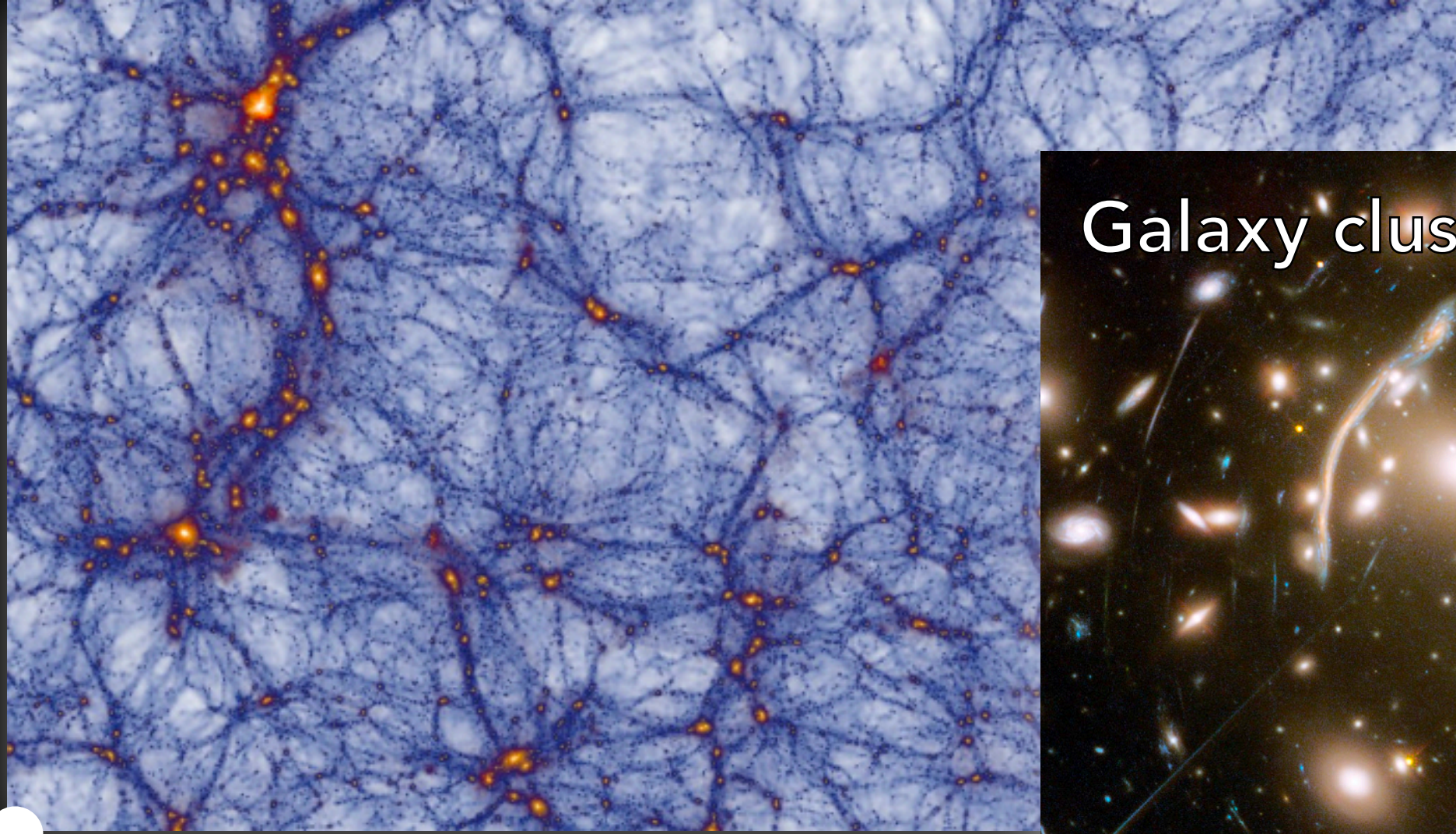


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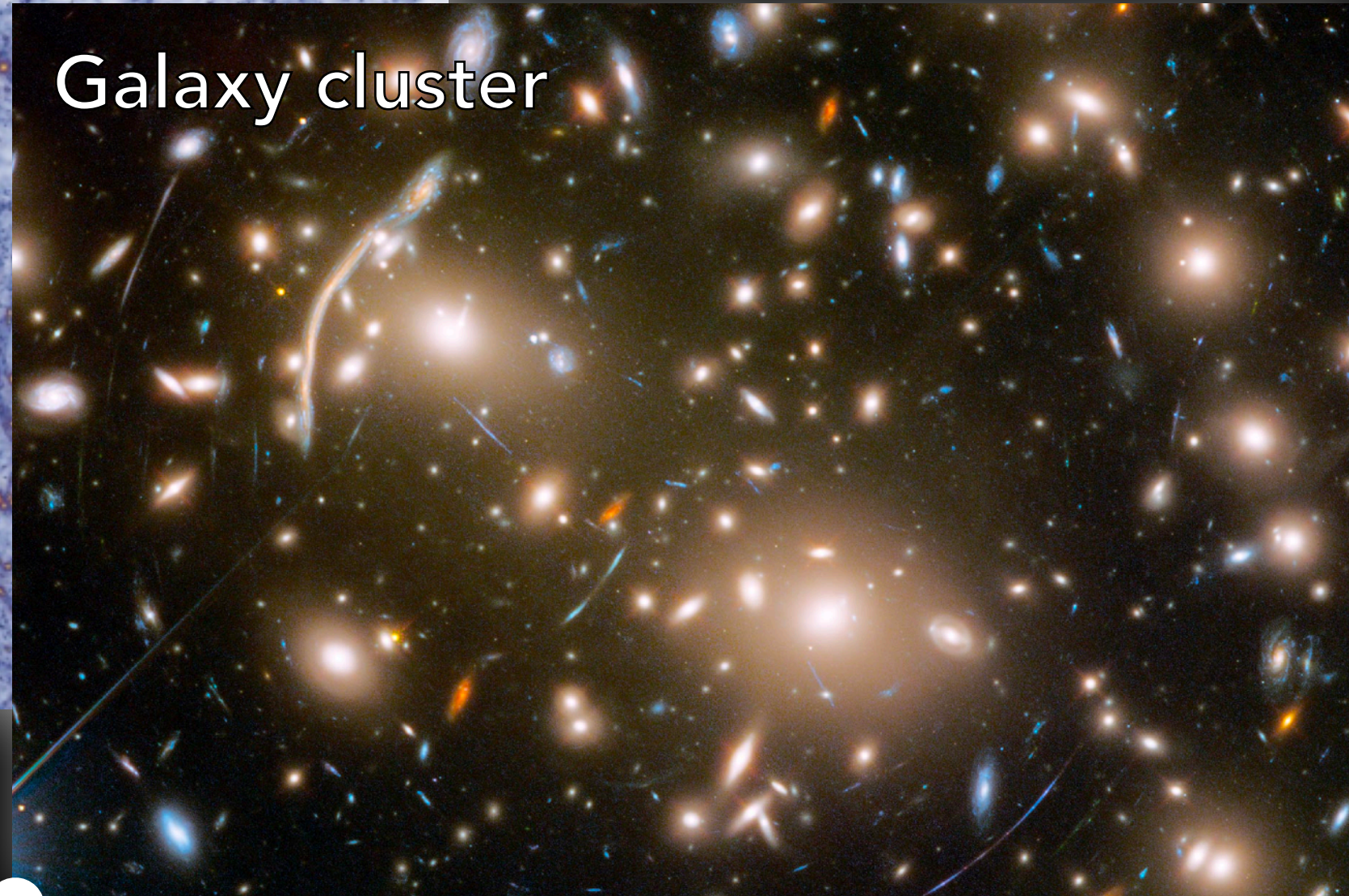
$L > \text{Gpc}$
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Large-scale structure: consistent
with hierarchical mergers of halos
seeded by collisionless matter

Large-scale structure



Galaxy cluster



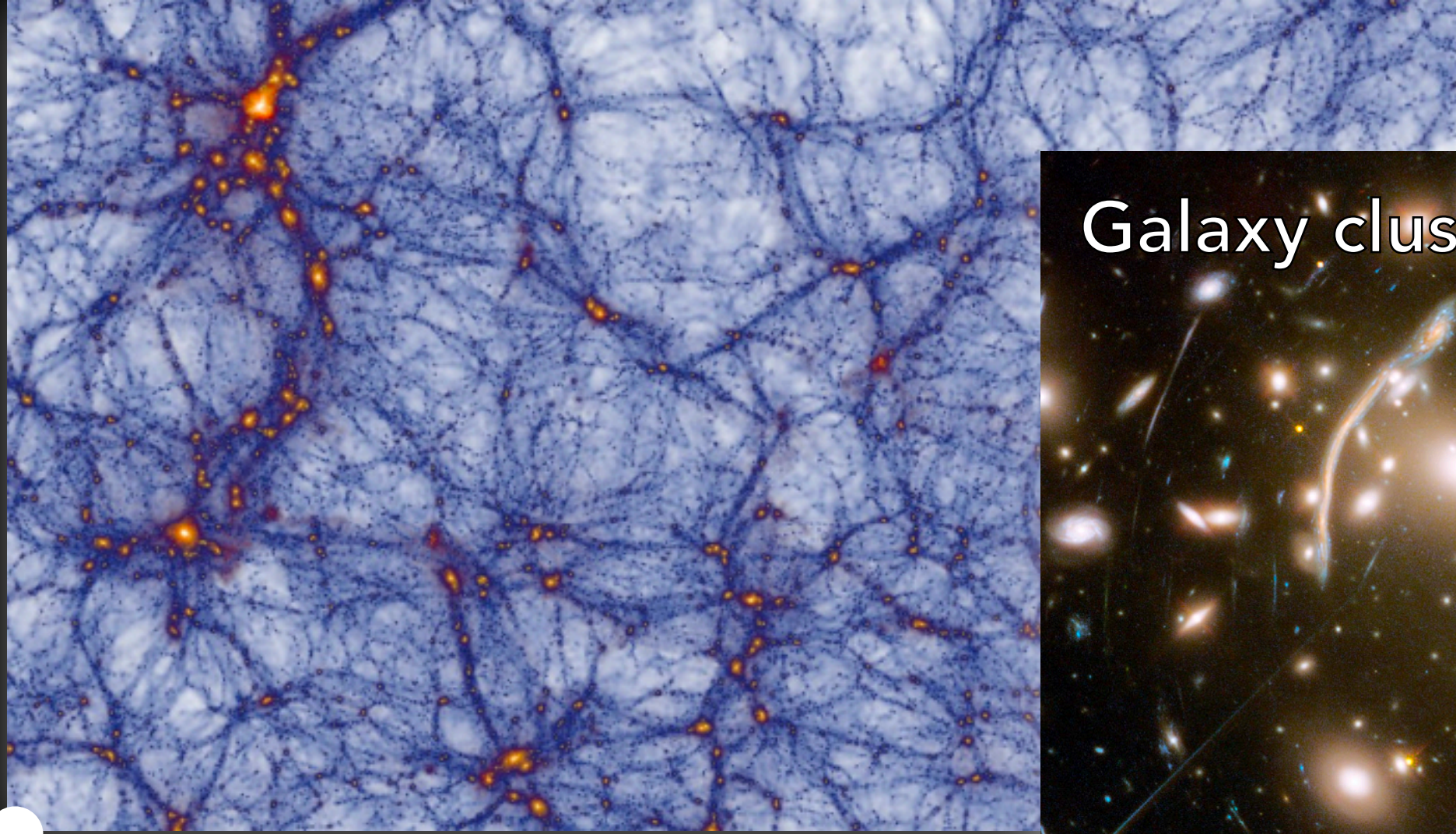
Evidence for dark matter across scales

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$L \sim \text{Mpc}$
 $M \sim 10^{14} M_{\odot}$

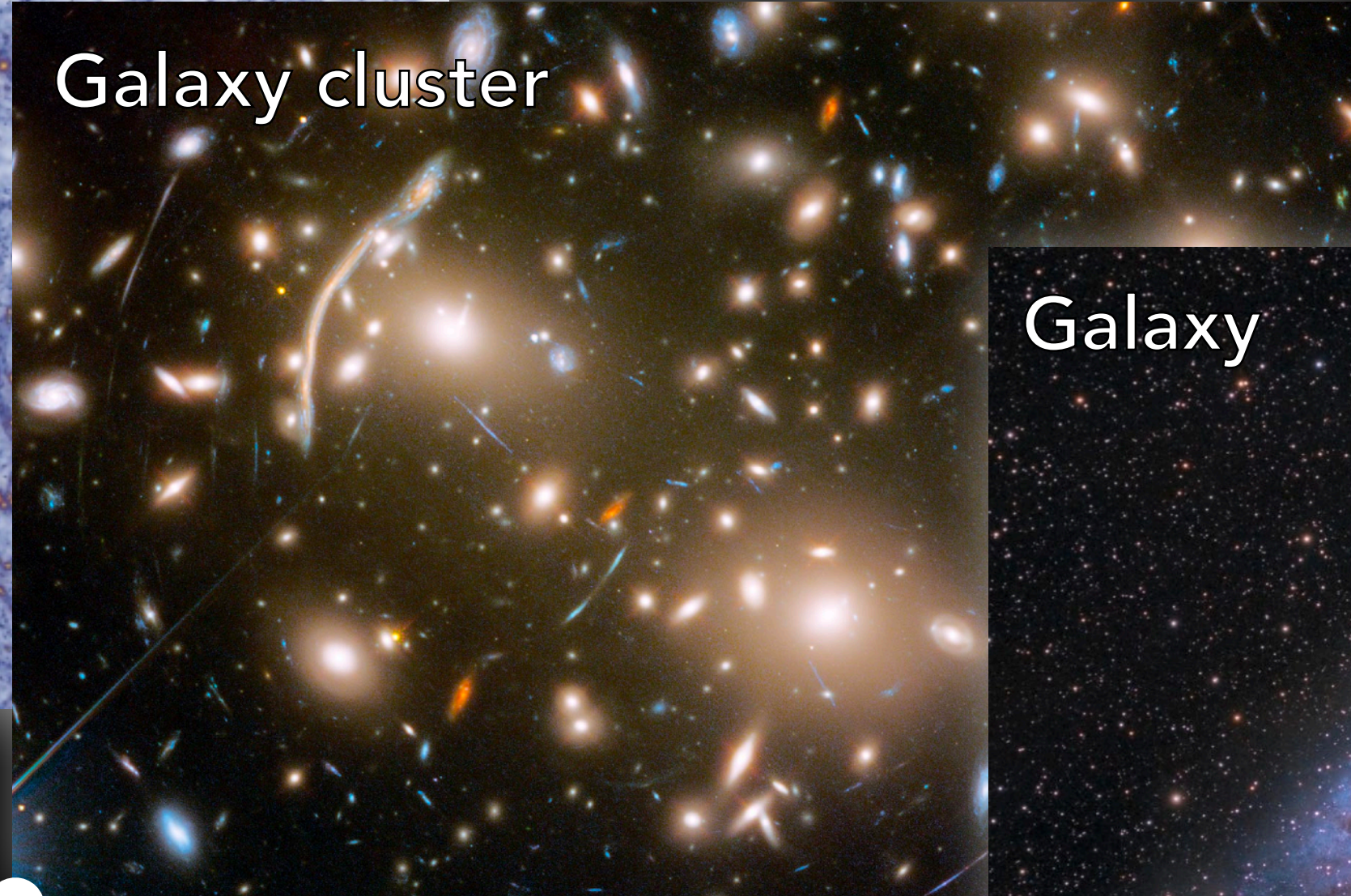
Galaxy clusters: strong gravitational
lensing reveals mass much larger than
one inferred from luminosity

Large-scale structure



Evidence for dark matter across scales

Galaxy cluster



Galaxy



$L > \text{Gpc}$
 $M > 10^{22} M_{\odot}$

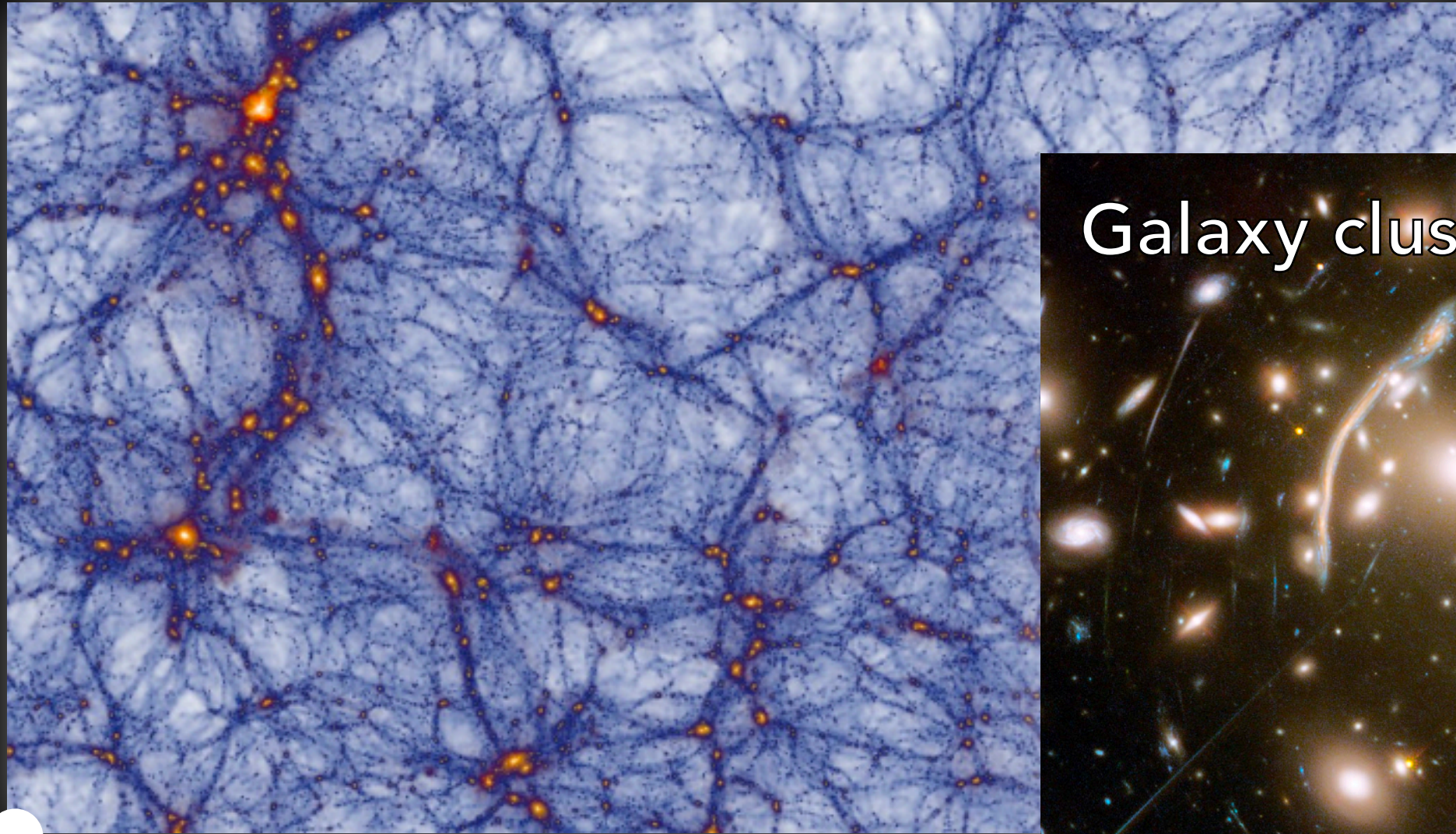
$L \sim \text{Mpc}$
 $M \sim 10^{14} M_{\odot}$

$L \sim 100 \text{ kpc}$
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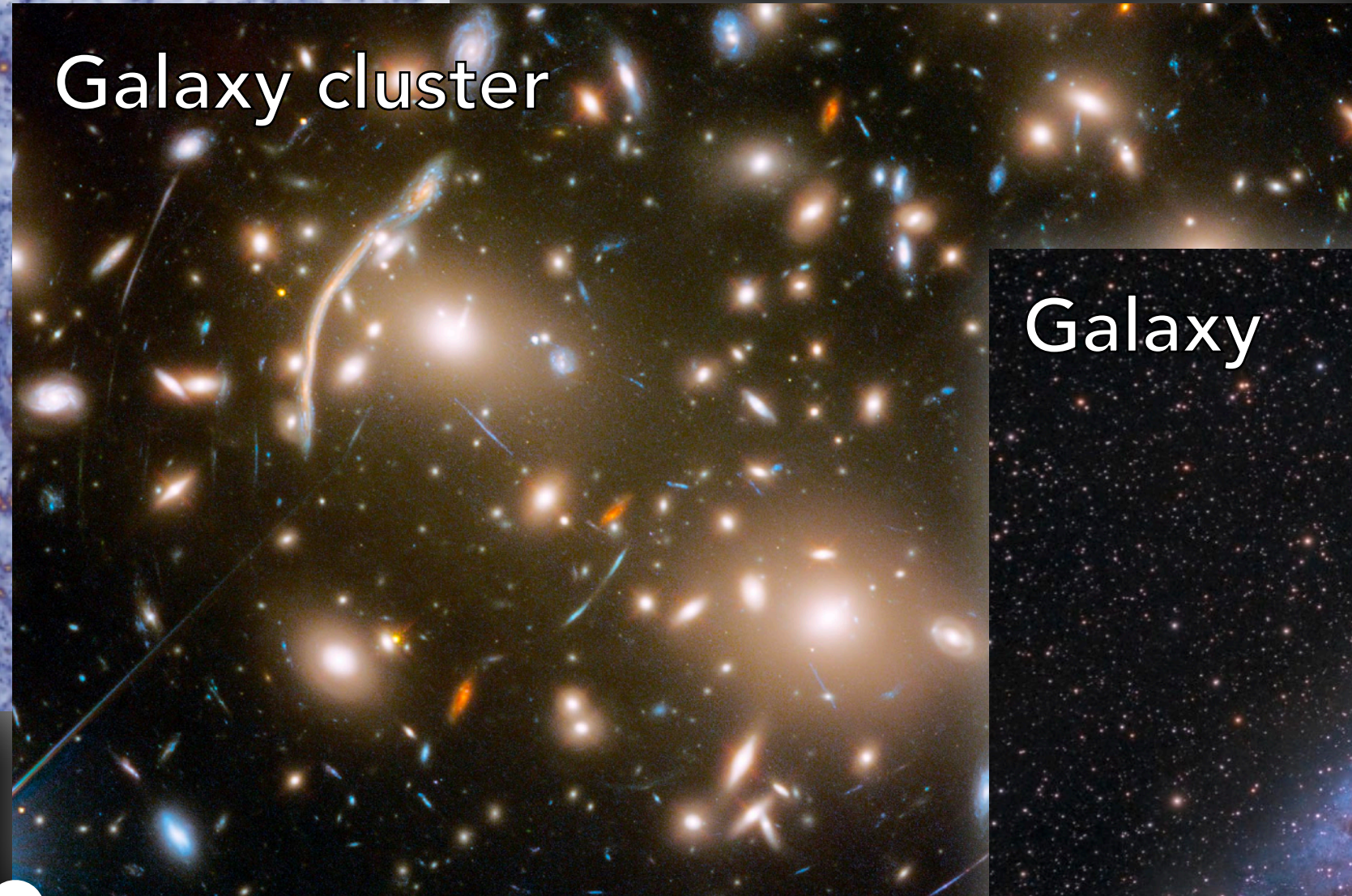
Galaxies: rotation profiles reveal
considerable unseen mass contribution
extending beyond visible structure

Evidence for dark matter across scales

Large-scale structure



Galaxy cluster



Galaxy



Dwarf galaxy



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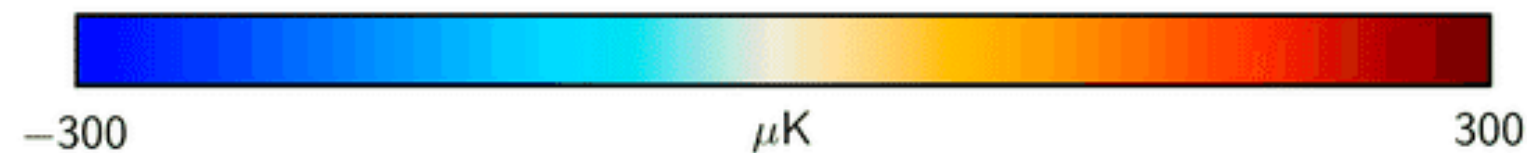
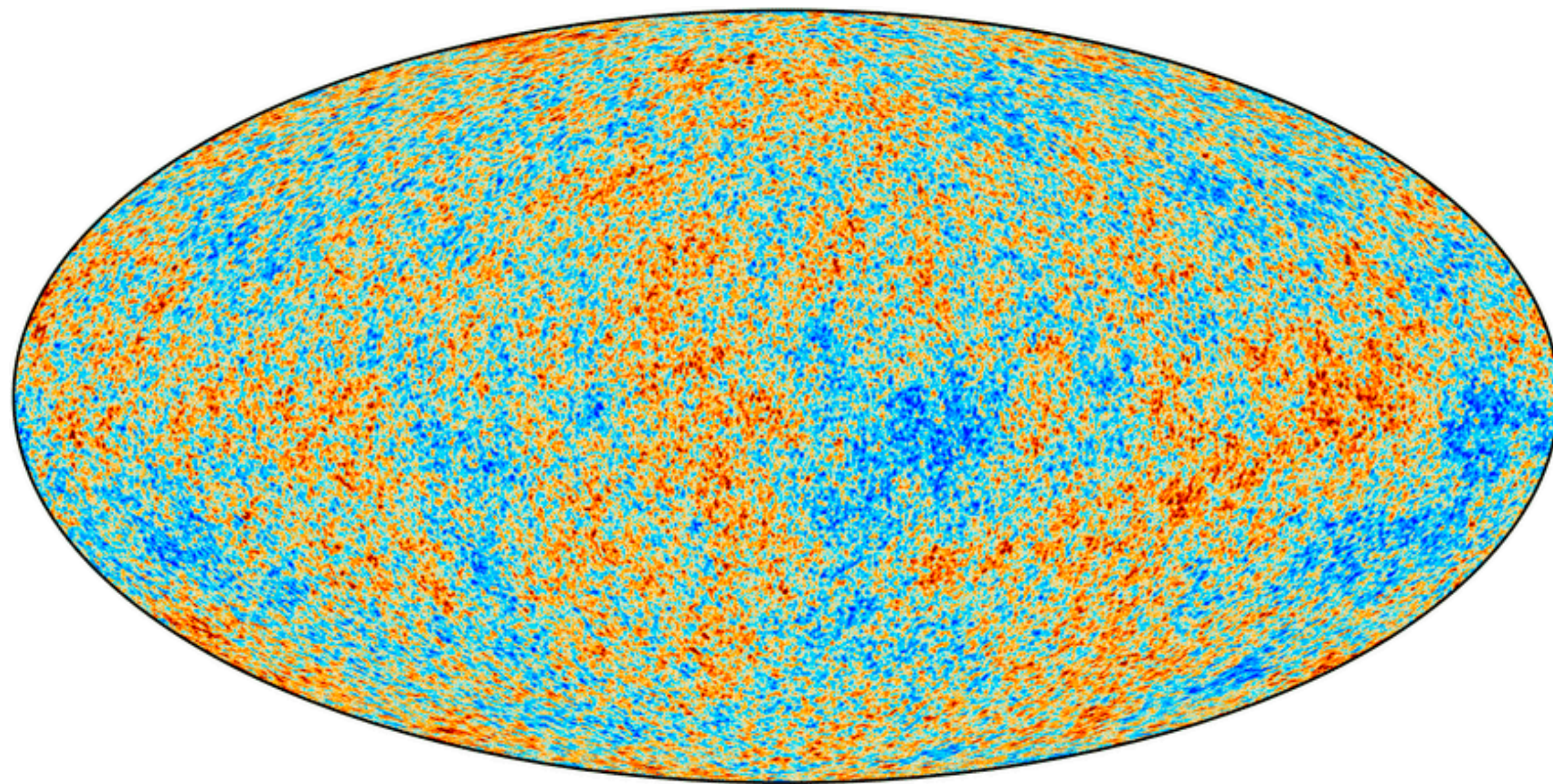
$L \sim 1\text{-}10 \text{ kpc}$
 $M \sim 10^{6-8} M_{\odot}$

Dwarf galaxies: Stellar kinematics
reveal mass-to-light ratios significantly
larger than $1M_{\odot}/L_{\odot}$

The cosmic microwave background

Fluctuations in photon temp. across sky

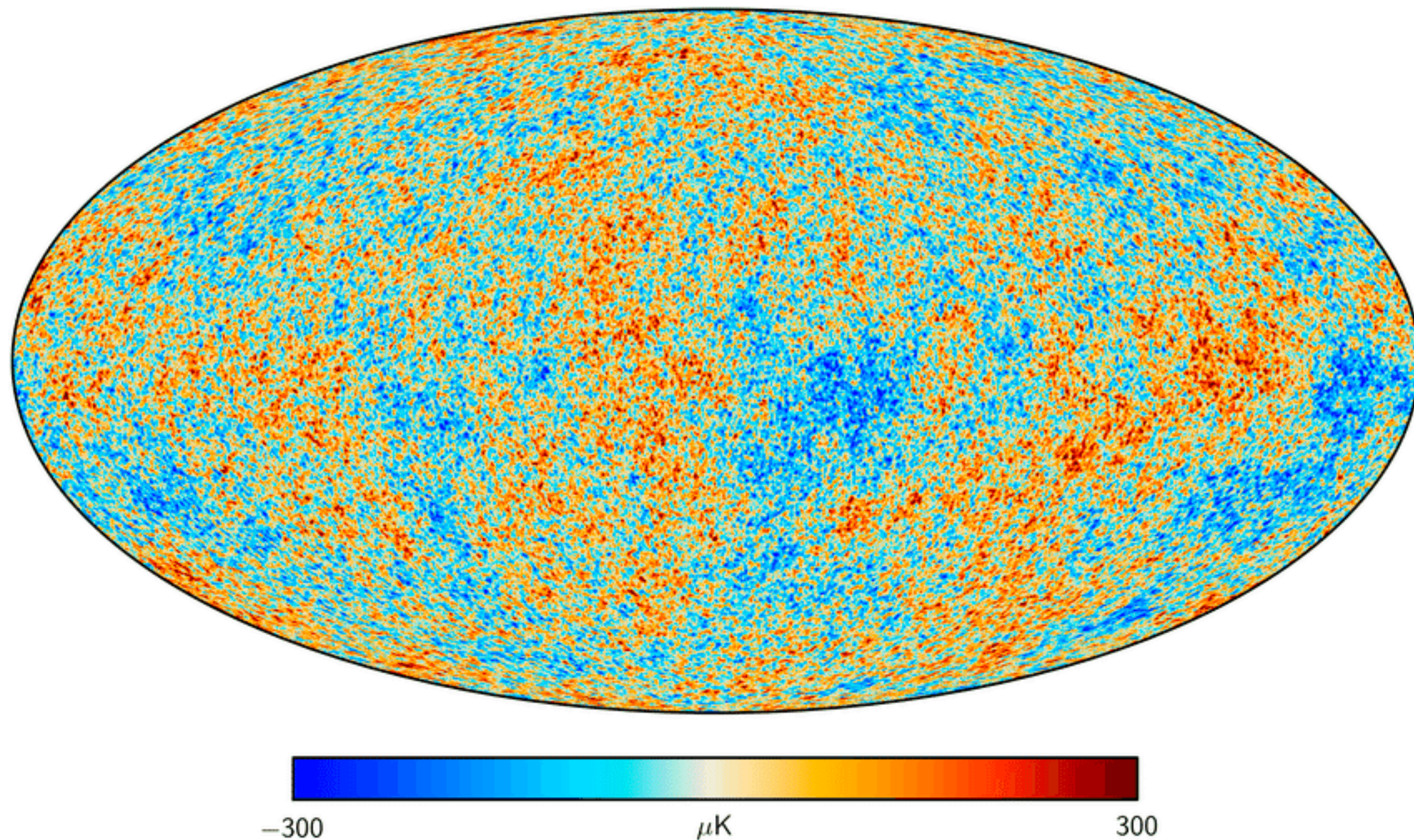
$$\frac{\delta T}{T} = \sum_{\ell, m} a_{\ell m} Y_{\ell m}(\theta, \phi)$$



The cosmic microwave background

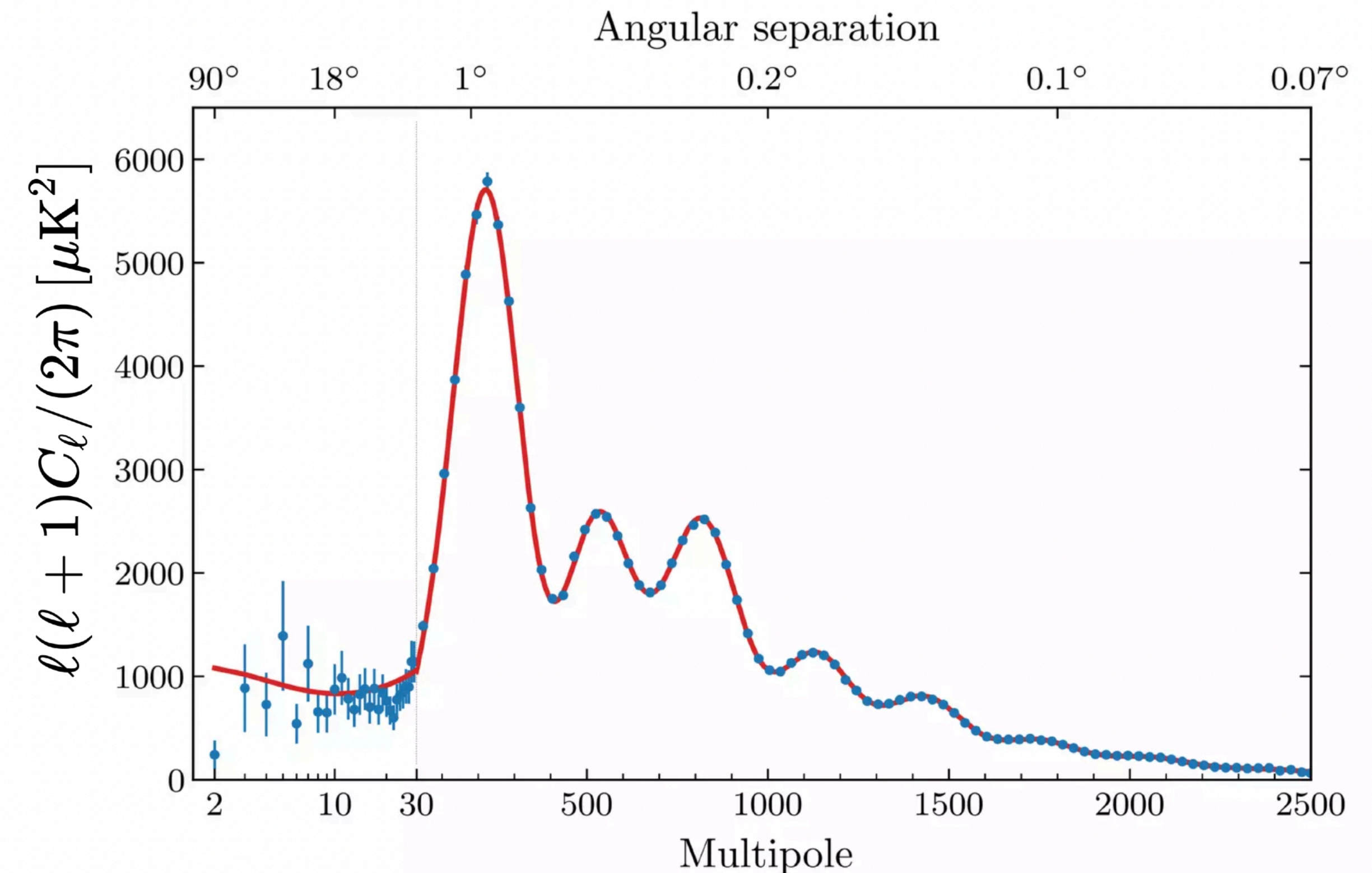
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Power spectrum of angular correlations vs multipole number

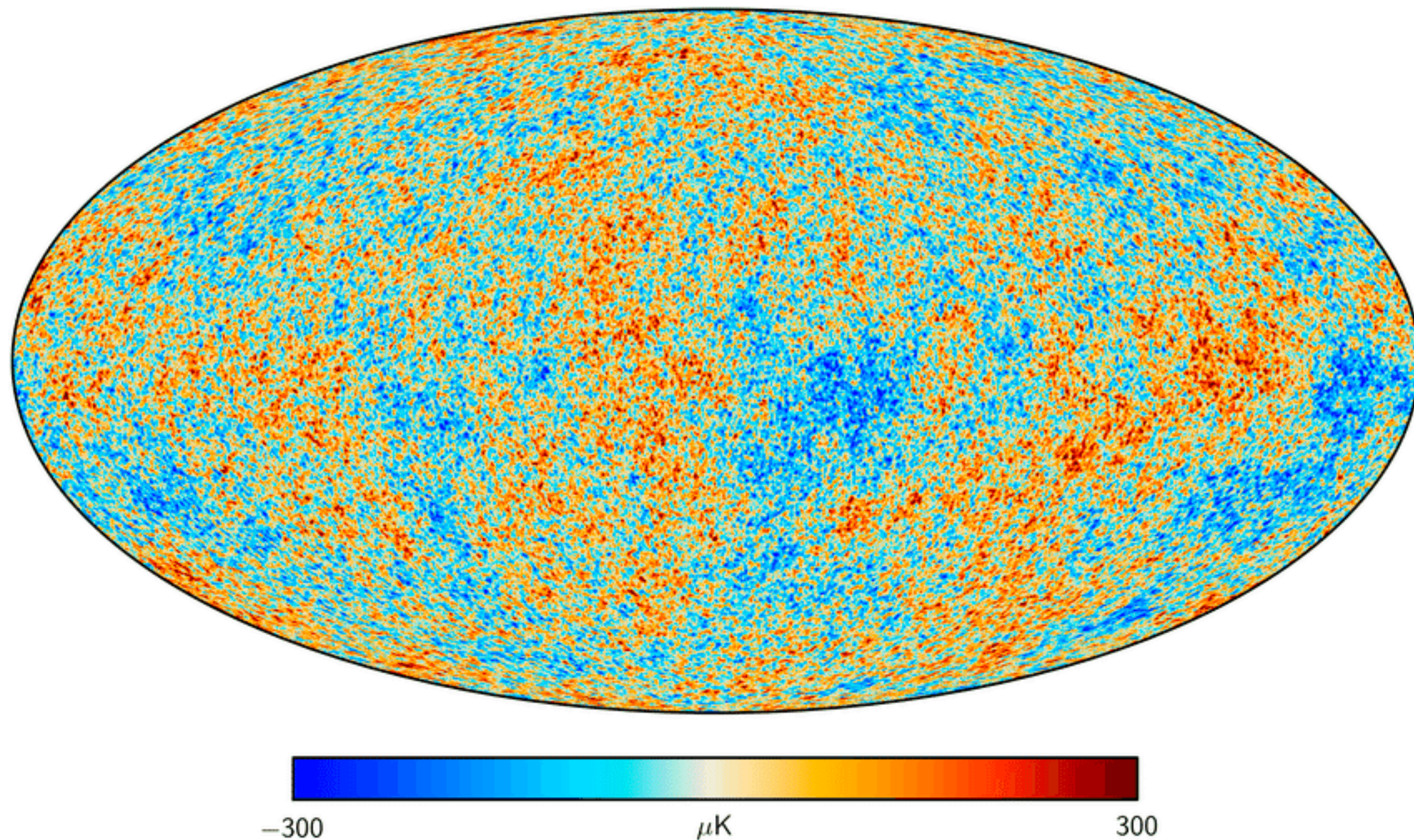
$$\langle a_{\ell m} a_{\ell' m'} \rangle = \delta_{\ell \ell'} \delta_{m m'} C_{\ell}$$



The cosmic microwave background

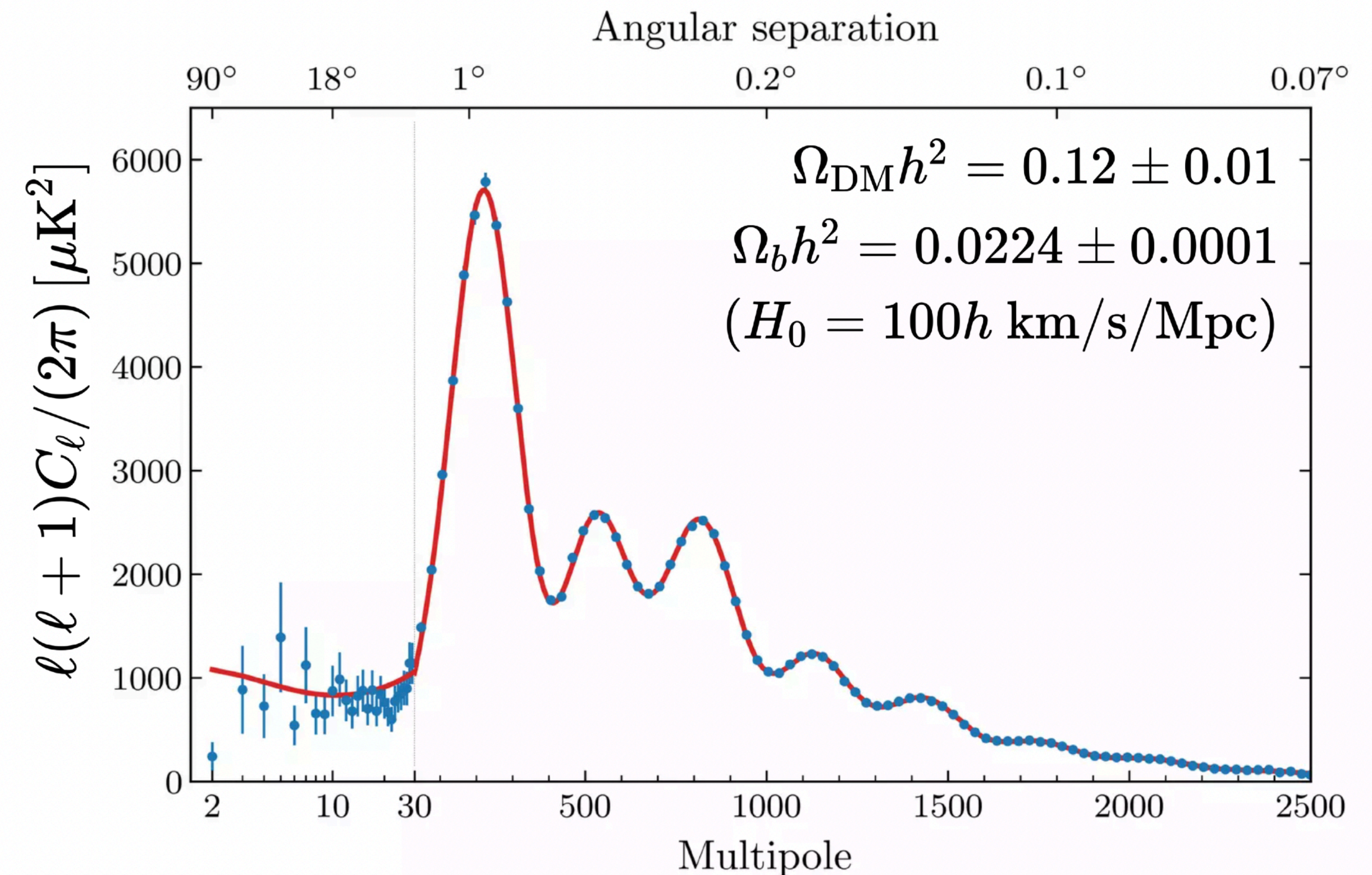
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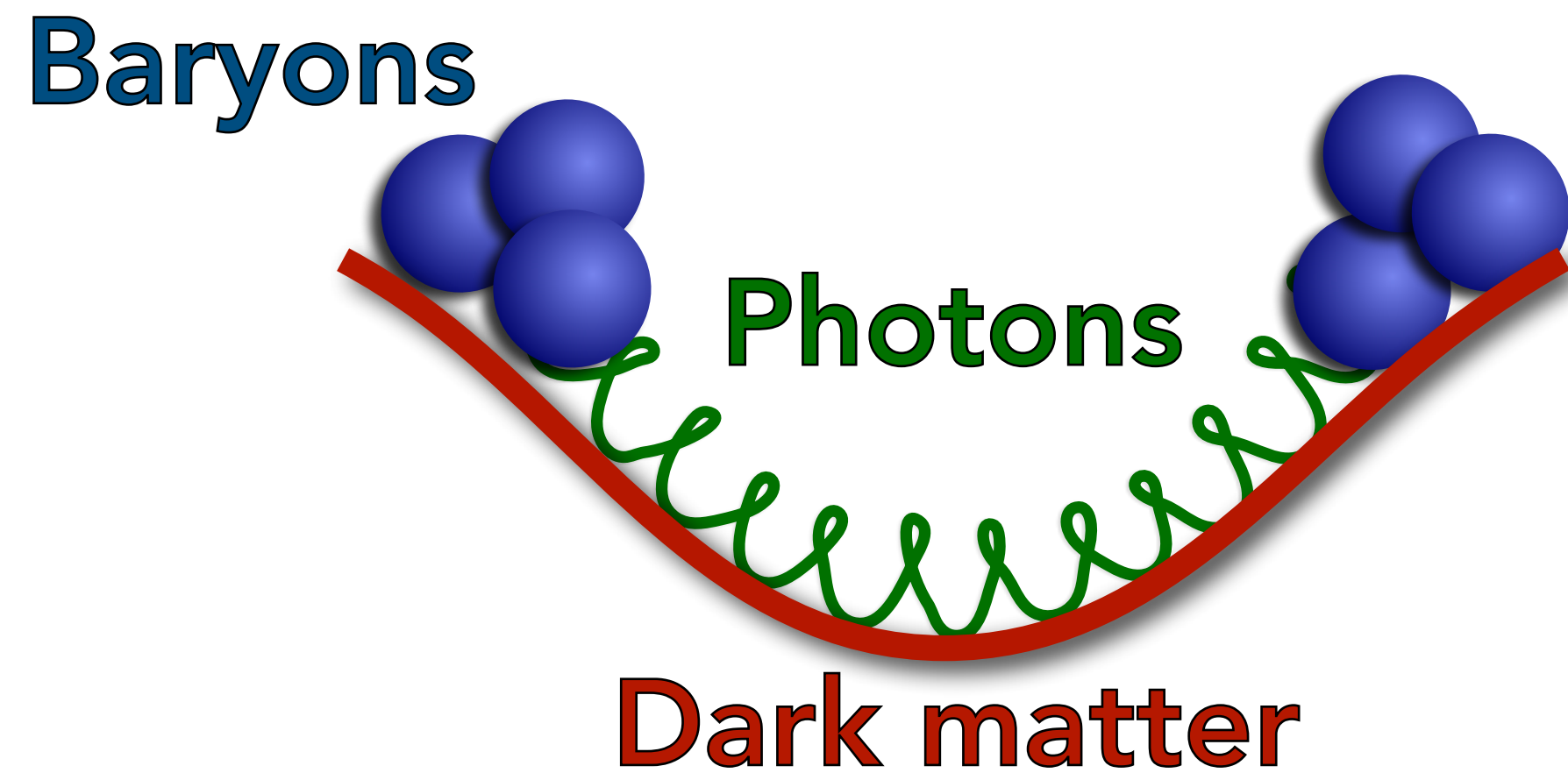
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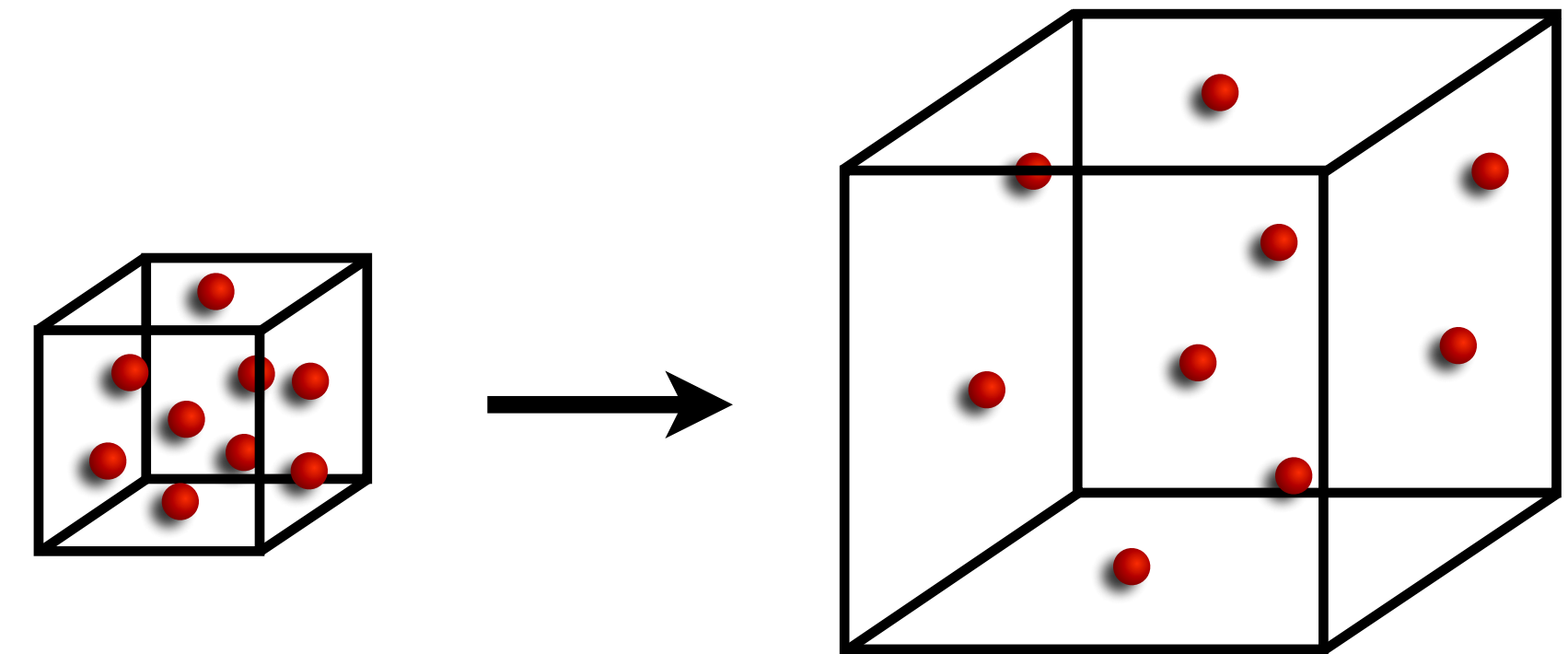


Minimal definition of dark matter

Dissipation-less, source of
gravitational potential
(dark)



Energy density dilutes
with expansion
(matter)



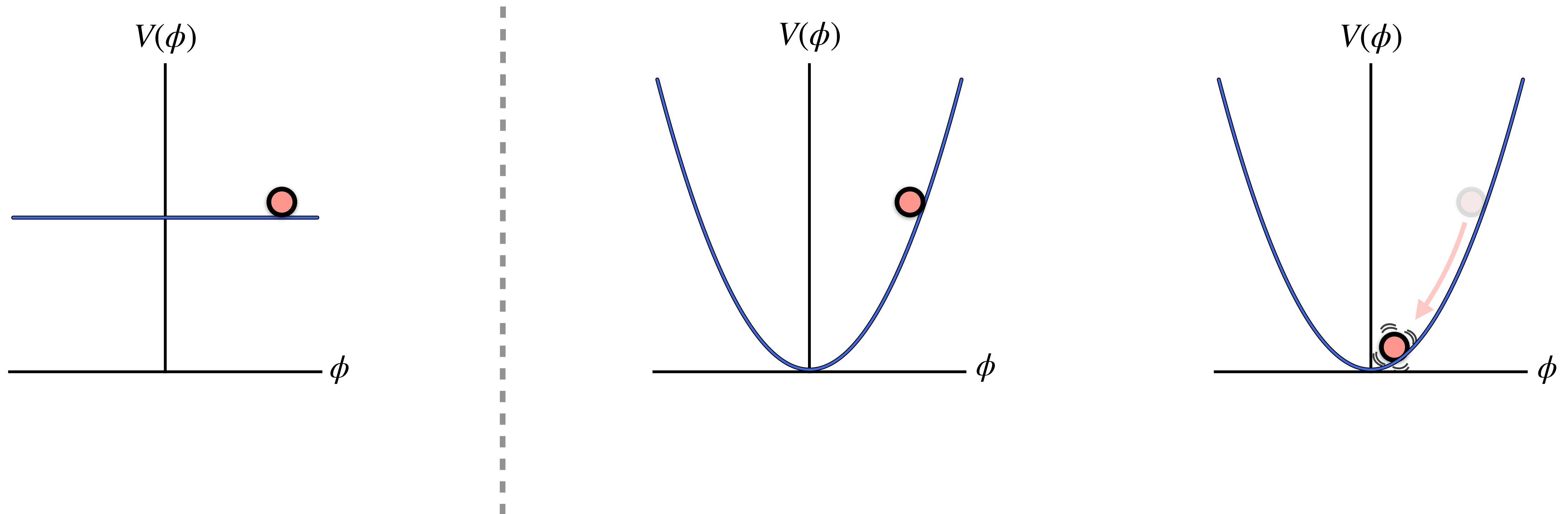
$$\rho \propto \frac{1}{a^3} = \frac{1}{V}$$

How can a scalar field be the dark matter?
→ the misalignment mechanism

Misalignment mechanism

Time $\longleftrightarrow$ Temperature

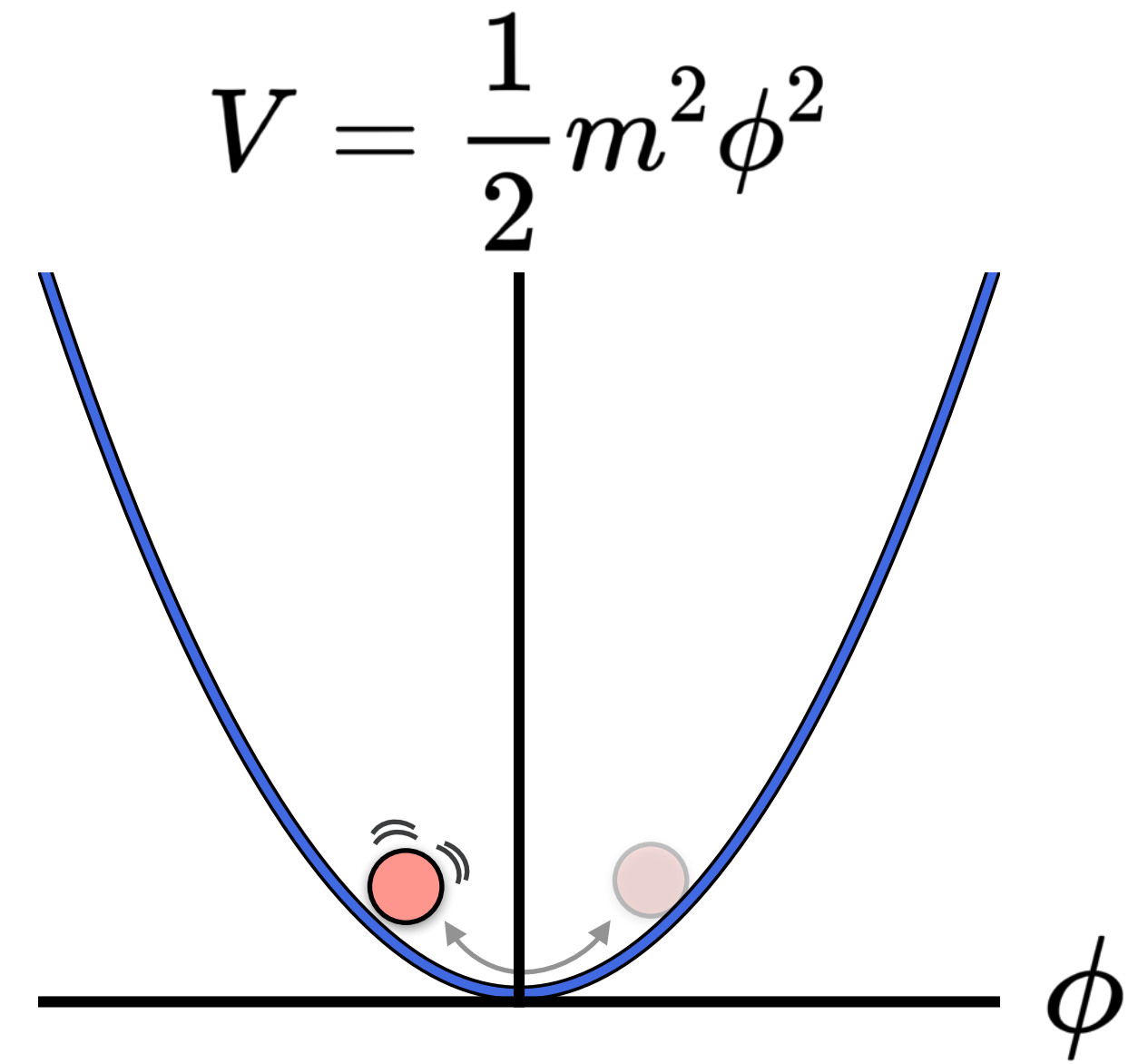
Symmetry
broken



Misalignment mechanism for a generic scalar

Imagine our scalar field's potential is just given by its mass

$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{2} (\partial_\mu \phi \partial^\mu \phi) - \frac{1}{2} m^2 \phi^2 \right]$$



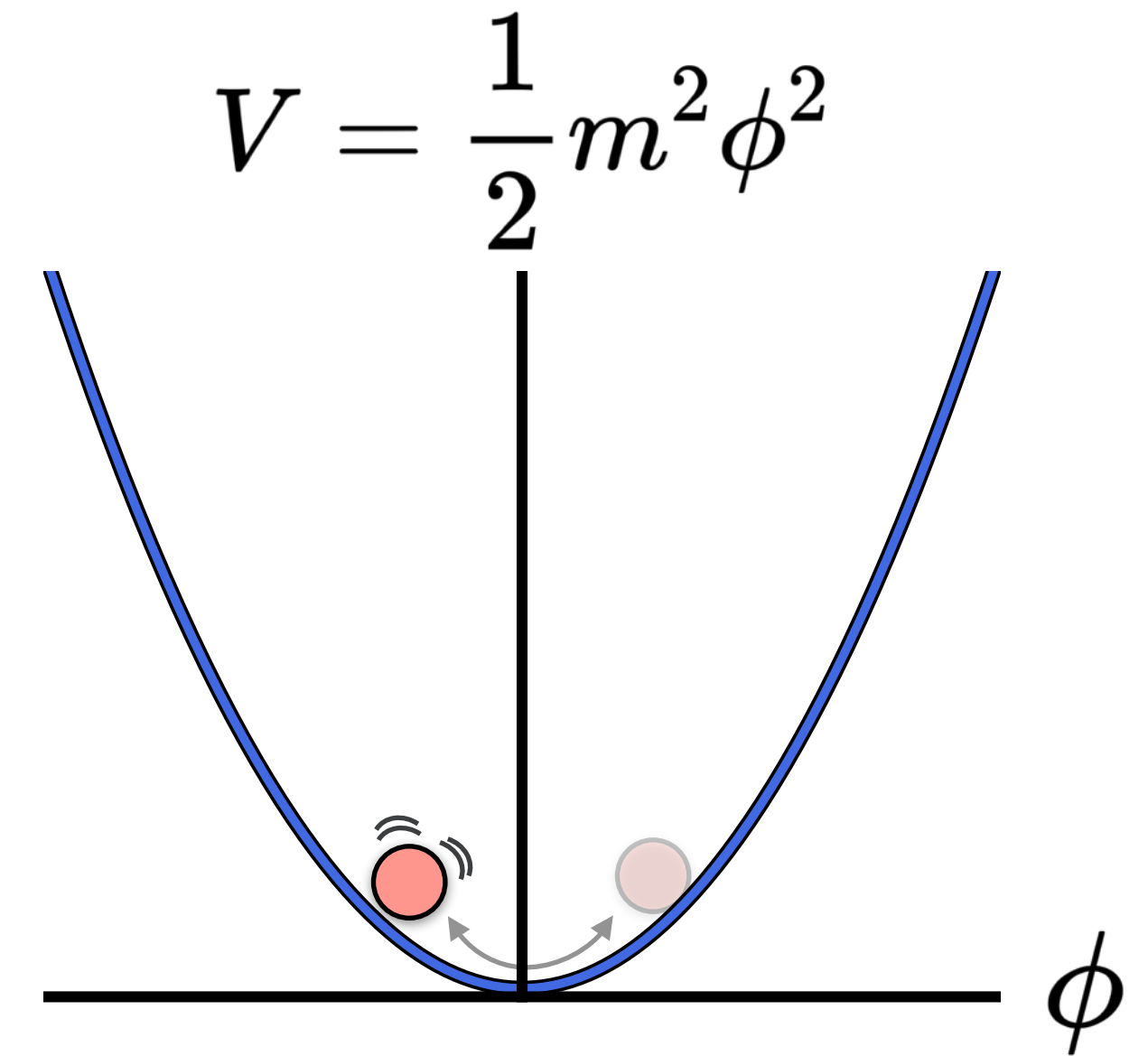
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Euler-Lagrange equation:

$$\partial_\mu \frac{\partial \sqrt{-g} \mathcal{L}}{\partial (\partial_\mu \phi)} = \frac{\partial \sqrt{-g} \mathcal{L}}{\partial \phi}$$



Misalignment mechanism for a generic scalar

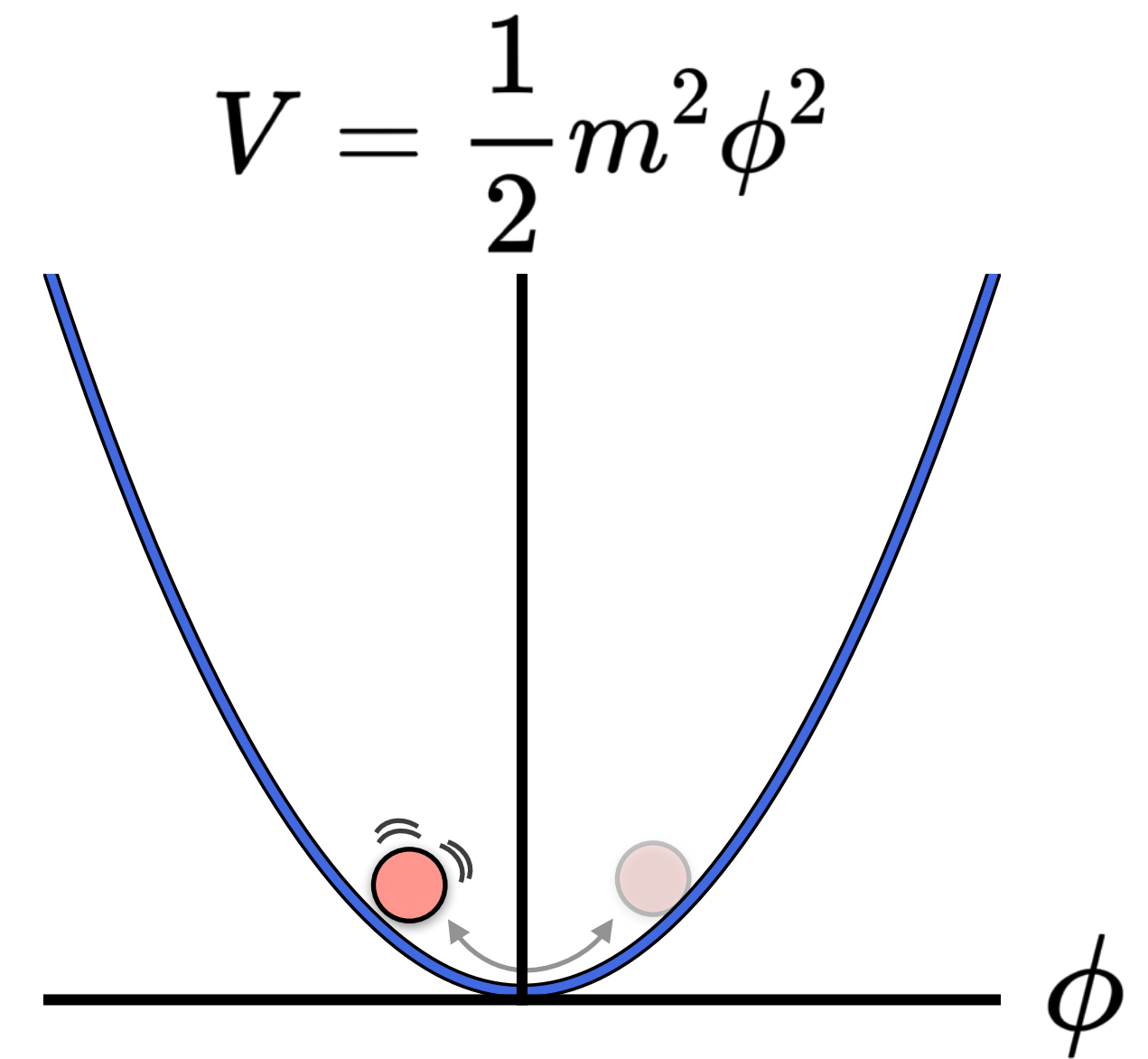
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(Klein-Gordon equation)



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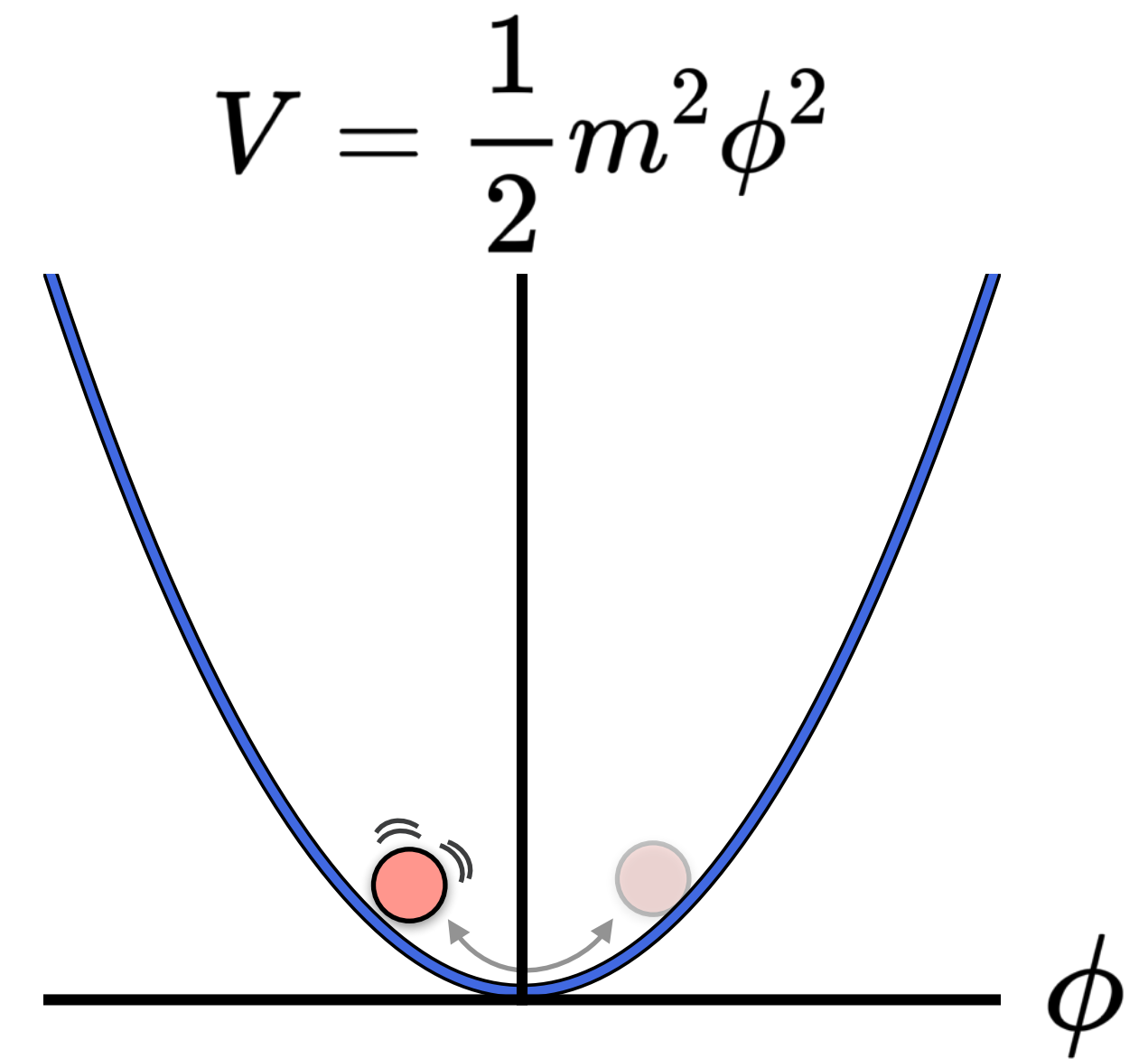
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Put in FRW metric $g = \text{diag}(-1, a^2, a^2, a^2)$ where $H = \dot{a}/a$

$$\square = -\partial_t^2 - 3H\partial_t$$



Misalignment mechanism for a generic scalar

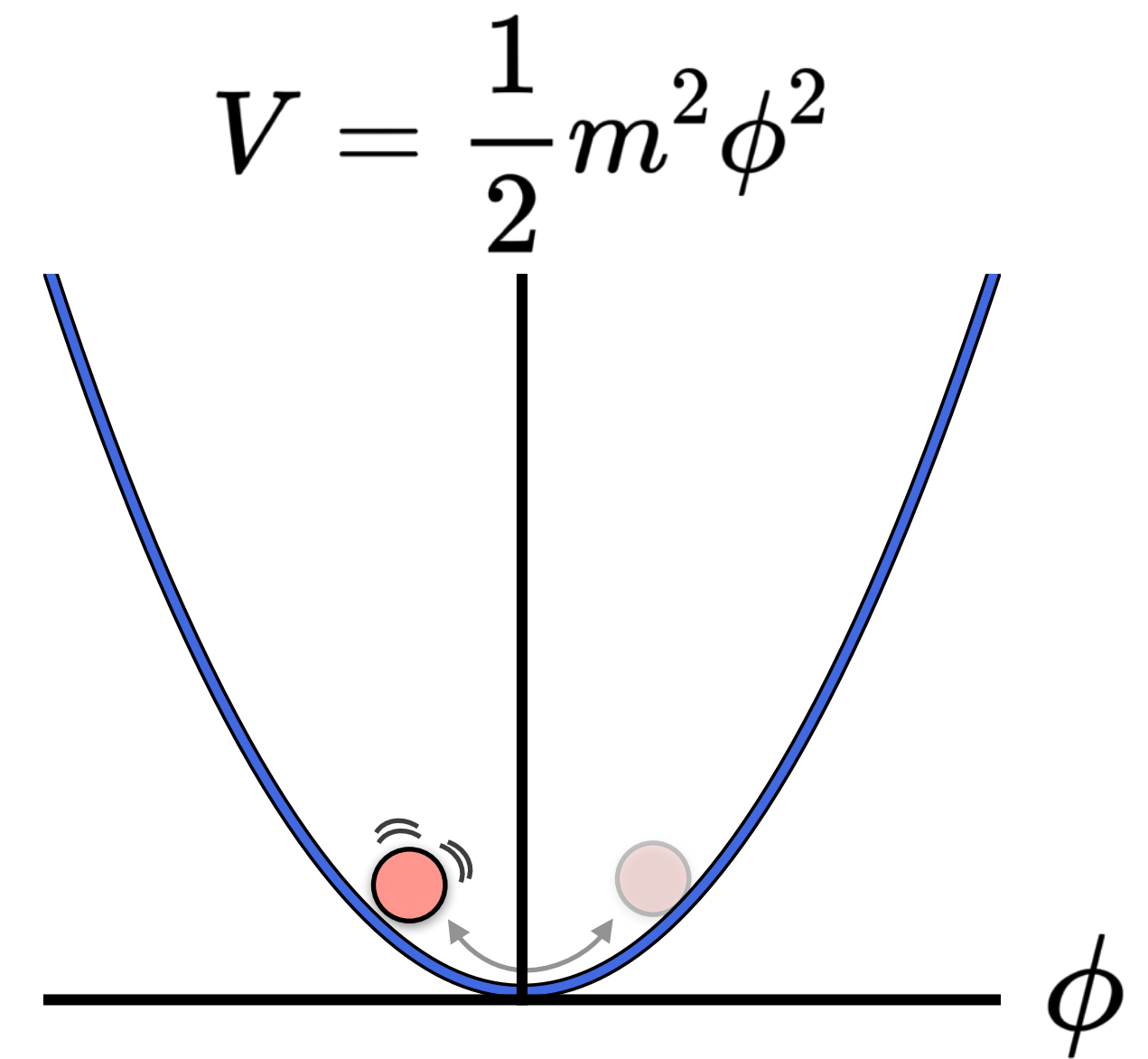
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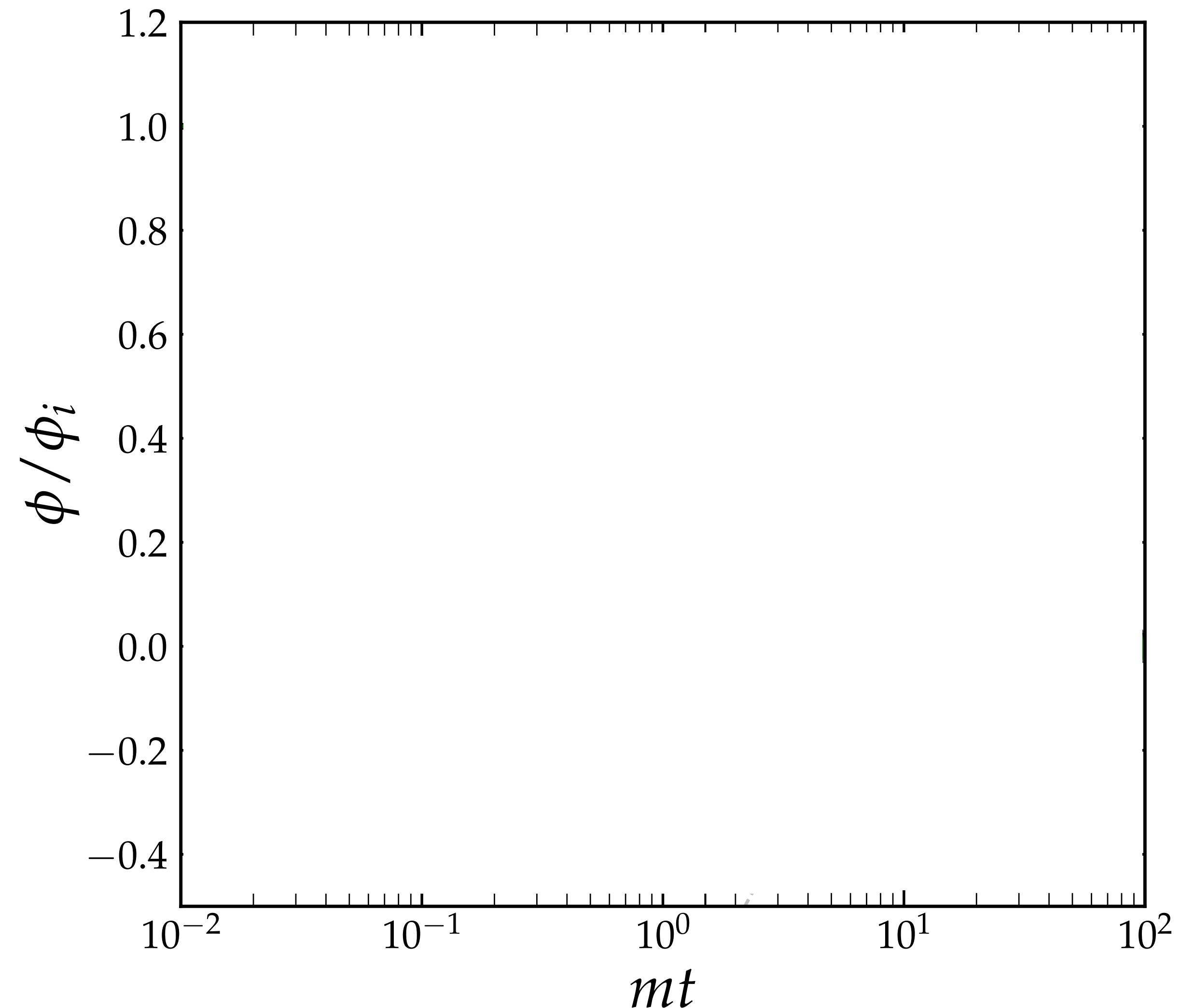
Damped harmonic oscillator

Misalignment mechanism for a generic scalar

$$\ddot{\phi} + 3H(t)\dot{\phi} + m^2\phi = 0$$

Taking radiation domination $H(t) = 1/2t$ and enforcing the boundary condition $\dot{\phi}(t_i) = 0$, we get a general solution:

$$\phi = \phi_i \left(\frac{2}{mt} \right)^{\frac{1}{4}} \Gamma\left(\frac{5}{4}\right) J_{\frac{1}{4}}(mt)$$



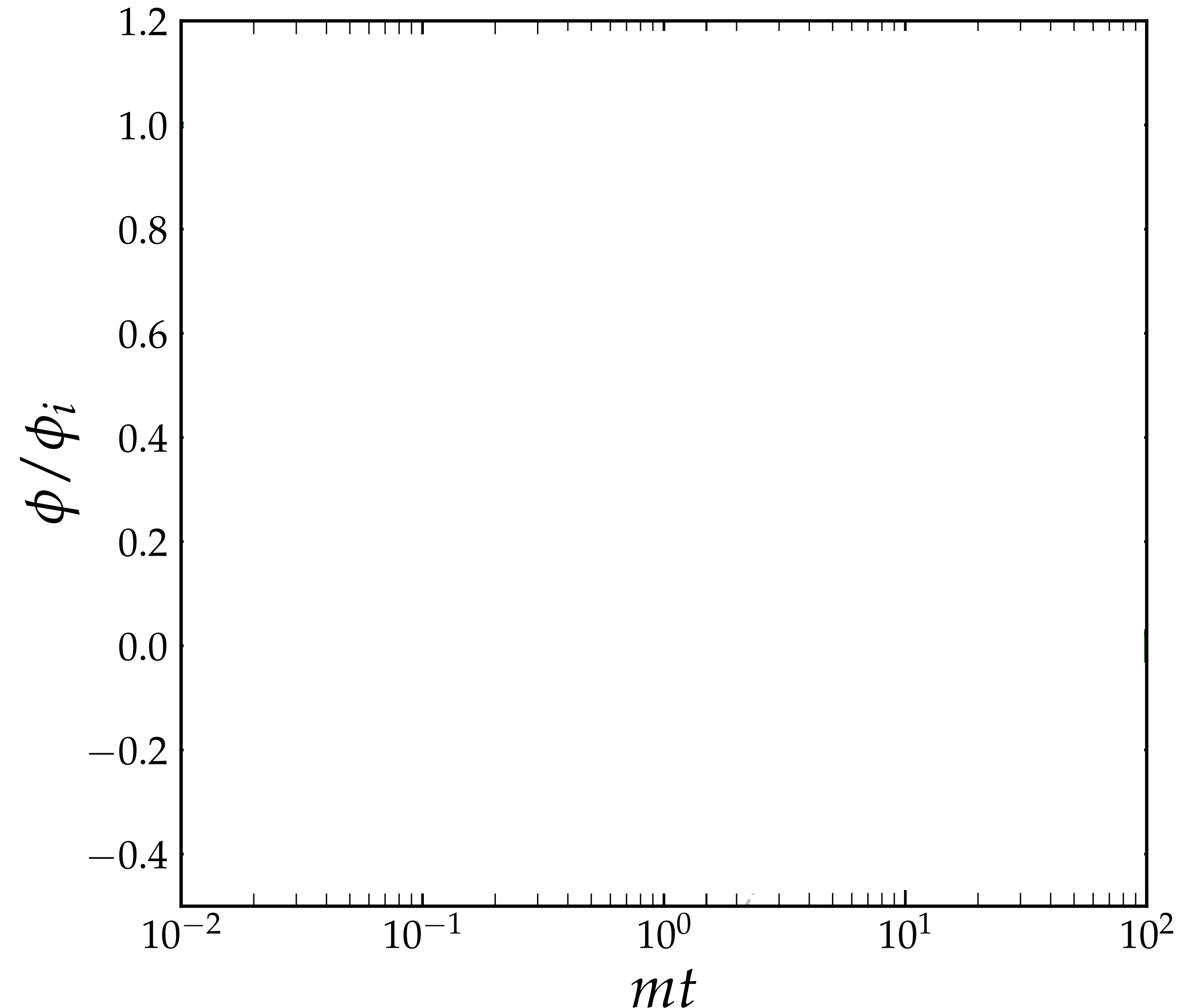
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Two regimes:



Misalignment mechanism for a generic scalar

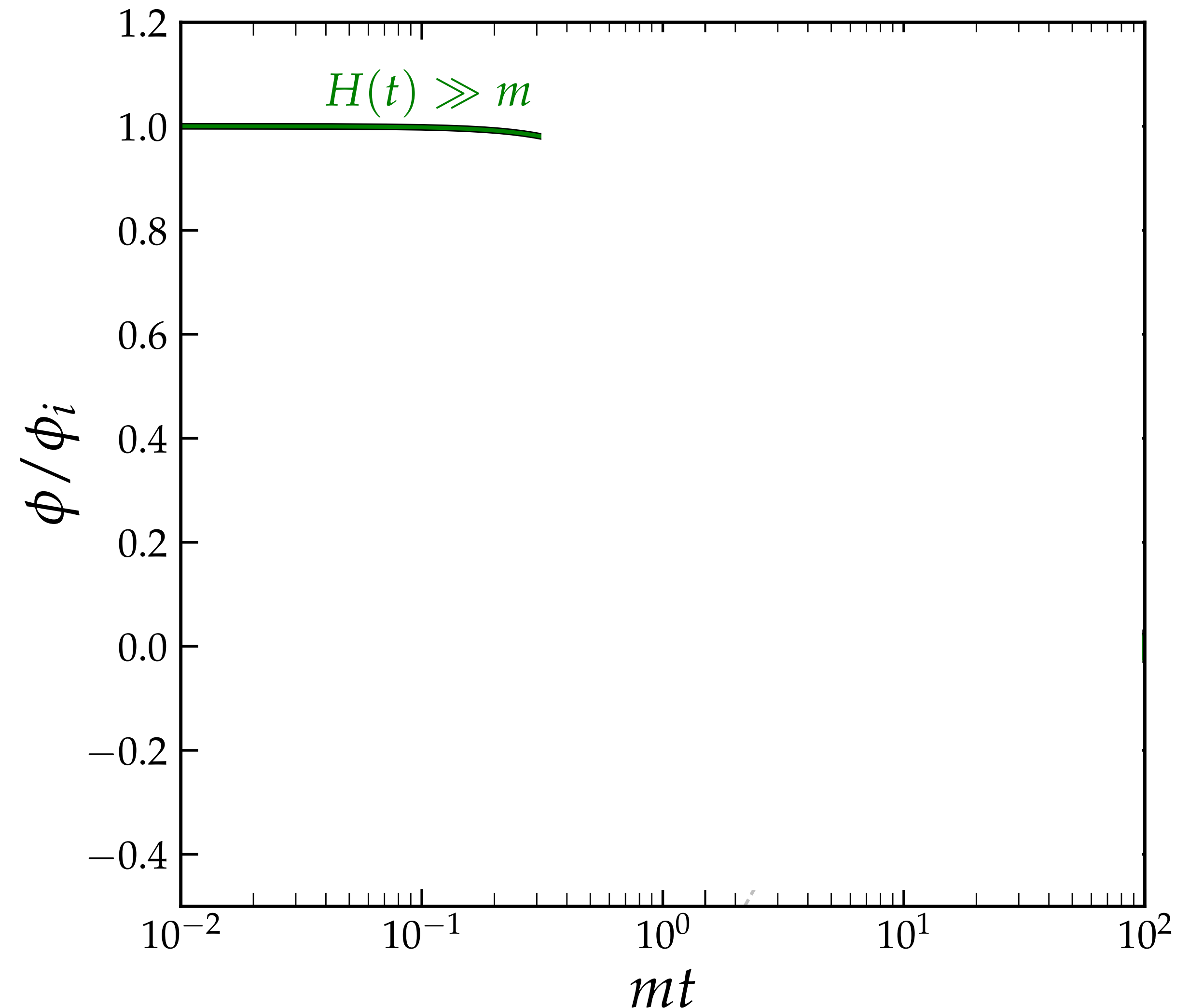
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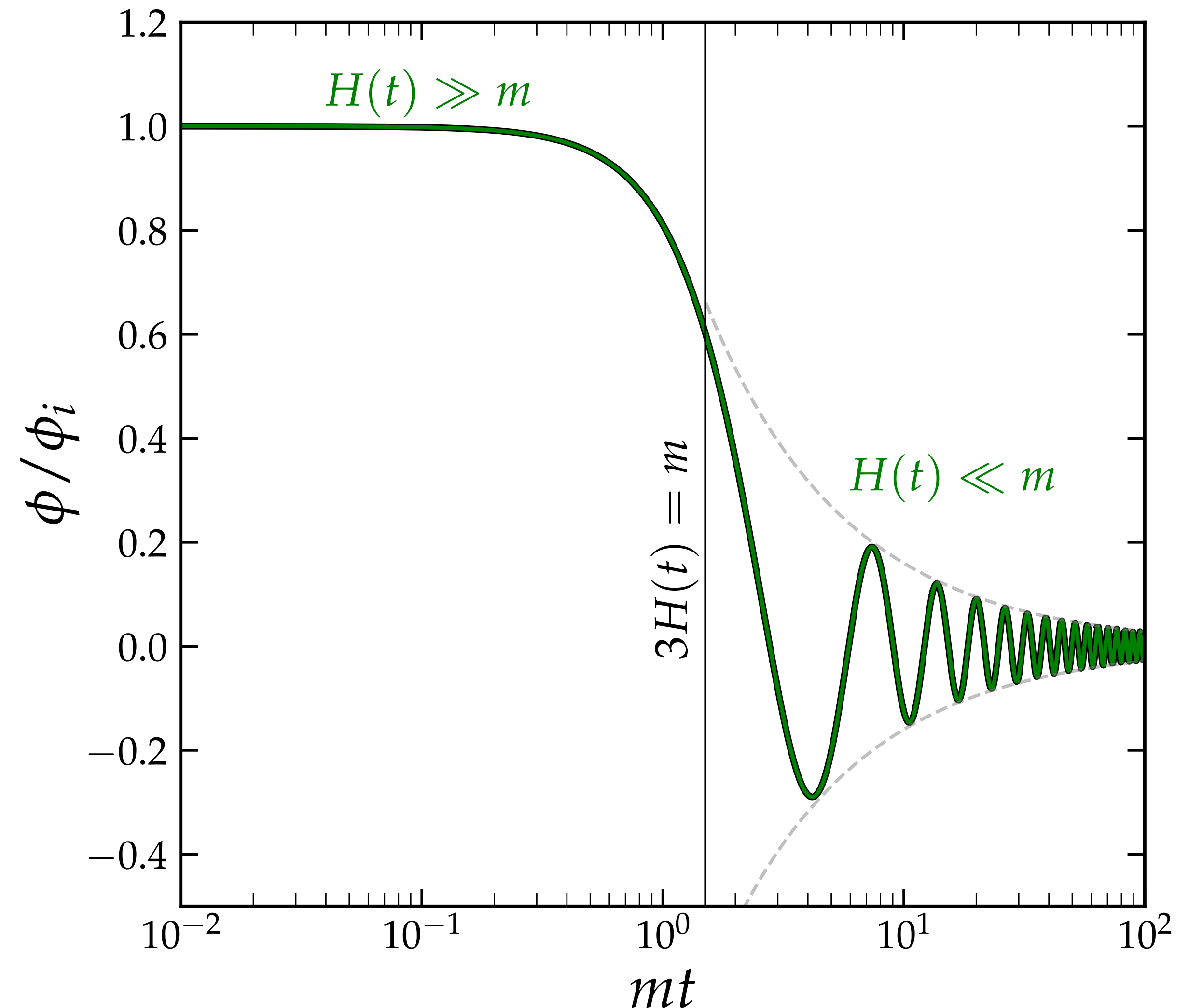
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- $3H(t) \ll m \rightarrow$ damped harmonic oscillator

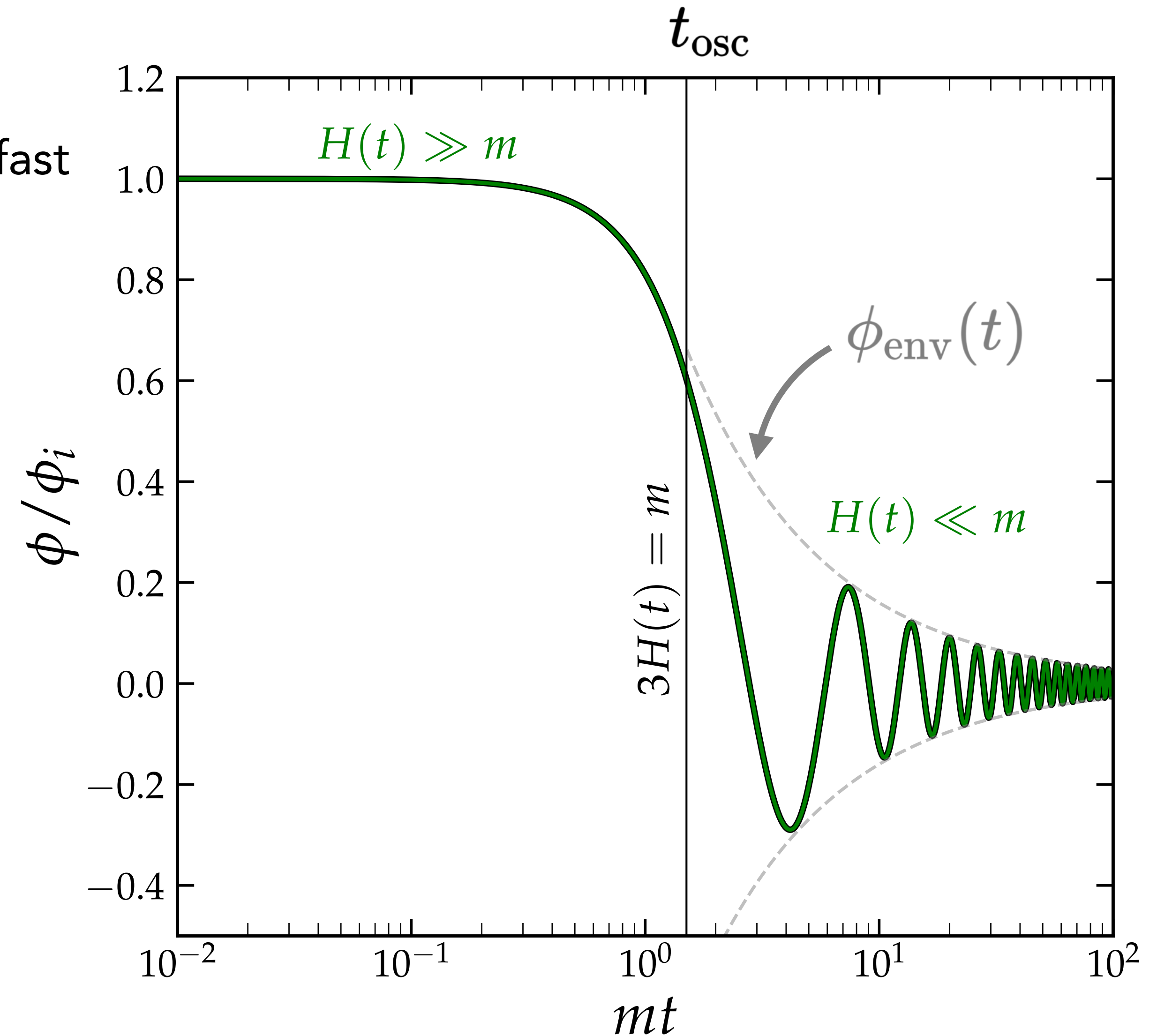


Misalignment mechanism for a generic scalar

At late times, the damping takes place over cosmological timescales, while the oscillations are fast
 → Make a WKB approximation $\dot{\phi}_{\text{env}}/\phi_{\text{env}} < m$

$$\phi(t) = \phi_{\text{env}}(t) \cos mt$$

$$\rightarrow \phi \propto \underbrace{\phi_i a(t)^{-3/2}}_{\substack{\text{Envelope} \\ \text{decays as } a^{-3/2}}} \cos(mt)$$



Misalignment mechanism for a generic scalar

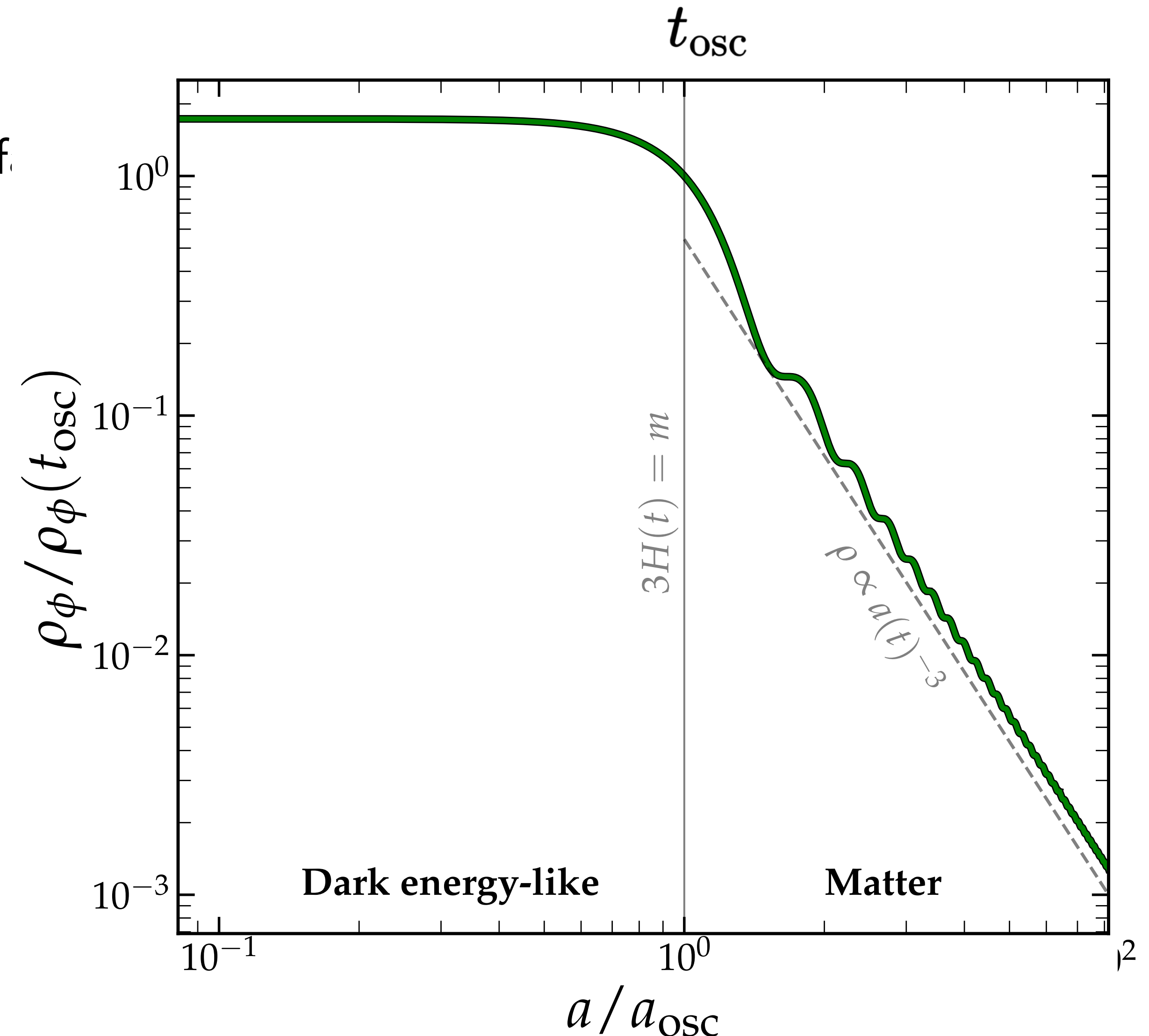
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Look at the energy density in the scalar field

$$\rho_\phi = \frac{1}{2} \dot{\phi}^2 + \frac{1}{2} m^2 \phi^2 \implies \rho_\phi \propto a^{-3}$$



Misalignment mechanism for a generic scalar

We want to force this to be equal to the abundance of dark matter today

$$\Omega_{\phi} h^2 = \frac{\rho_{\phi}}{\rho_c} h^2 = \frac{\rho_{\phi}}{3H_0^2 M_{\text{Pl}}^2} h^2 \rightarrow 0.12$$

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Assume we're late enough that $g_* = 3.4 = \text{const.}$

To get these temperatures, use Friedmann eq: $3H^2 M_{\text{Pl}}^2 = \frac{\pi^2}{30} g_* T^4$

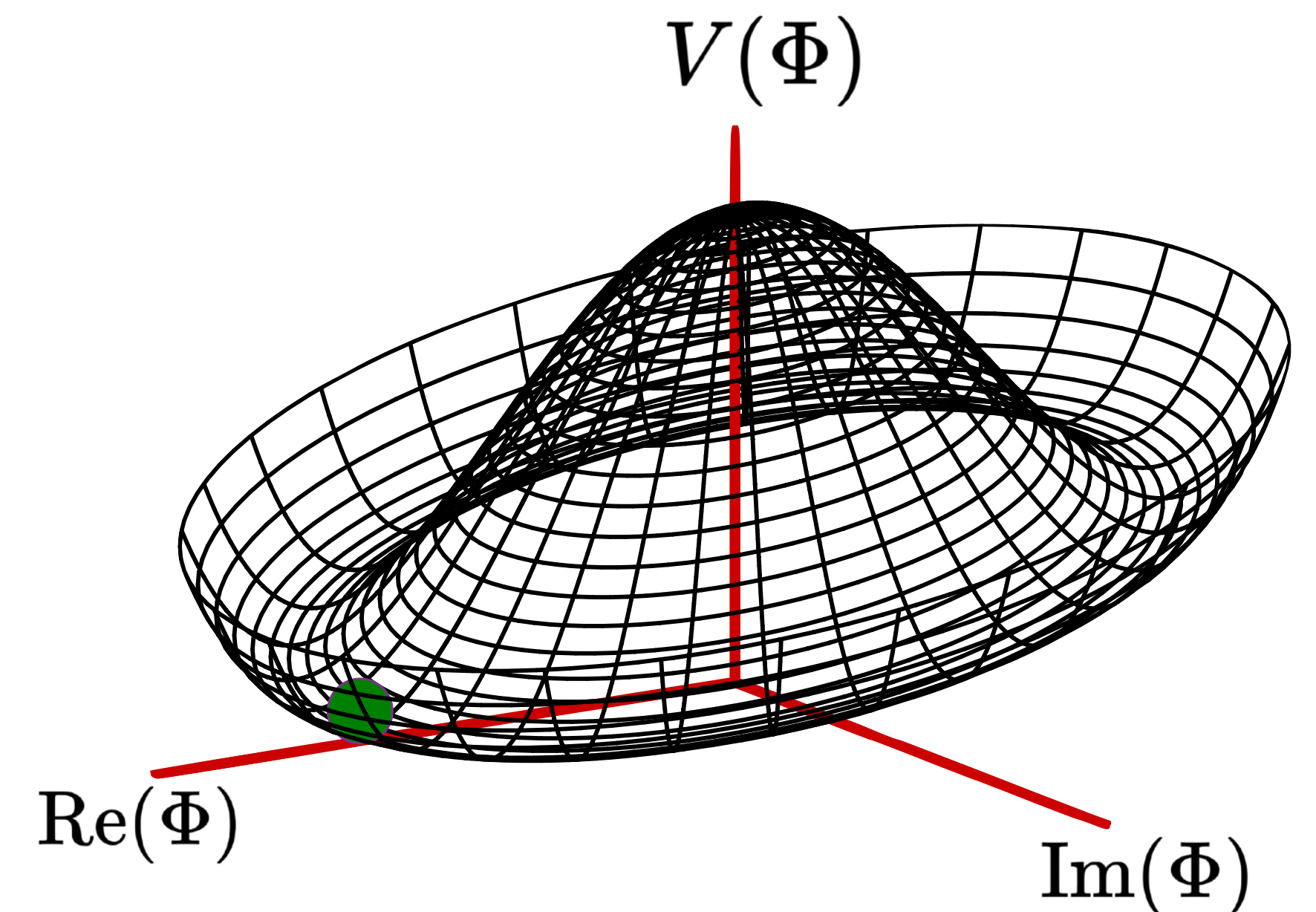
$$\Rightarrow \Omega_\phi h^2 = 0.12 \left(\frac{\phi_i}{4.7 \times 10^{16} \text{ GeV}} \right)^2 \left(\frac{m}{10^{-21} \text{ eV}} \right)^{\frac{1}{2}}$$

Now, the axion case: How do we adapt this?

Axion is the Goldstone (θ) appearing after the $U(1)_{PQ}$ is broken at scale f_a .

We write it as the phase of a complex scalar field: $\Phi(t) = \rho e^{i\theta}$

$$V(\Phi) = V_{PQ}(\rho) + V_{QCD}(\theta) = \frac{\lambda}{8} (\rho^2 - f_a^2)^2 + \Lambda_{QCD}^4 (1 - \cos \theta)$$



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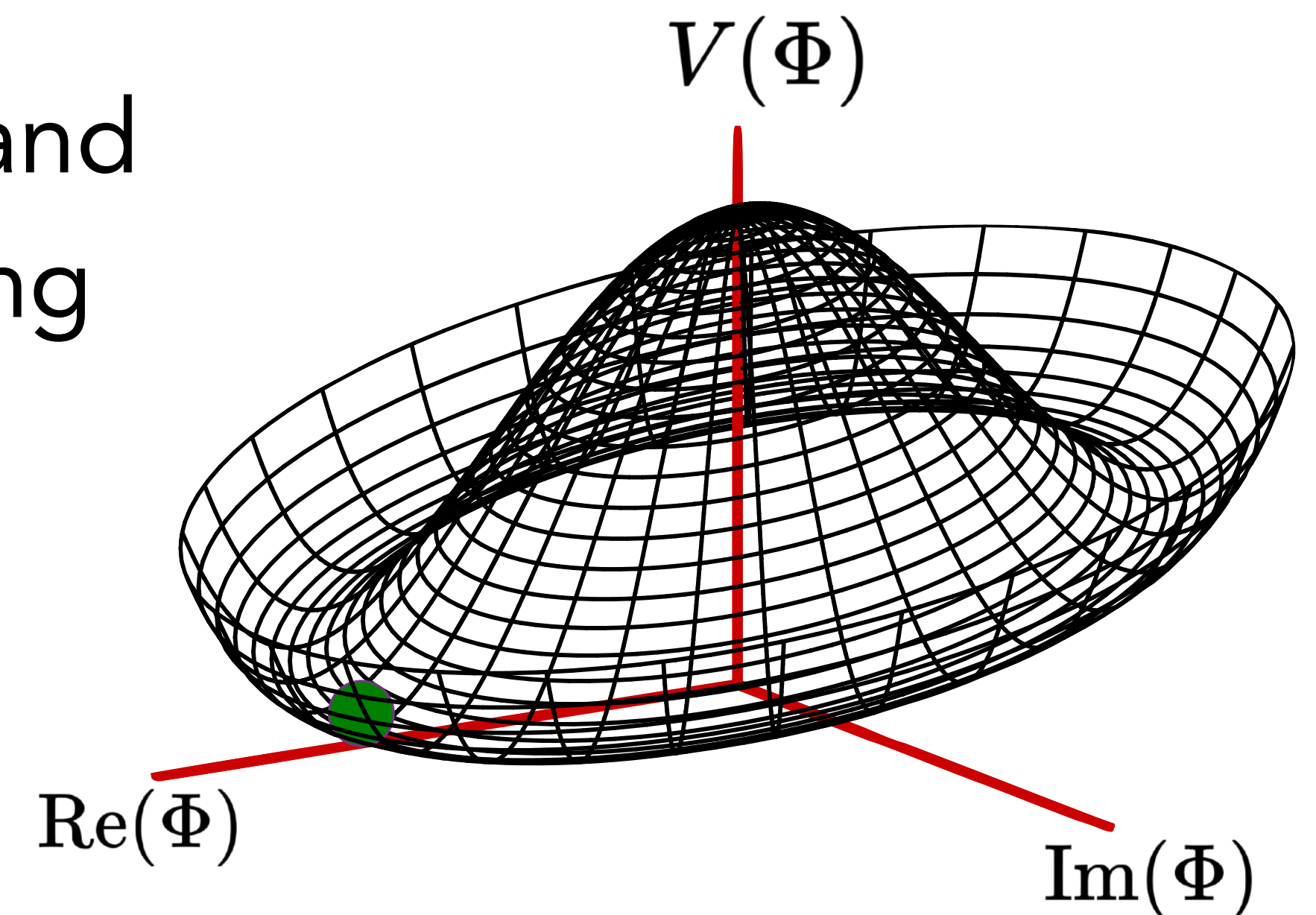
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So our scalar field is now an angle in units of f_a and with a mass set by the explicit symmetry-breaking term generated by QCD

$$\phi \rightarrow f_a \theta \qquad m \rightarrow m_a \approx \frac{\Lambda_{QCD}^2}{f_a}$$



The QCD axion abundance, is it correct?

For the generic scalar we got,

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To convert this to the axion case, can I just replace:

$$\phi_i \rightarrow \theta_i f_a \approx \mathcal{O}(1) f_a \quad m \rightarrow m_a \approx 6 \mu\text{eV} \left(\frac{10^{12} \text{ GeV}}{f_a} \right) ?$$

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No! If you do this, you will not get what is quoted in the literature

$$\Omega_a h^2 \approx 0.12 \theta_i^2 \left(\frac{f_a}{9 \times 10^{11} \text{ GeV}} \right)^{1.165} \implies \textbf{Why?}$$

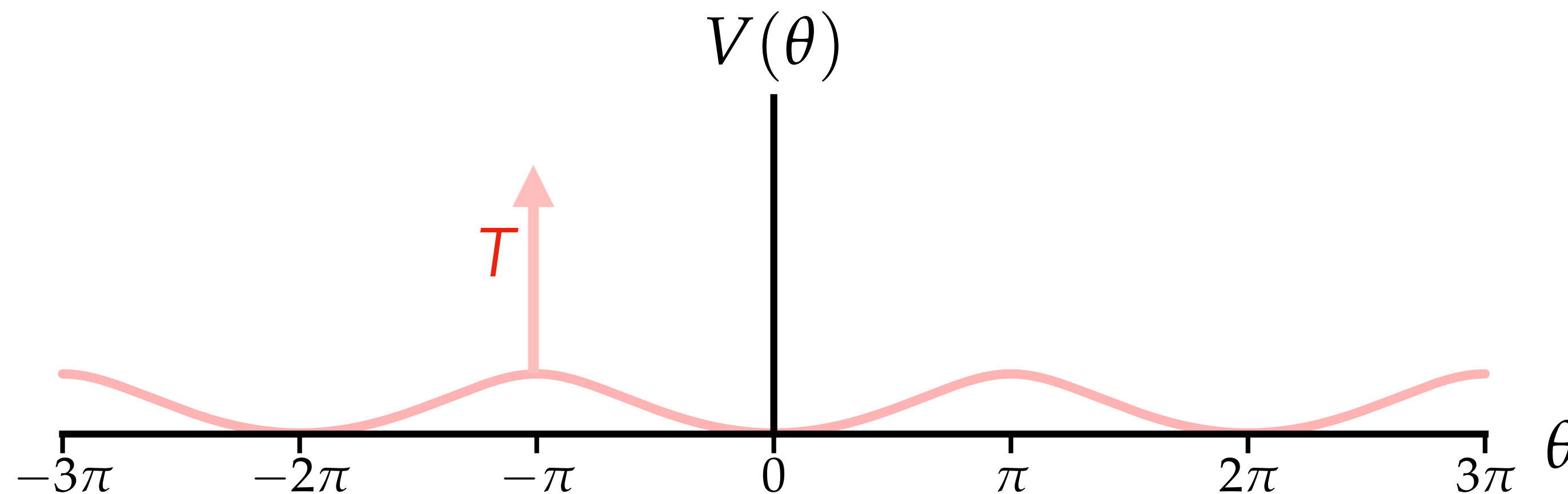
The QCD axion

Mass is generated by instantons whose effects are temperature-dependent

In the literature this dependence is called the “topological susceptibility”, $\chi(T)$

$$V(\theta) \approx \chi(T)(1 - \cos \theta) = m_a^2(T) f_a^2 (1 - \cos \theta)$$

Axion mass grows as temperature drops,
reaching a constant when $T < T_{\text{QCD}}$



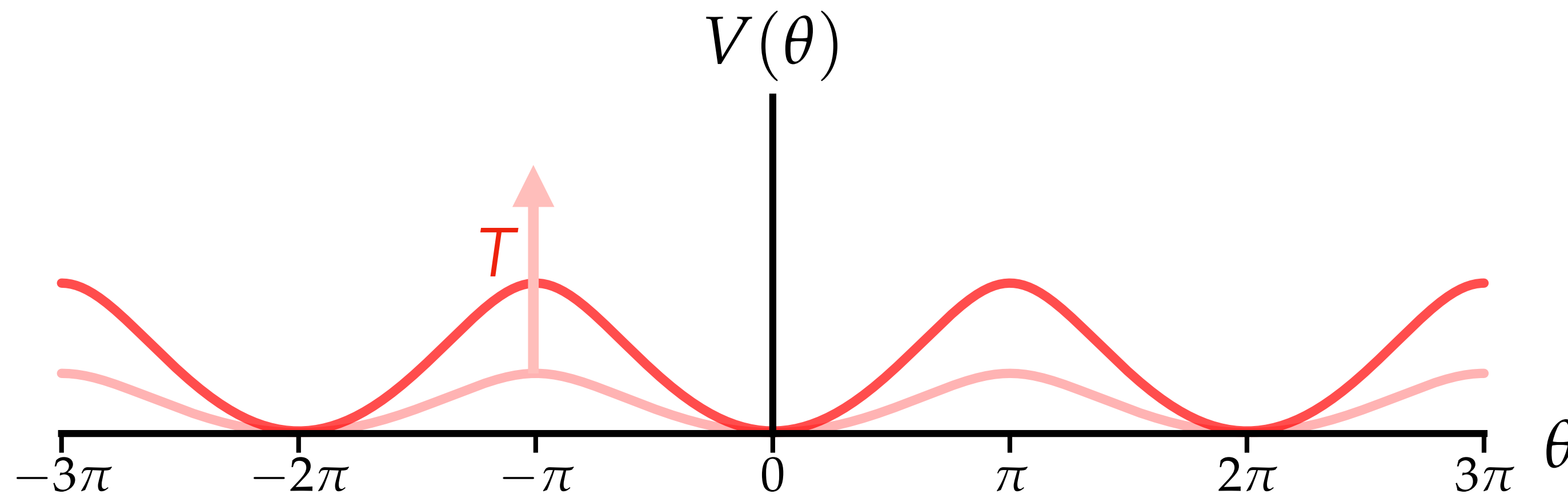
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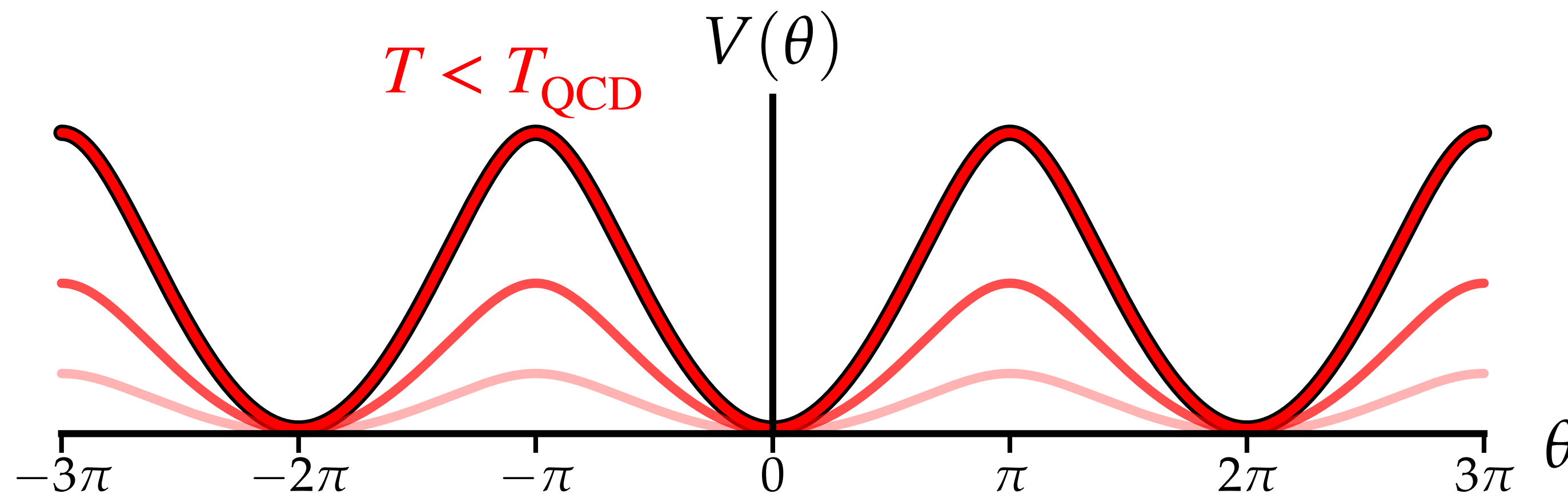
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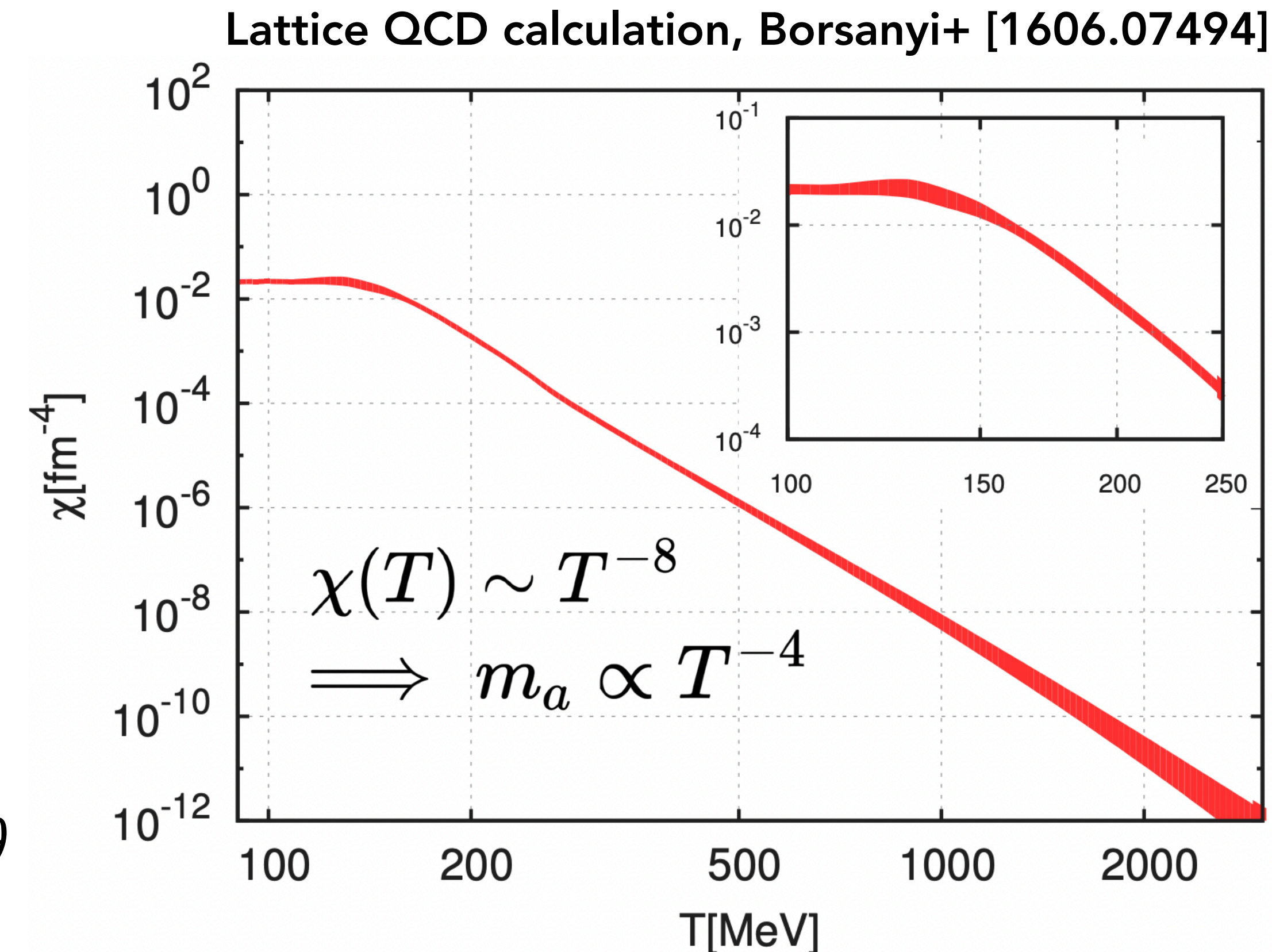
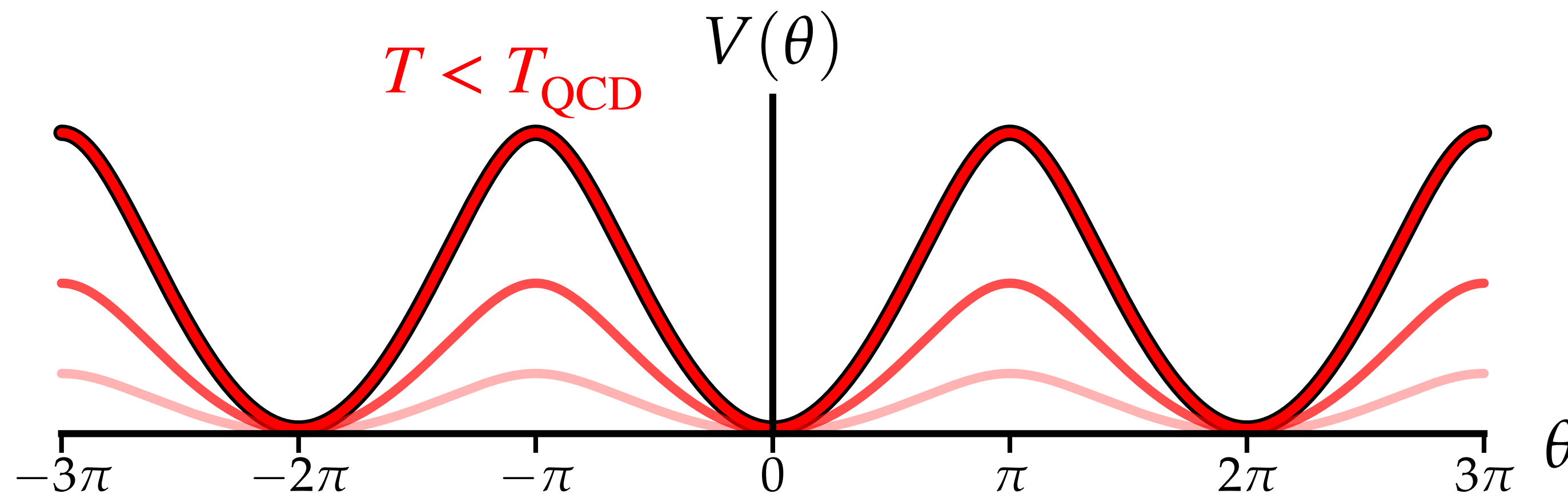
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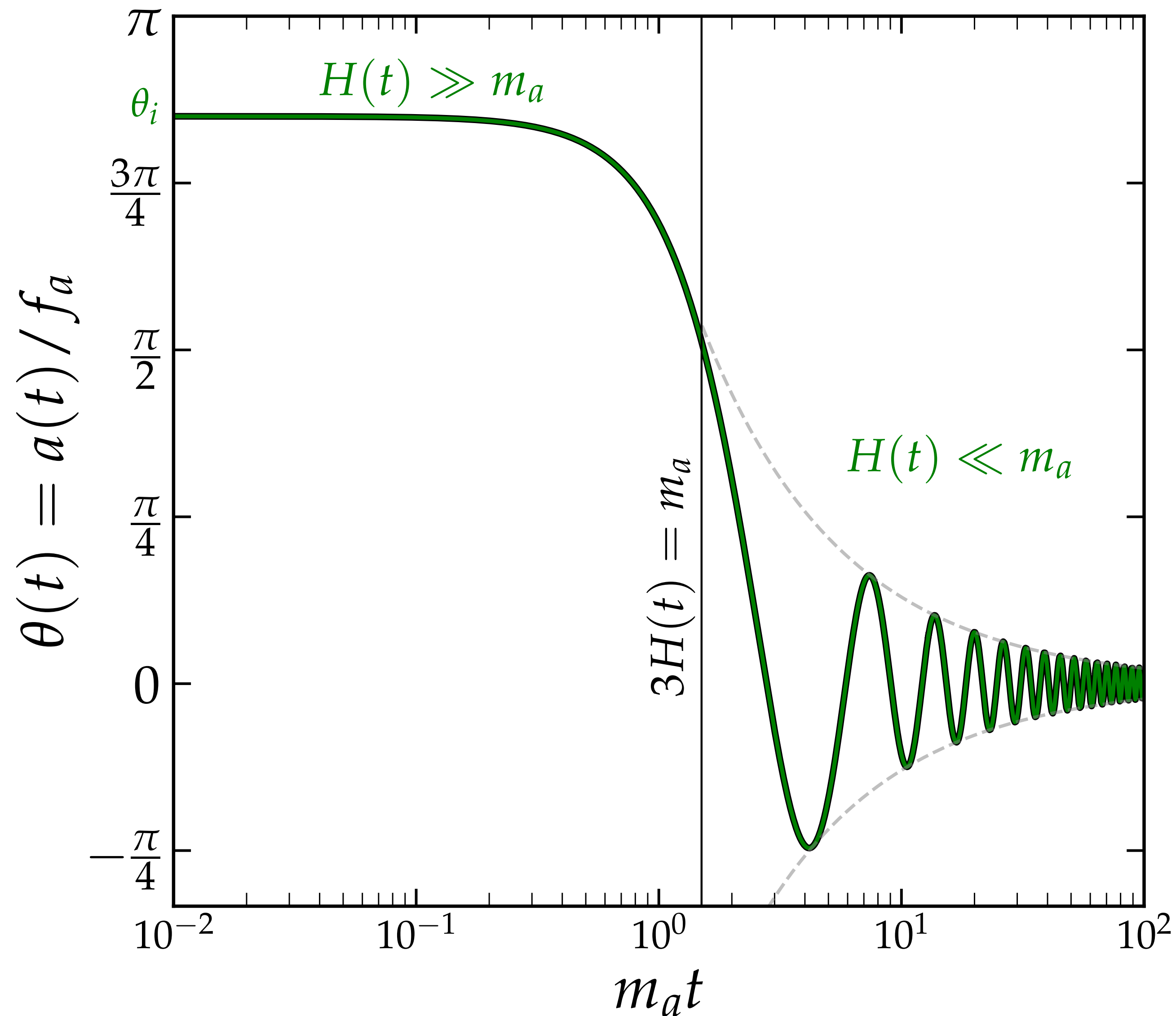
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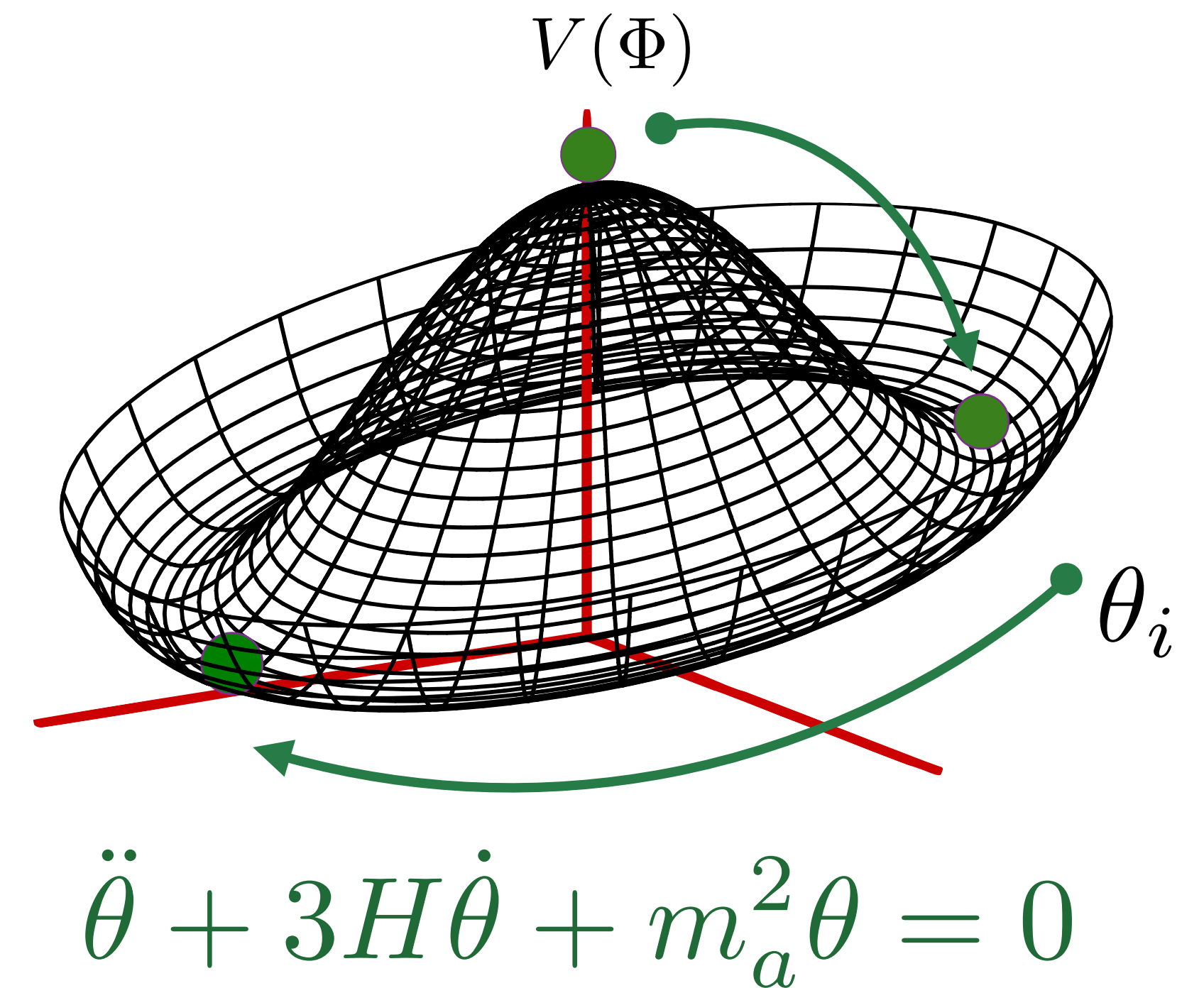
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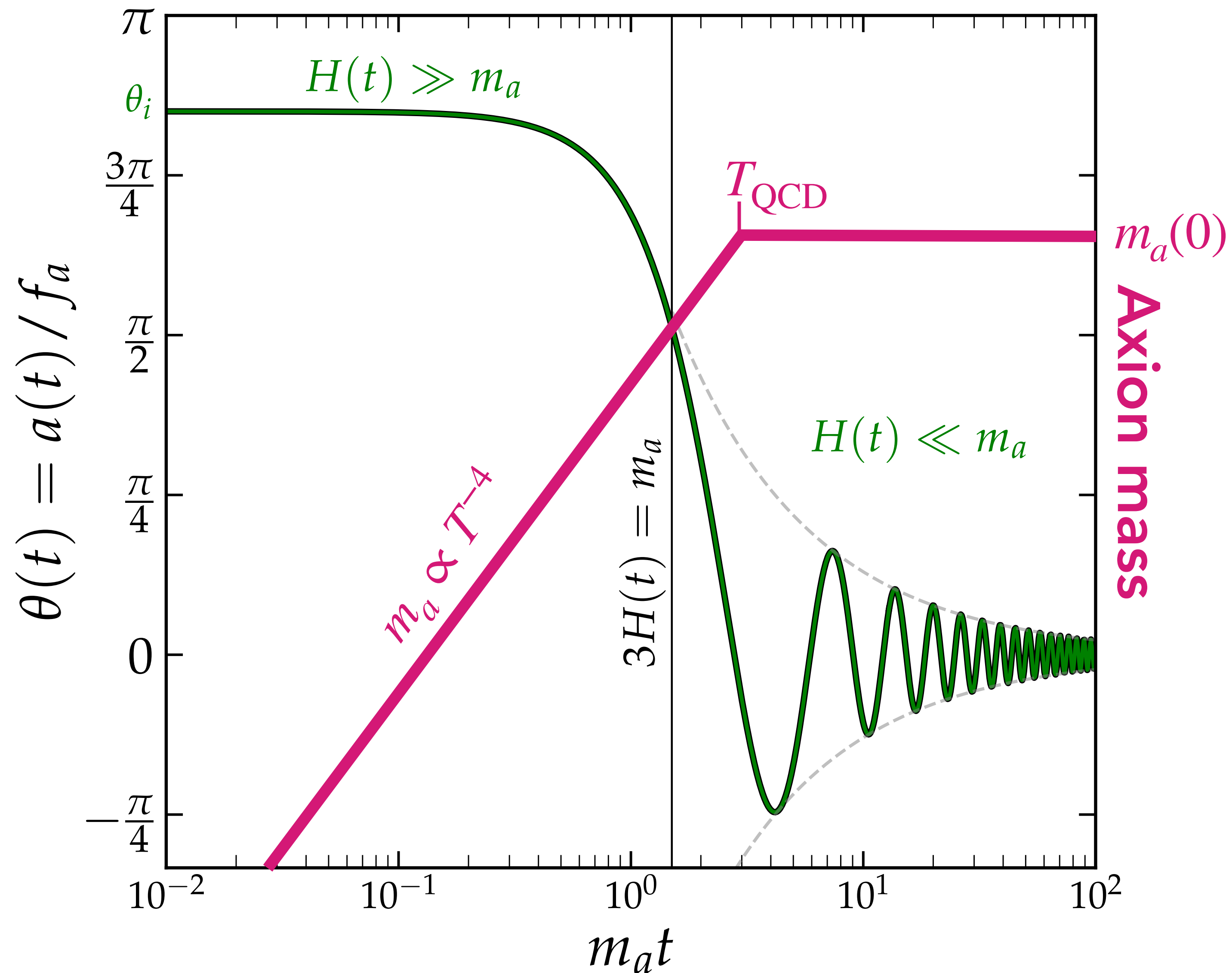
The QCD axion mass



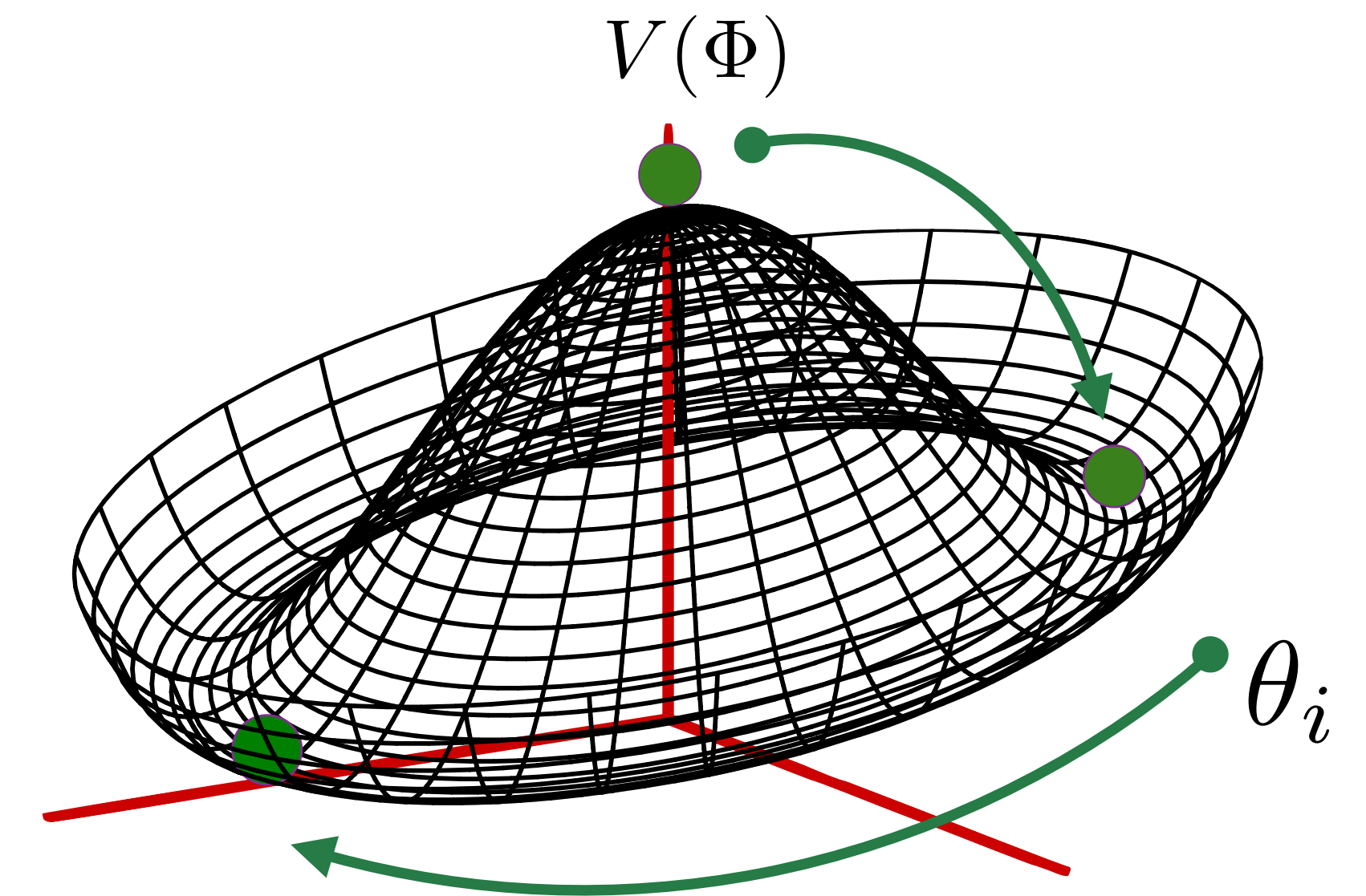
The tilt comes on gradually as the temperature drops



The QCD axion mass



The tilt comes on gradually as the temperature drops



$$\ddot{\theta} + 3H\dot{\theta} + m_a^2\theta = 0$$

$$\ddot{\theta} + 3H\dot{\theta} + m_a(T)^2 \sin(\theta) = 0$$

QCD axion abundance

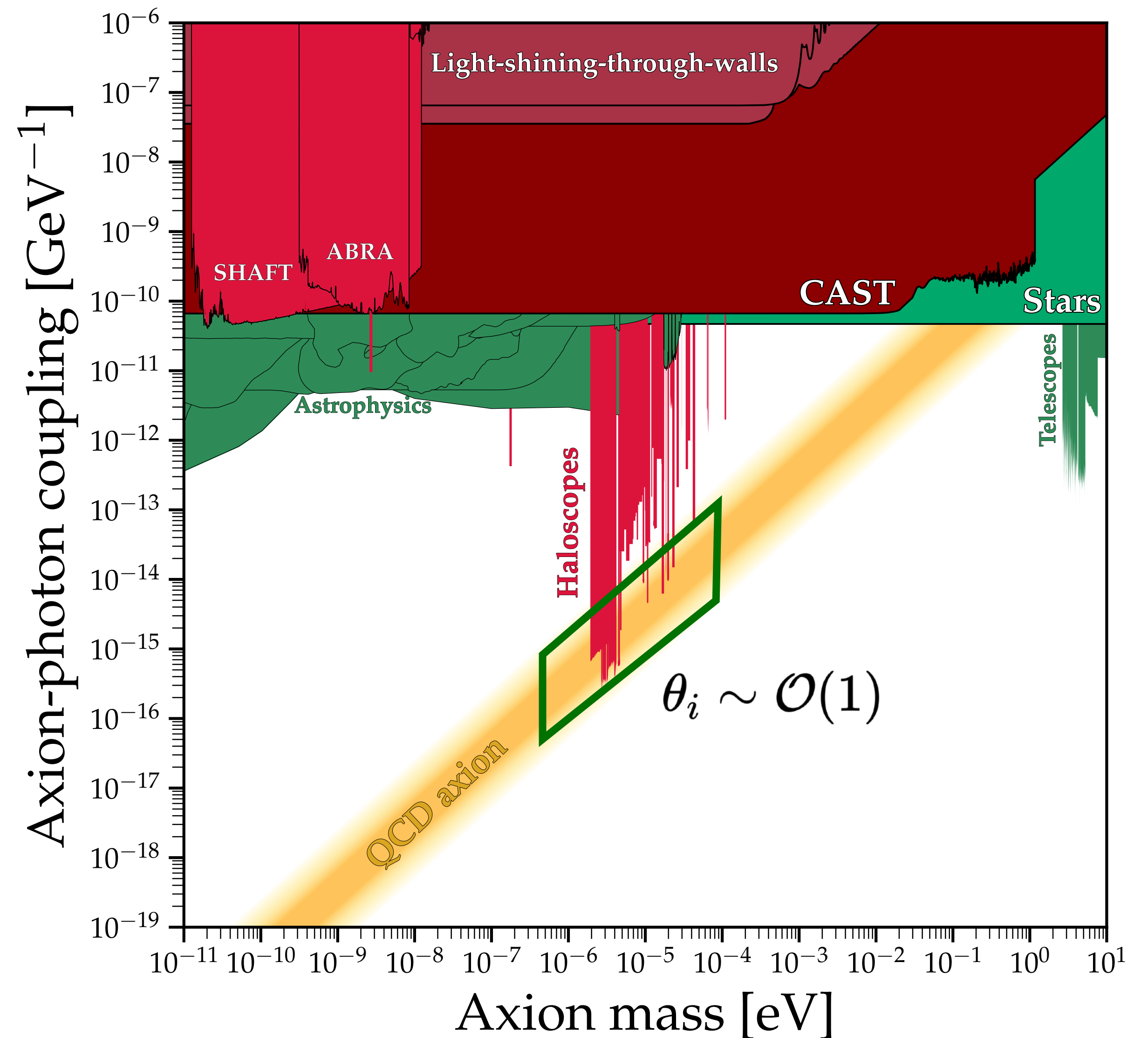
- Full calculation leads to:

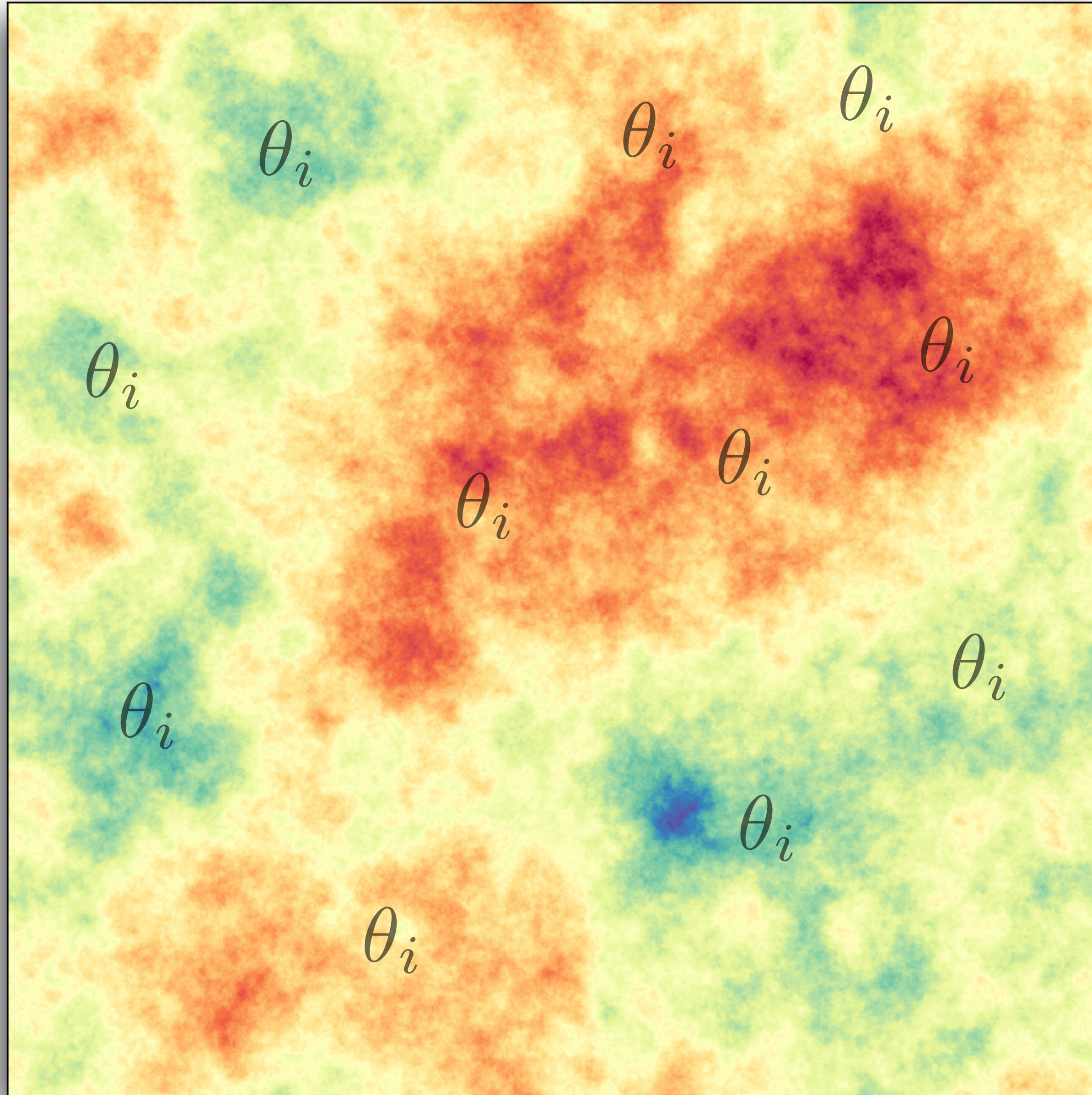
$$\Omega_a h^2 \approx 0.12 \theta_i^2 \left(\frac{7.26 \mu\text{eV}}{m_a} \right)^{\frac{n+6}{n+4}}$$

where $n \sim 8$

- Seems to prefer the "classic QCD axion window": $\mathcal{O}(1\text{--}10) \mu\text{eV}$

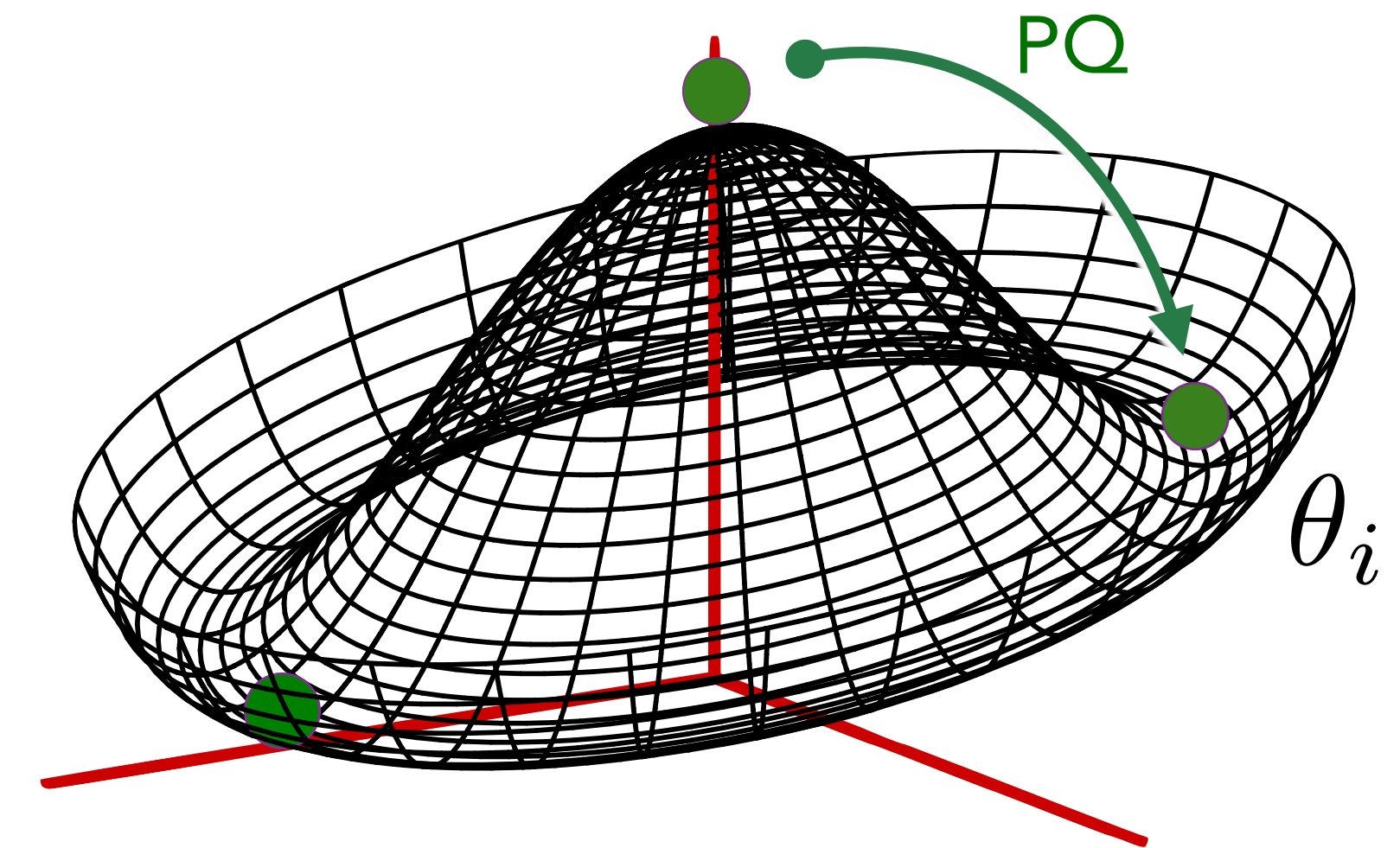
→ but what should we pick for θ_i ?





The issue is that $T_{\text{PQ}} \gg T_{\text{QCD}}$

The Universe should be filled with random θ_i everywhere since the axion didn't know about the preferred angle when the PQ phase transition happened

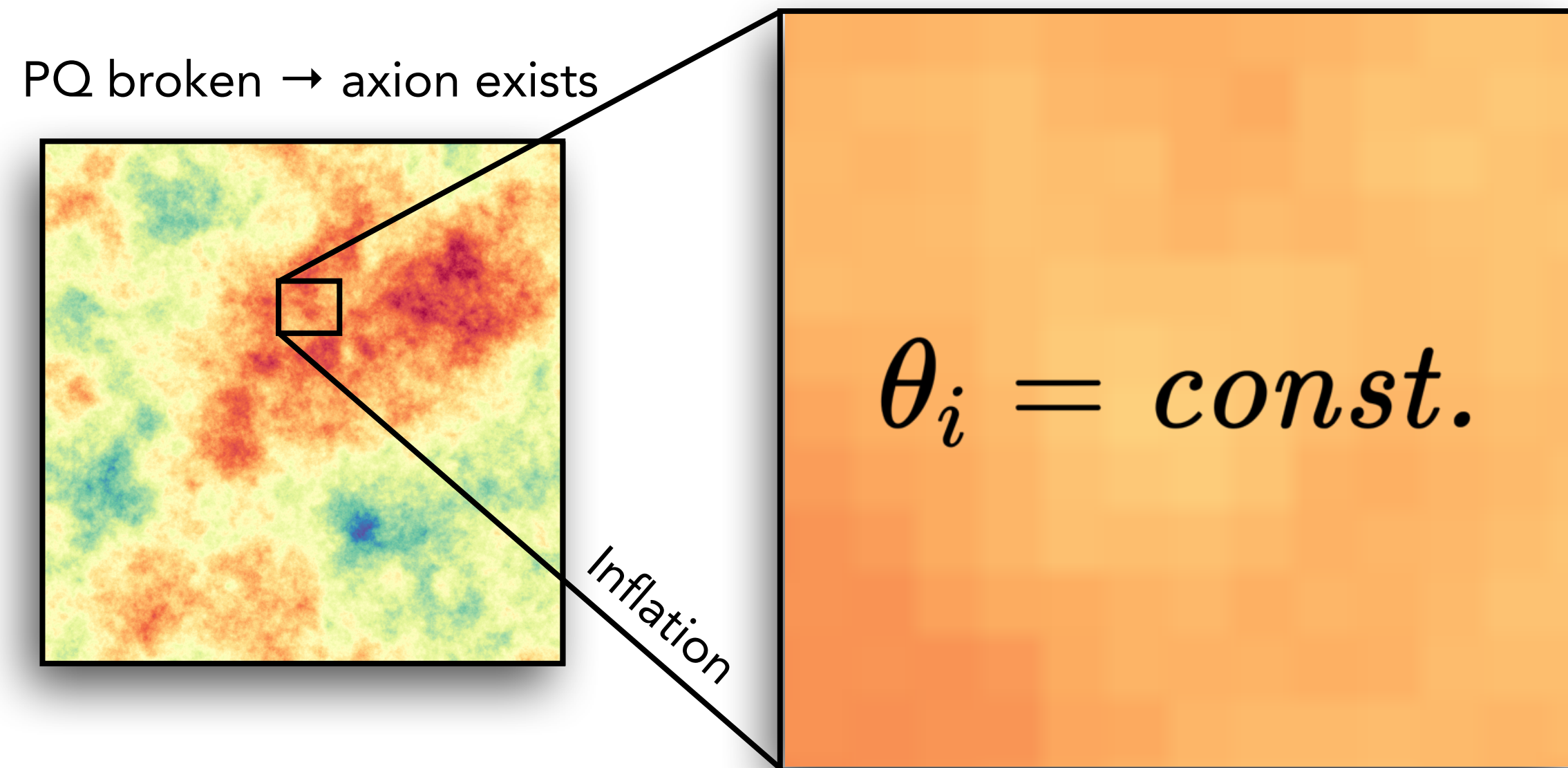


When did inflation happen?

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Option 1:

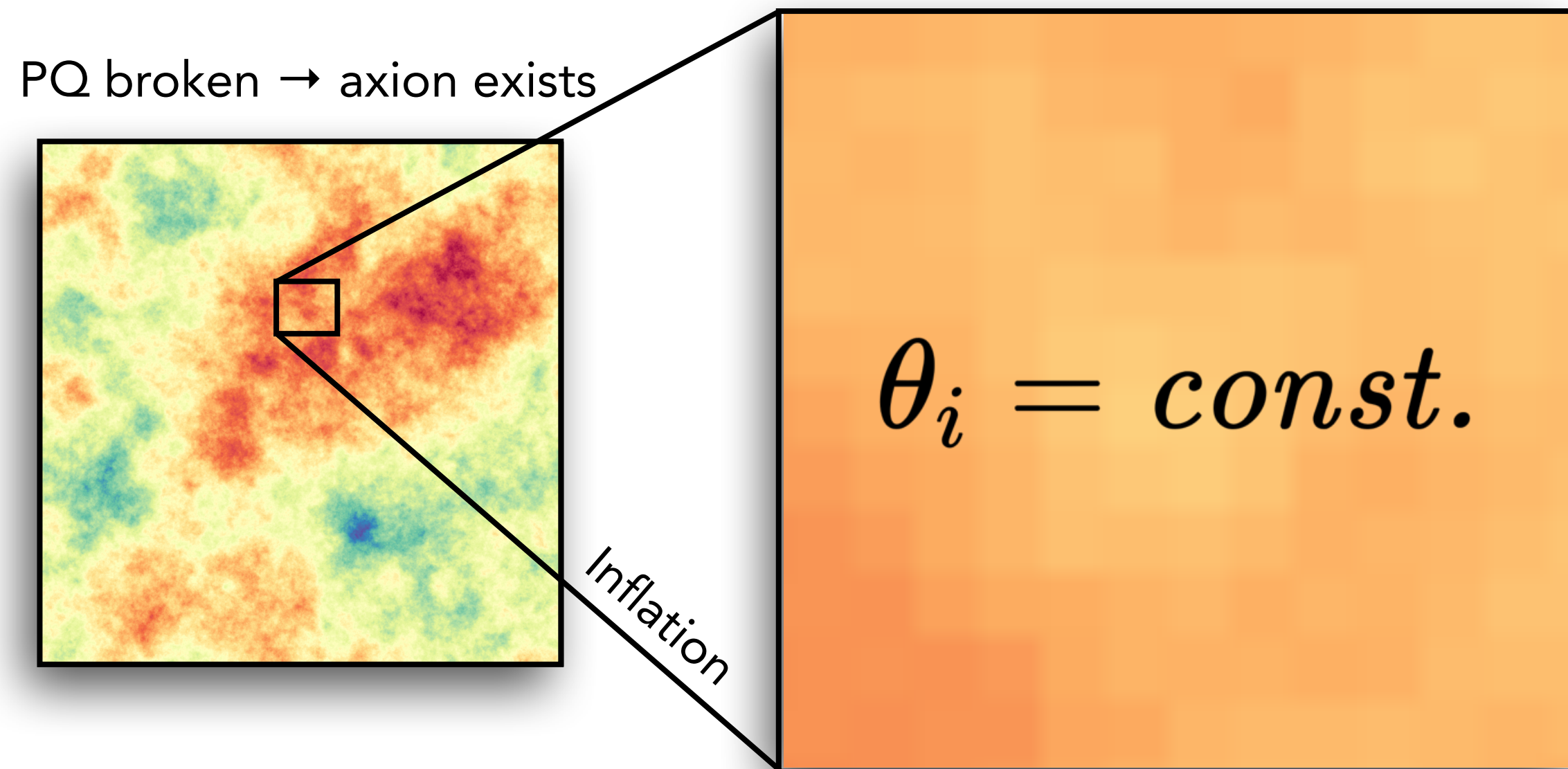
PQ is broken *before and during* inflation



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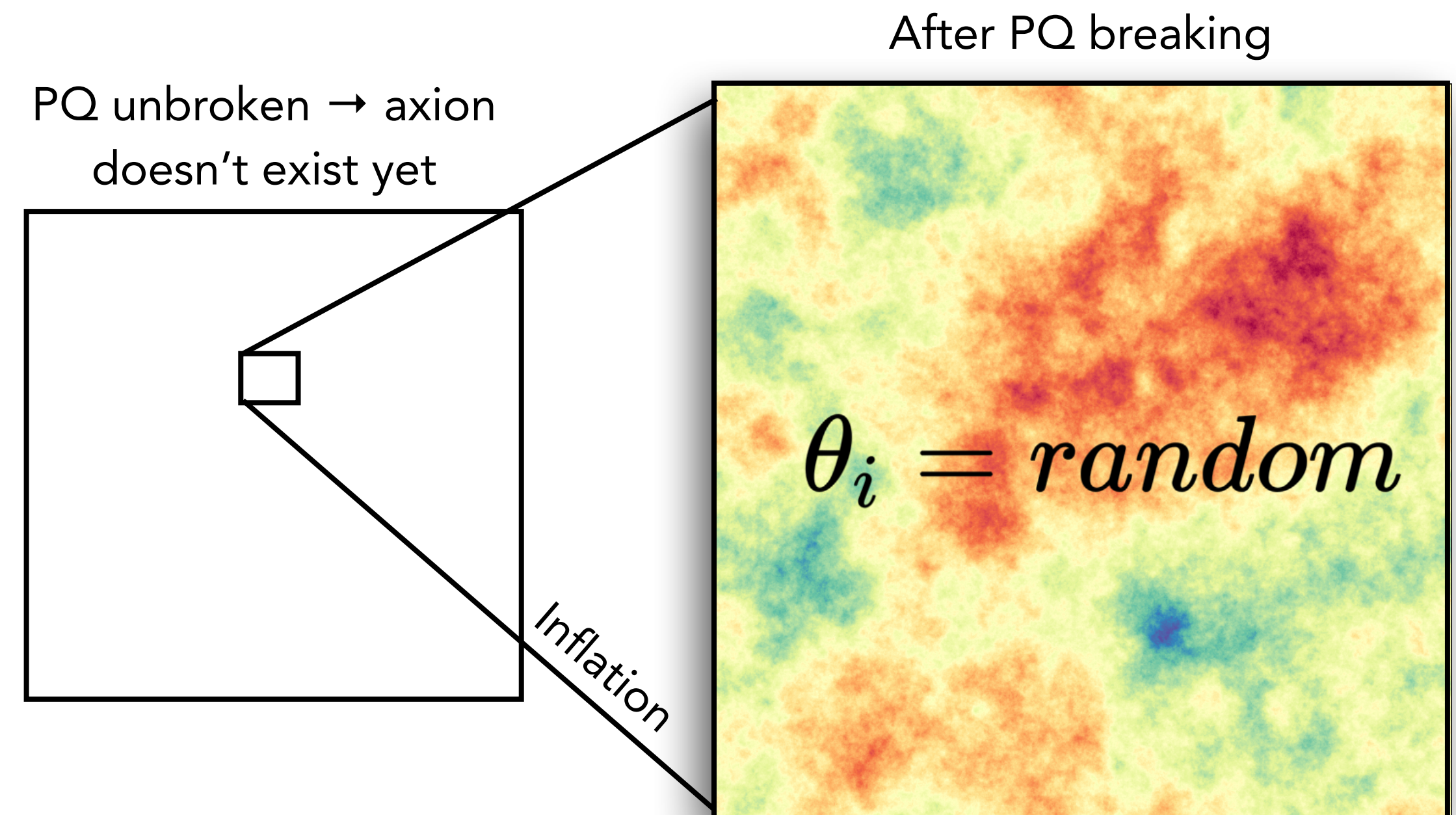
Option 1:

PQ is broken *before and during* inflation



Option 2:

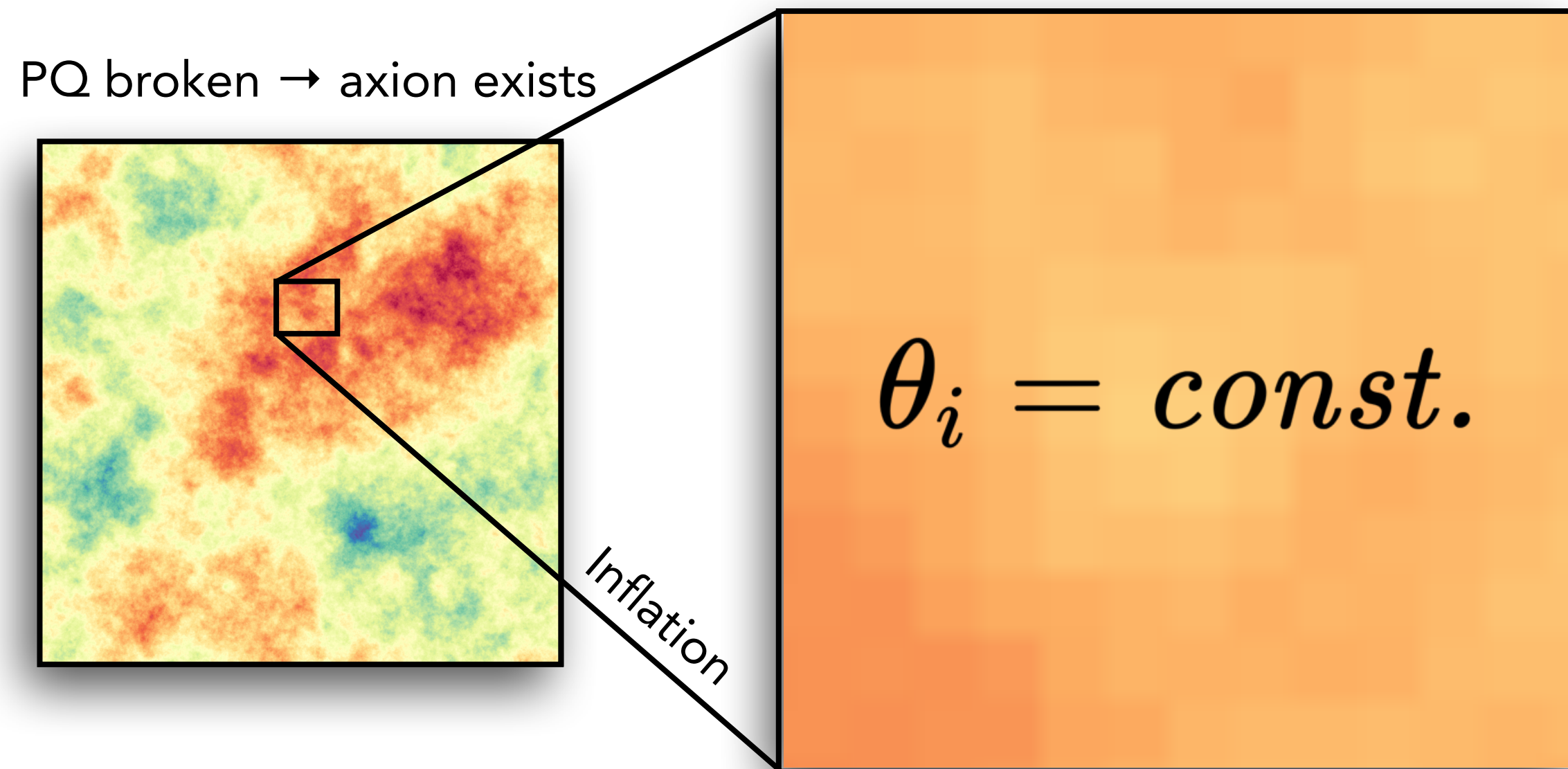
PQ is broken *after* inflation



When did inflation happen?

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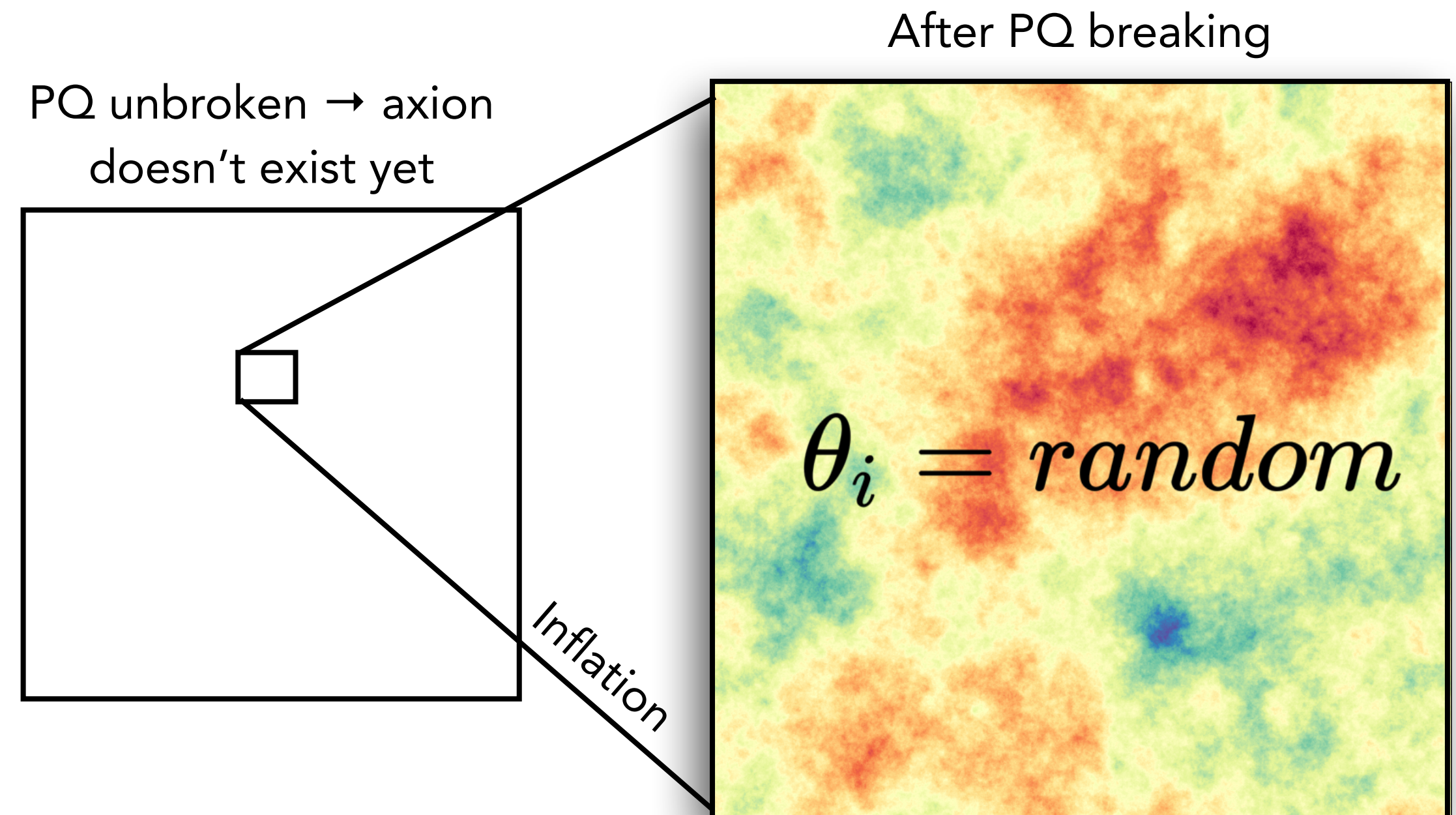
PQ is broken *before and during* inflation



Pre-inflationary scenario

Option 2:

PQ is broken *after* inflation



Post-inflationary scenario

Pre-inflationary axions

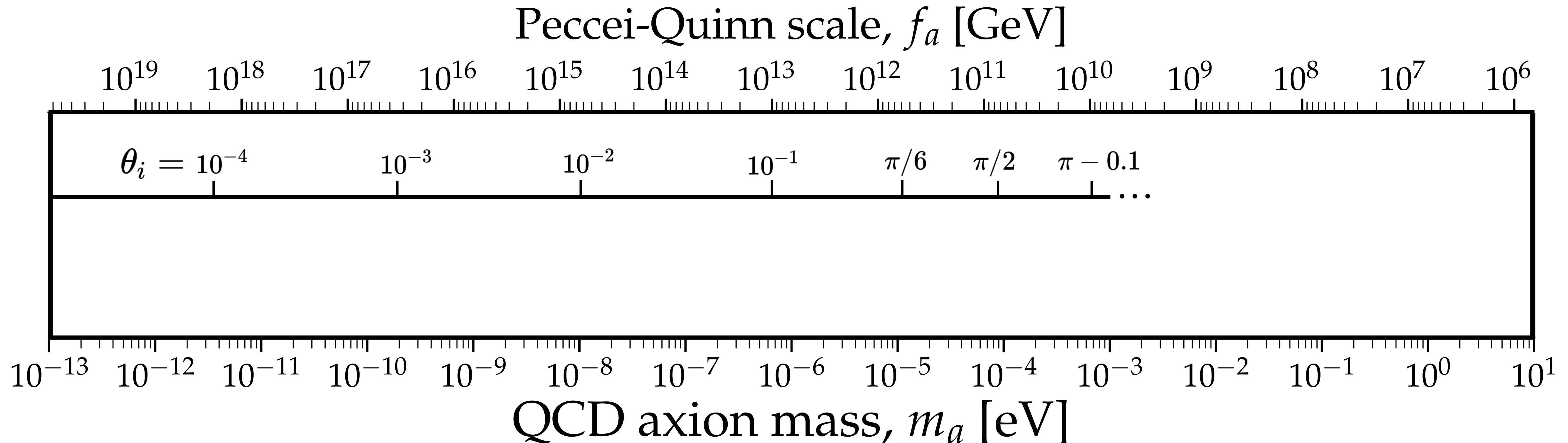
$$\Omega_a h^2 \approx 0.12 \theta_i^2 \left(\frac{7.26 \mu\text{eV}}{m_a} \right)^{\frac{n+6}{n+4}}$$

Our Universe could have been given any value of $\theta_i \in [-\pi, \pi]$. So we can make any axion mass work as long as we choose the θ_i that gives $\Omega_a h^2 = 0.12$

Pre-inflationary axions

$$\Omega_a h^2 \approx 0.12 \theta_i^2 \left(\frac{7.26 \mu\text{eV}}{m_a} \right)^{\frac{n+6}{n+4}}$$

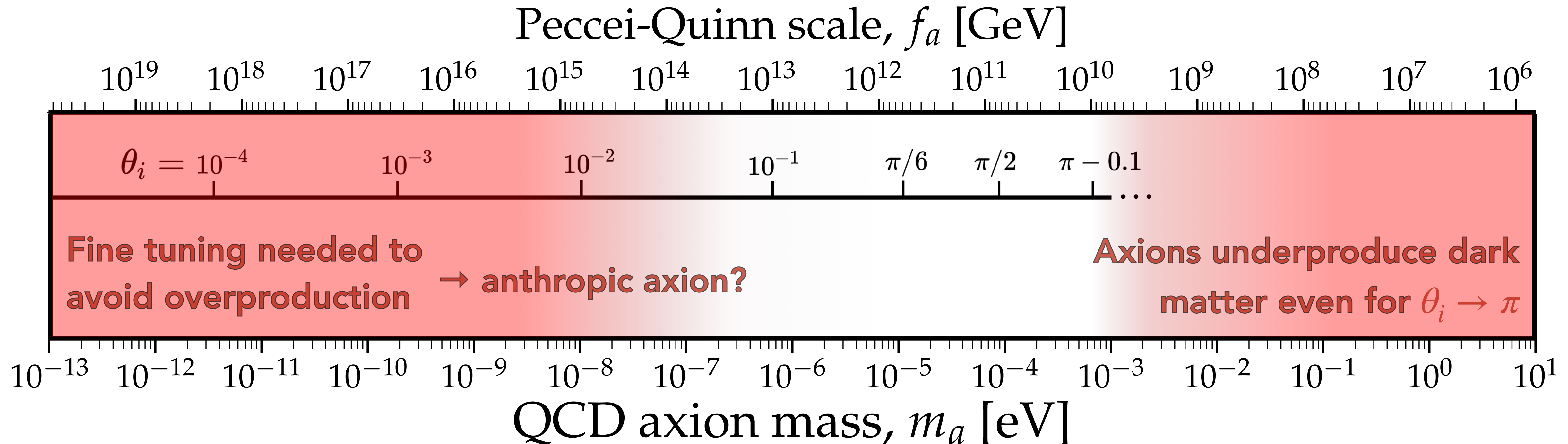
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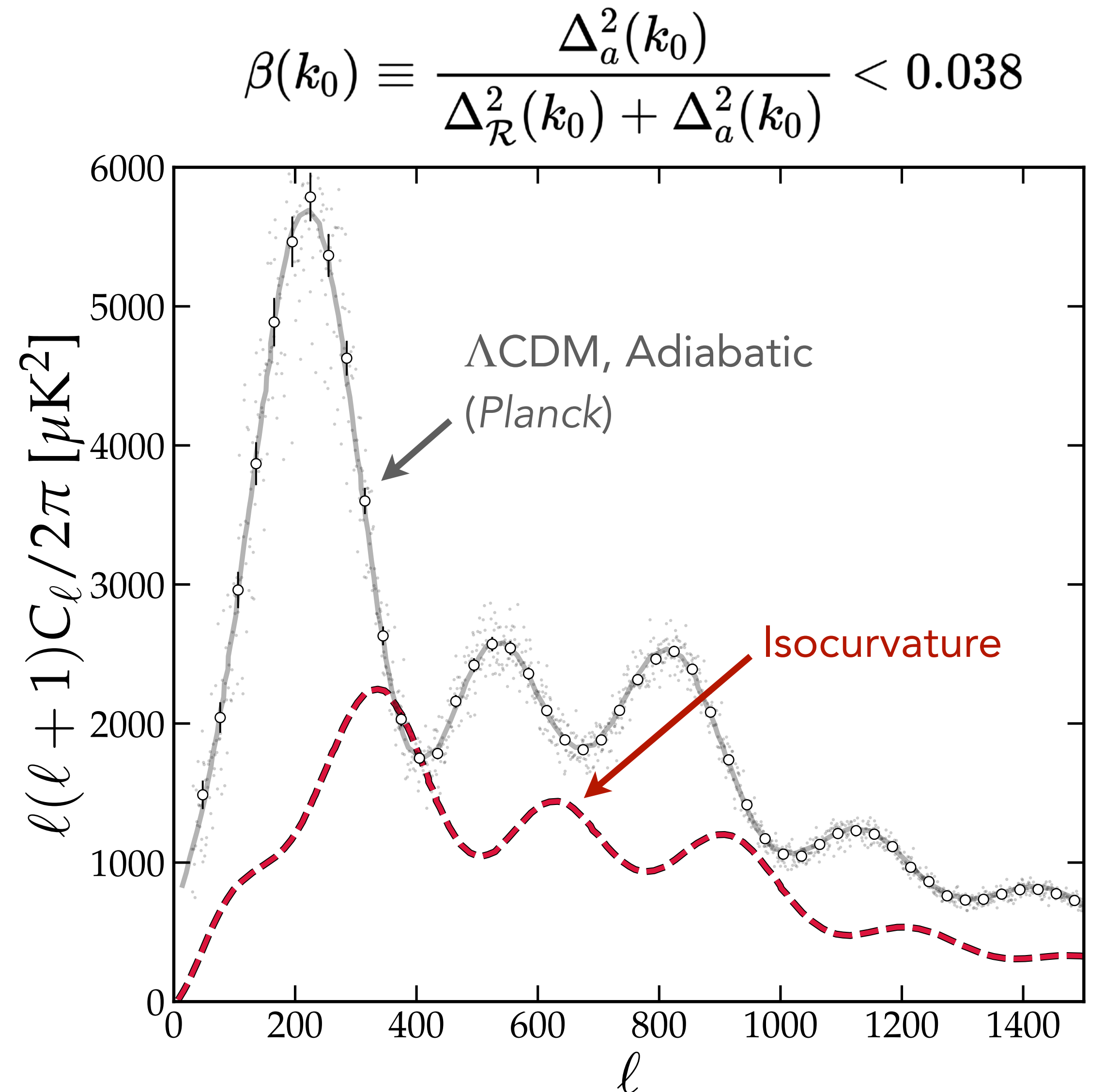


Pre-inflationary axions: isocurvature

- Axion exists as scalar d.o.f *during* inflation
→ quantum fluctuations are also inflated

$$\sigma_{\text{axion}} \sim H_I/2\pi$$

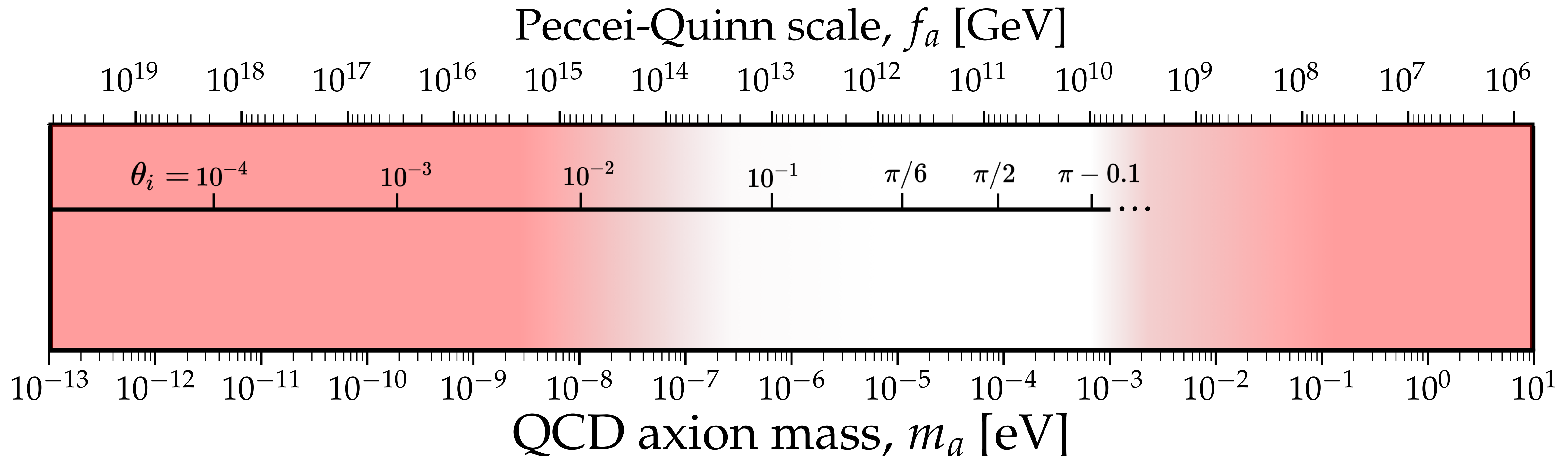
- Fluctuations eventually become matter perturbations when the axion gets a mass
- This is bad: they will be *uncorrelated* with curvature fluctuations from inflaton. Such fluctuations are called **isocurvature**
- *Planck* bounds power in isocurvature to be less than <4% compared to primordial curvature perturbations.



Pre-inflationary axions

For a given f_a the scale of inflation must be below some maximum value or axion produces too much isocurvature

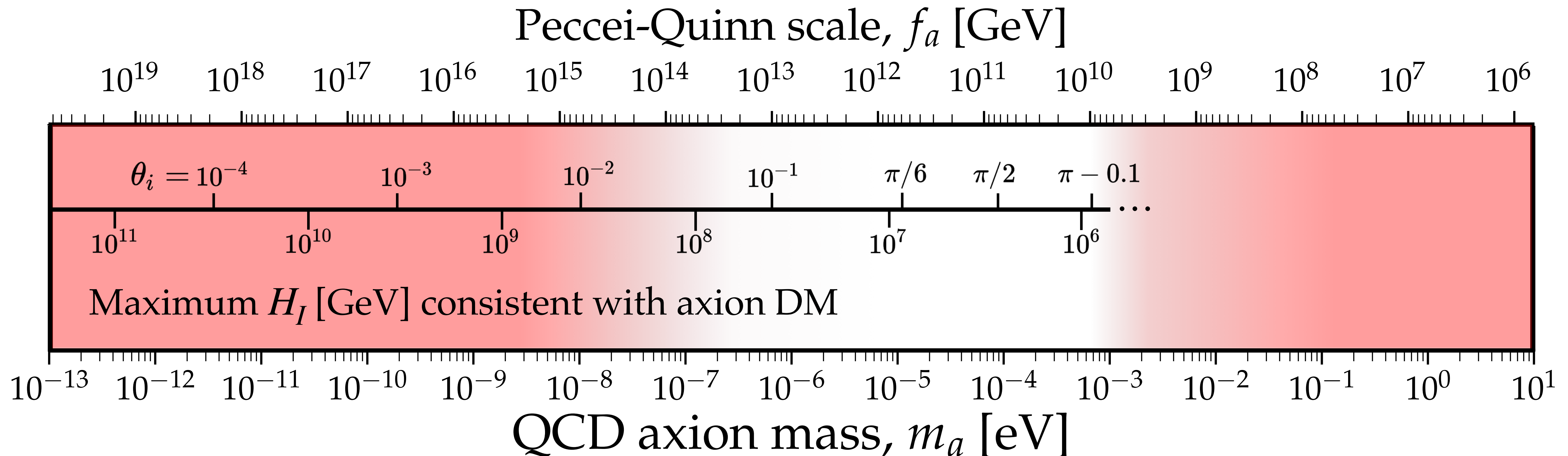
$$H_I \lesssim 2.8 \times 10^8 \text{ GeV} \times \theta_i \left(\frac{f_a}{10^{11} \text{ GeV}} \right)$$



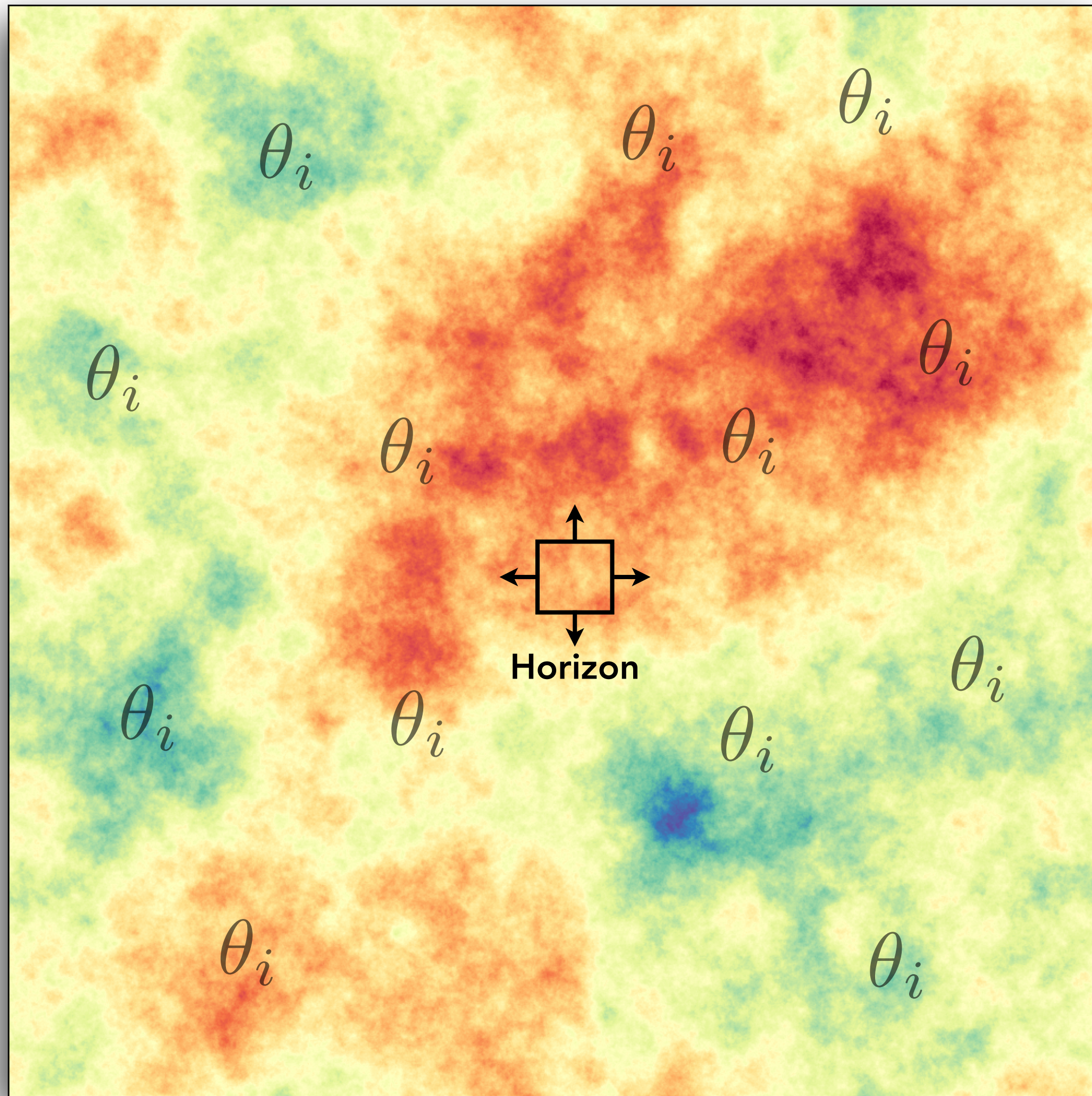
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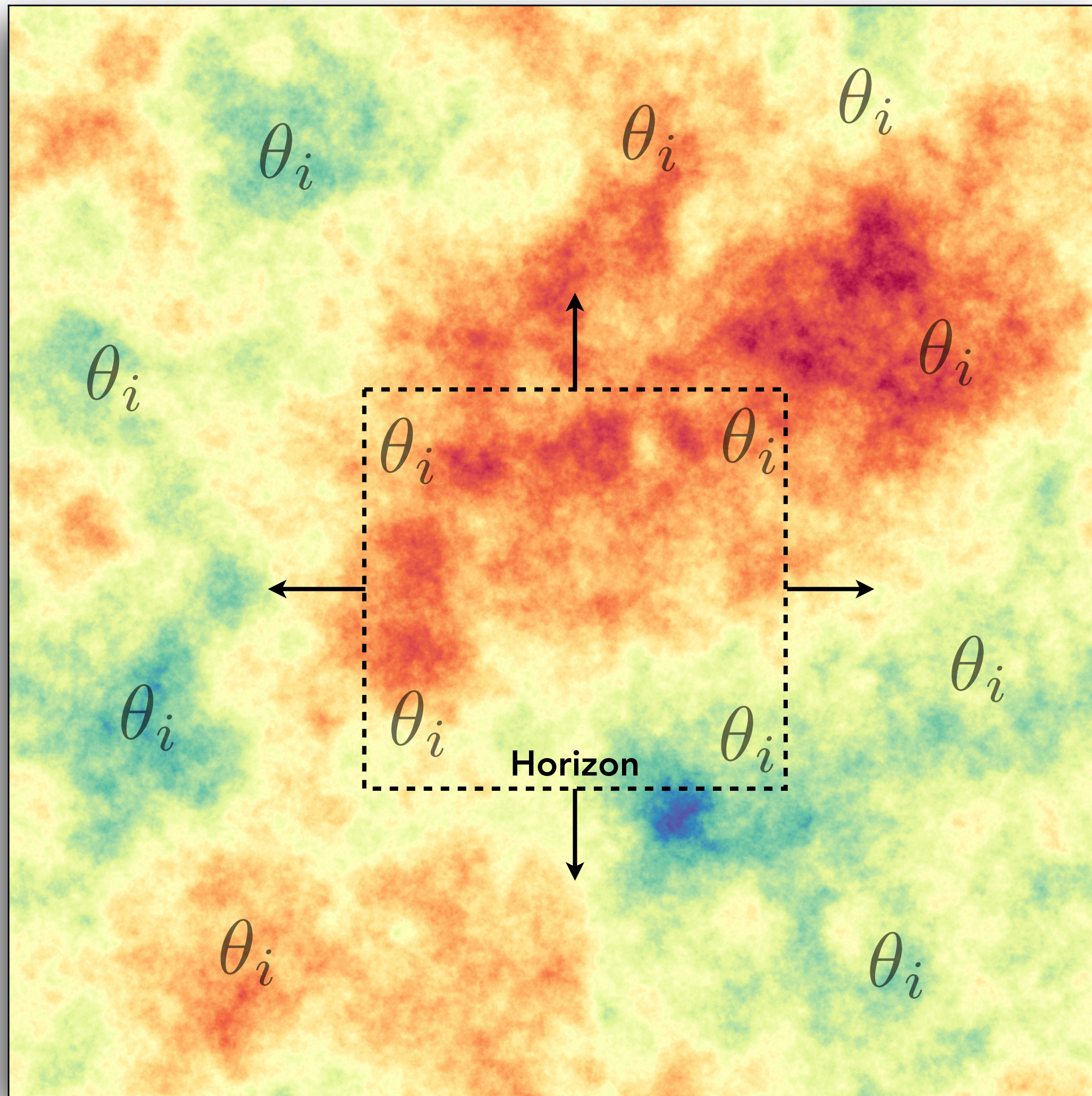
Post-inflationary scenario



Inflation has already happened *before* axion was born

- Universe filled with many values of θ_i
- Different value in every causal patch

Post-inflationary scenario



Inflation has already happened *before* axion was born

- Universe filled with many values of θ_i
- Different value in every causal patch
- Patches come into contact as horizon grows.

Post-inflationary scenario

- We have an ensemble of every possible θ_i sampled across our Universe.
- Stochastic average:

$$\langle \theta_i^2 \rangle \approx \left(\frac{\pi}{\sqrt{3}} \right)^2 \approx (1.81)^2$$

$$\Omega_a h^2 \approx 0.12 \frac{\langle \theta_i^2 \rangle}{(1.81)^2} \left(\frac{20 \mu\text{eV}}{m_a} \right)^{\frac{n+6}{n+4}}$$

Peccei-Quinn scale, f_a [GeV]

10^{19} 10^{18} 10^{17} 10^{16} 10^{15} 10^{14} 10^{13} 10^{12} 10^{11} 10^{10} 10^9 10^8 10^7 10^6

10^{-13} 10^{-12} 10^{-11} 10^{-10} 10^{-9} 10^{-8} 10^{-7} 10^{-6} 10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 10^0 10^1

QCD axion mass, m_a [eV]

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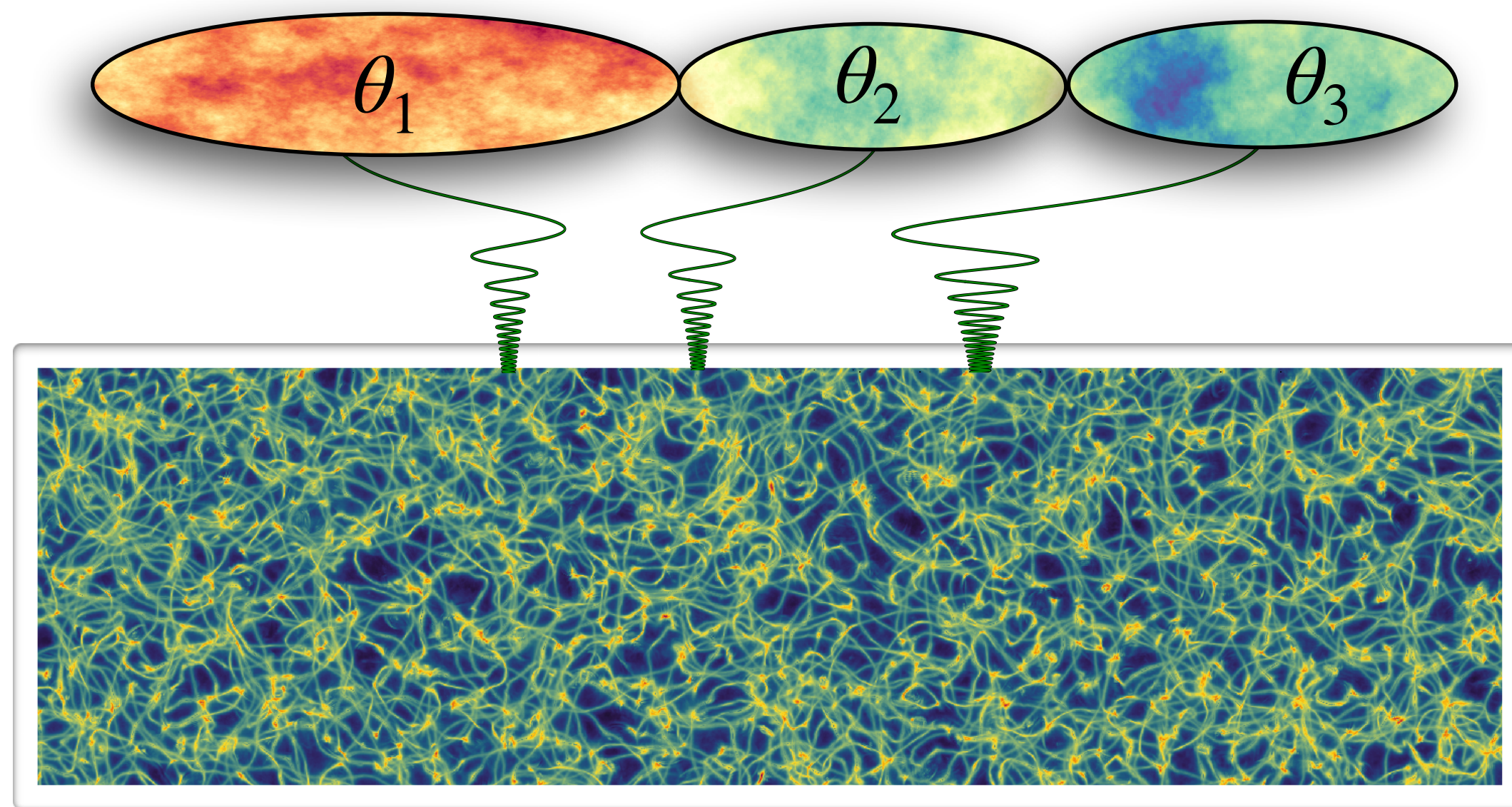
**In the post-inflationary scenario only one mass
is consistent with observed DM abundance**
(Up to theoretical uncertainties)

Overabundant ← → Underabundant

10^{-13} 10^{-12} 10^{-11} 10^{-10} 10^{-9} 10^{-8} 10^{-7} 10^{-6} 10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 10^0 10^1

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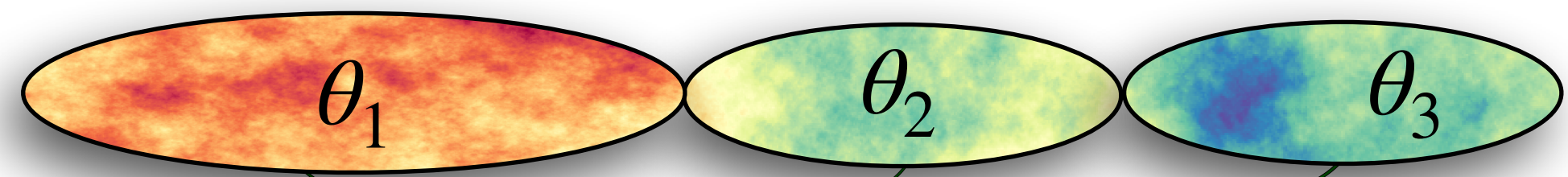
But there's a complication: $\nabla \theta$



Different patches meet up
→ Field gradients!

$$\leftarrow \ddot{\theta} + 3H\dot{\theta} \left[-\frac{1}{a^2} \nabla^2 \theta \right] + m_a^2 \theta = 0$$

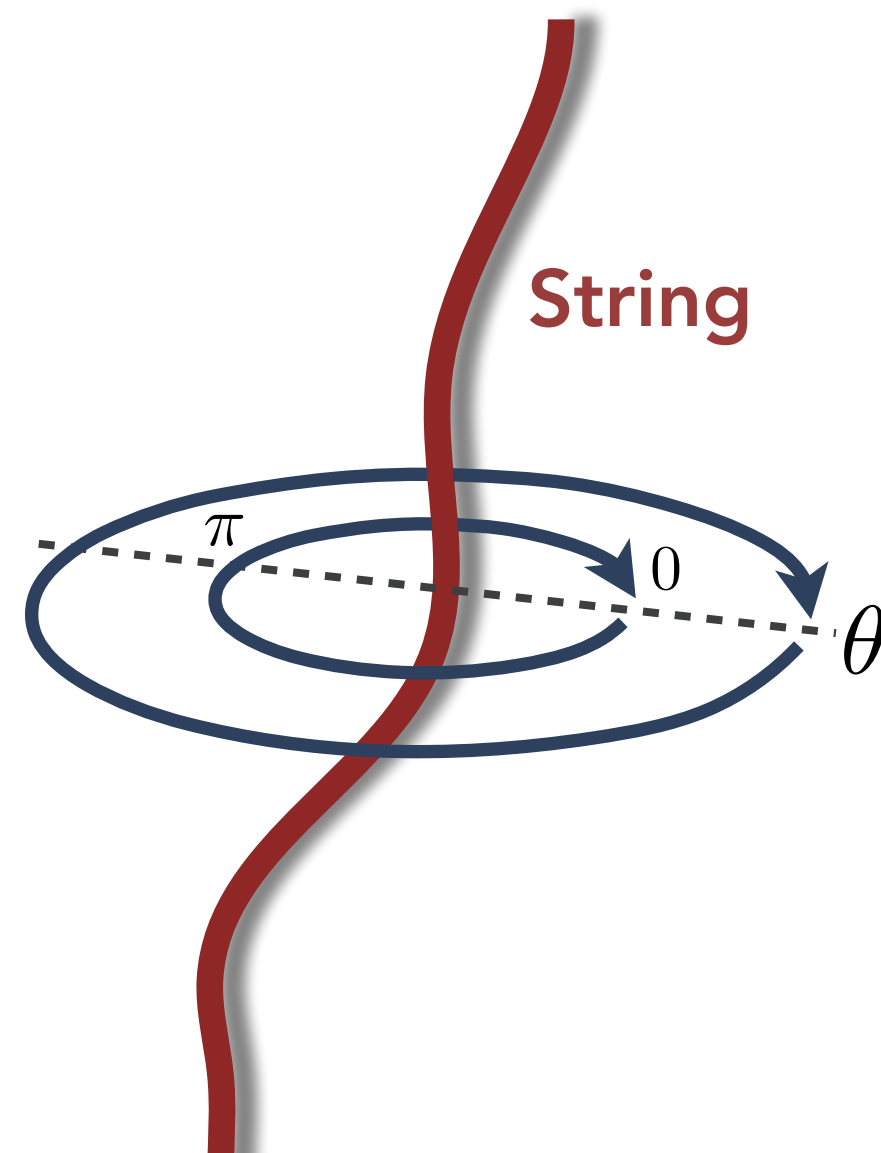
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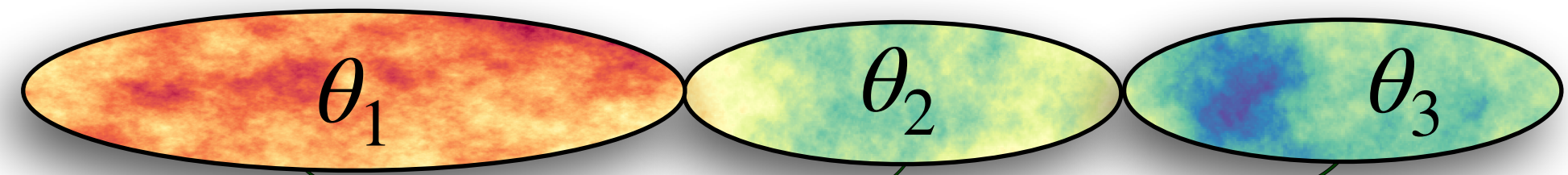
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⇒ Cosmic strings
from axion field
winding around 2π



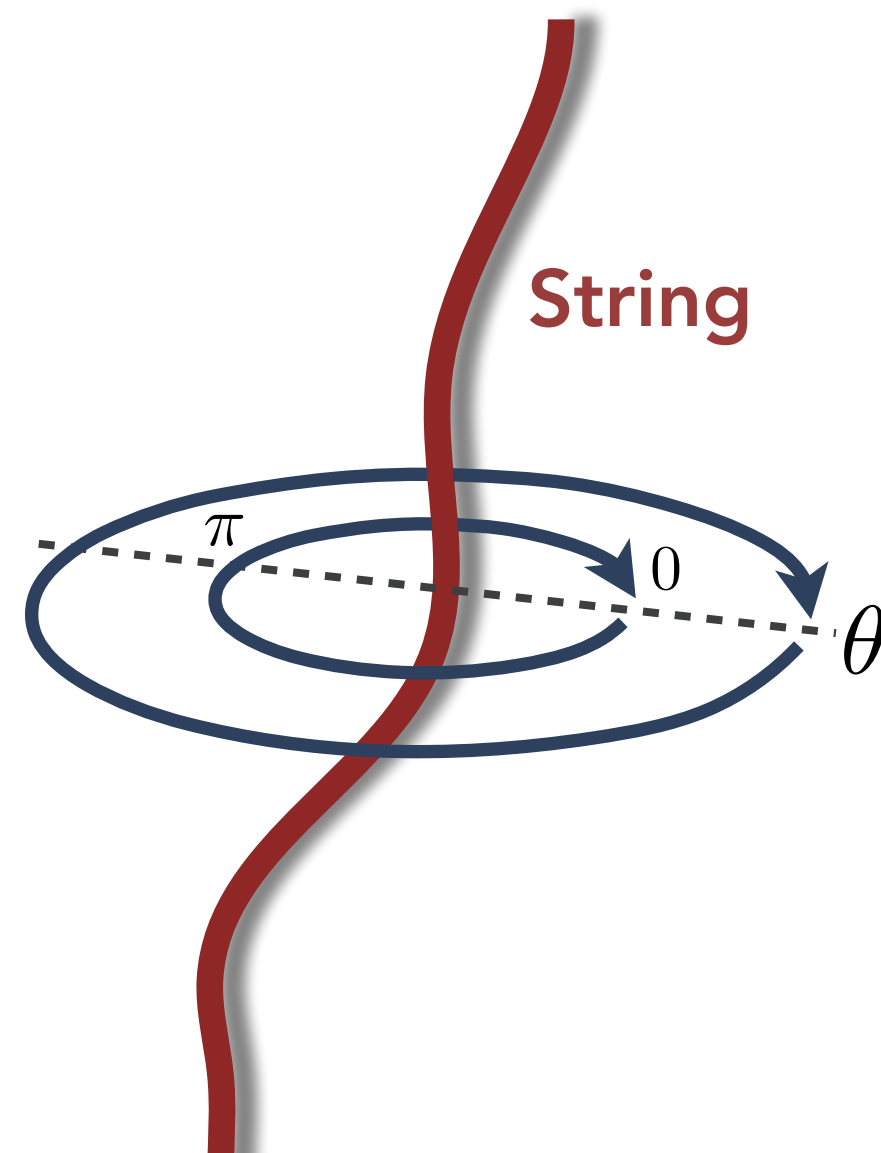
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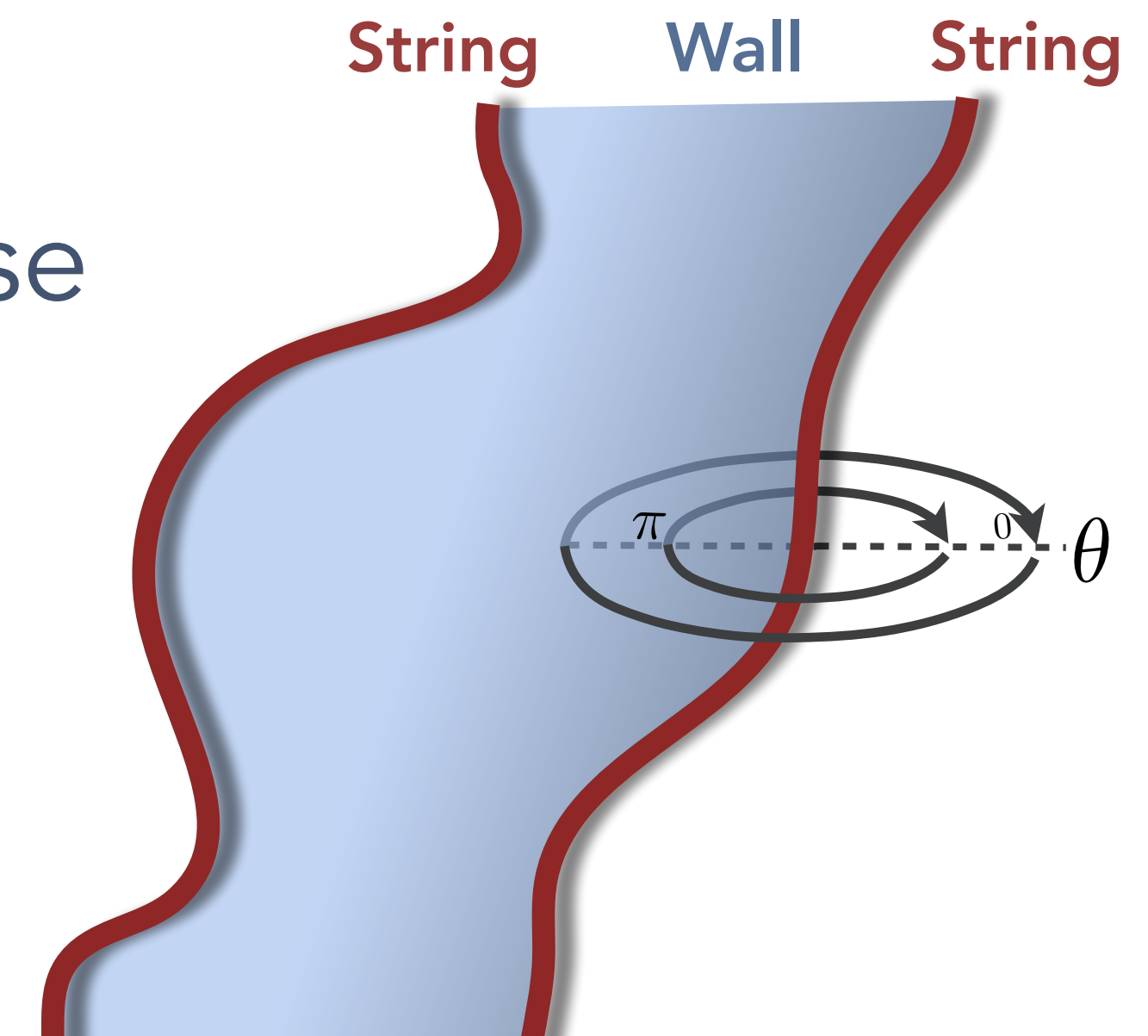
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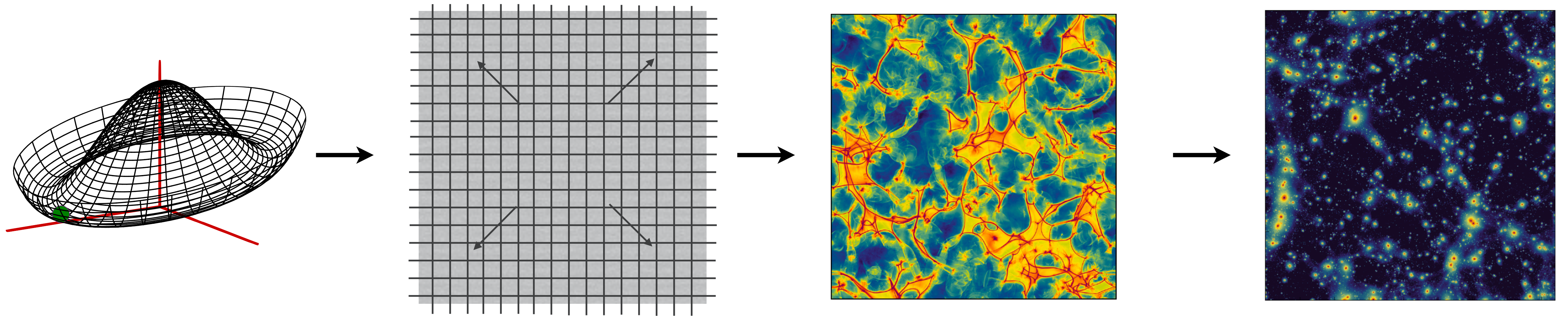
⇒ Cosmic strings
from axion field
winding around 2π



⇒ Domain walls
between true/false
vacuum (0 and π)



Brute force solution: simulate



Evolve the
axion field...

...on an
expanding
lattice...

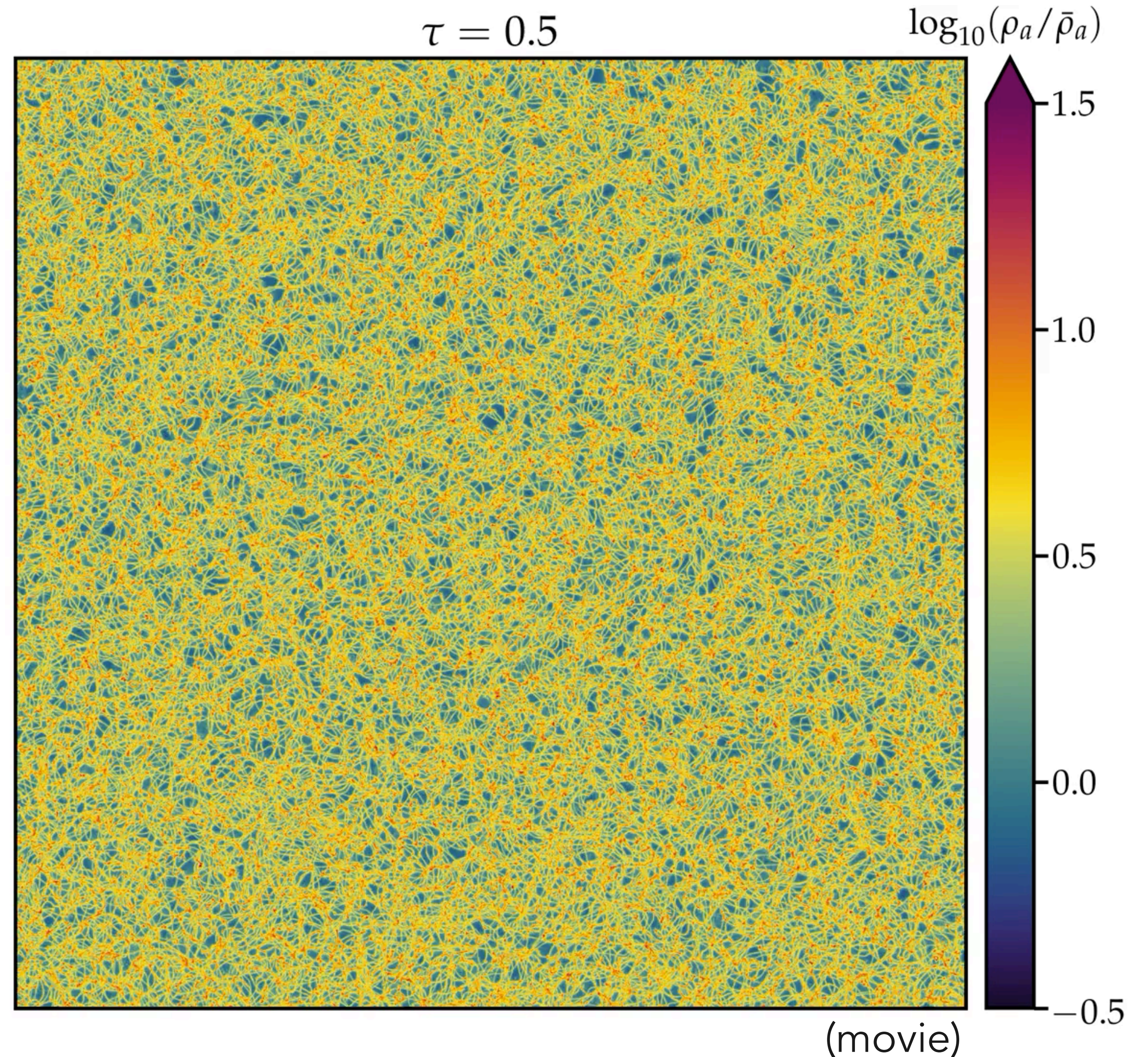
...to measure the
relic abundance
of axions...

...and predict its
present day
distribution

Evolution of the axion field in the post-inflationary scenario

Projection through 3D co-moving box, coloured by integrated axion energy density:

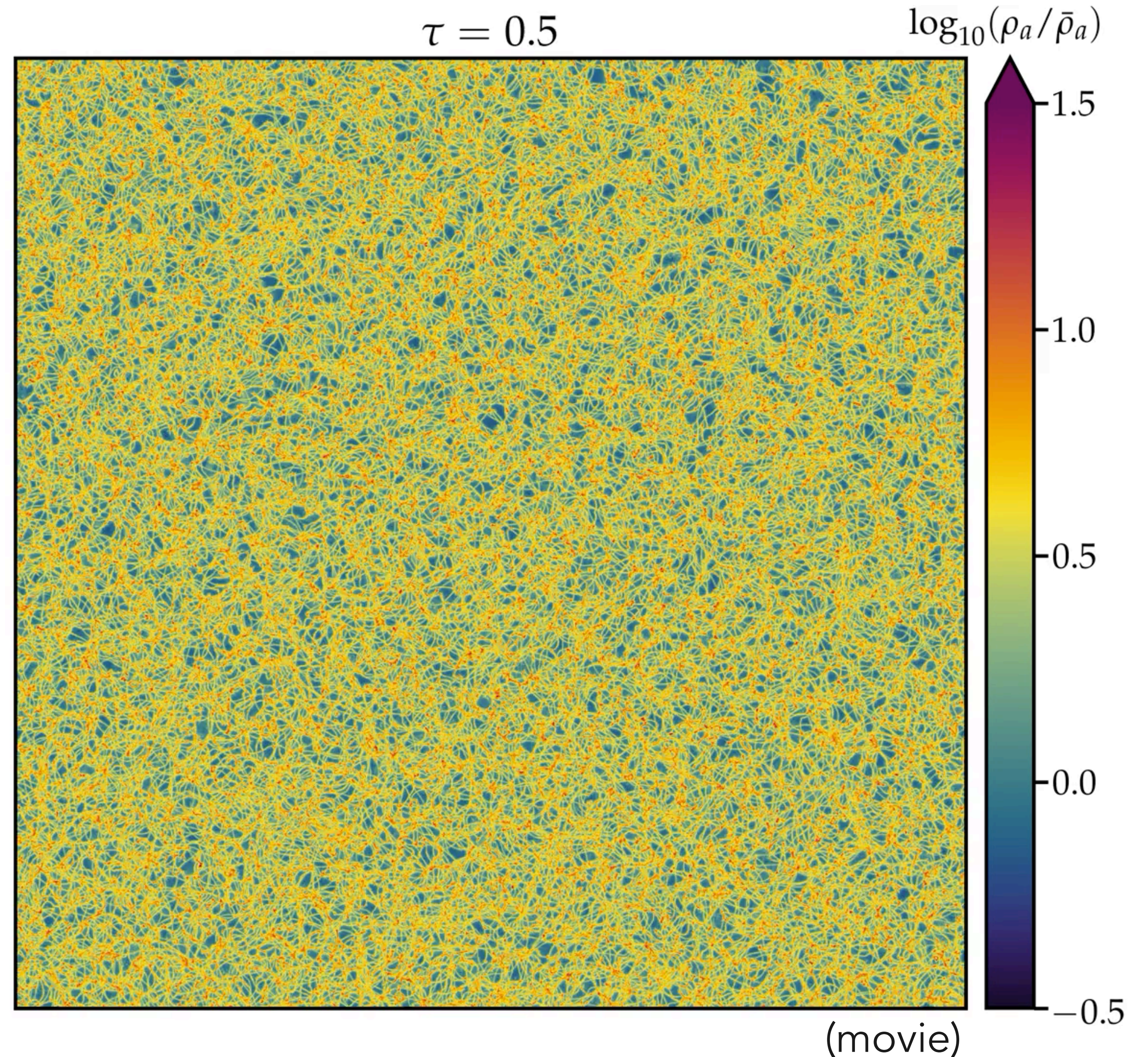
$$\rho_a = \frac{1}{2}\dot{a}^2 + \frac{1}{2R^2}(\nabla a)^2 + \chi(1 - \cos a/f_a)$$



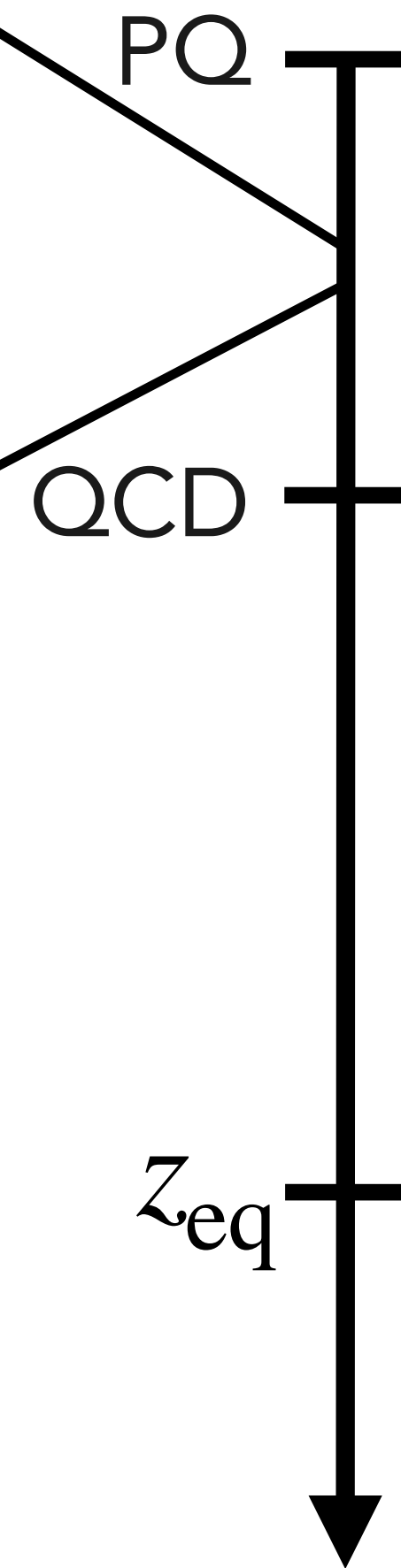
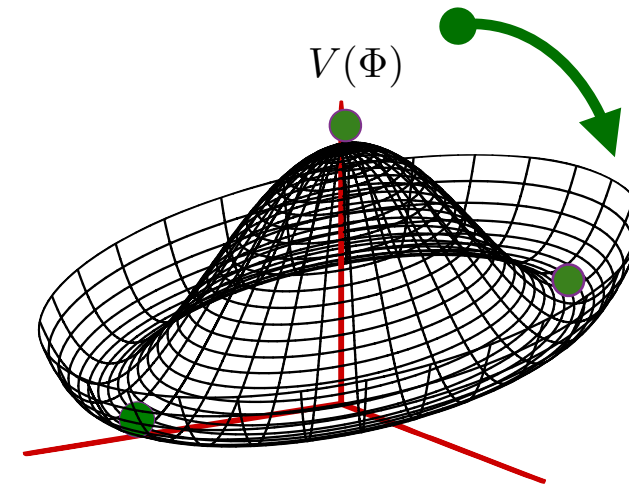
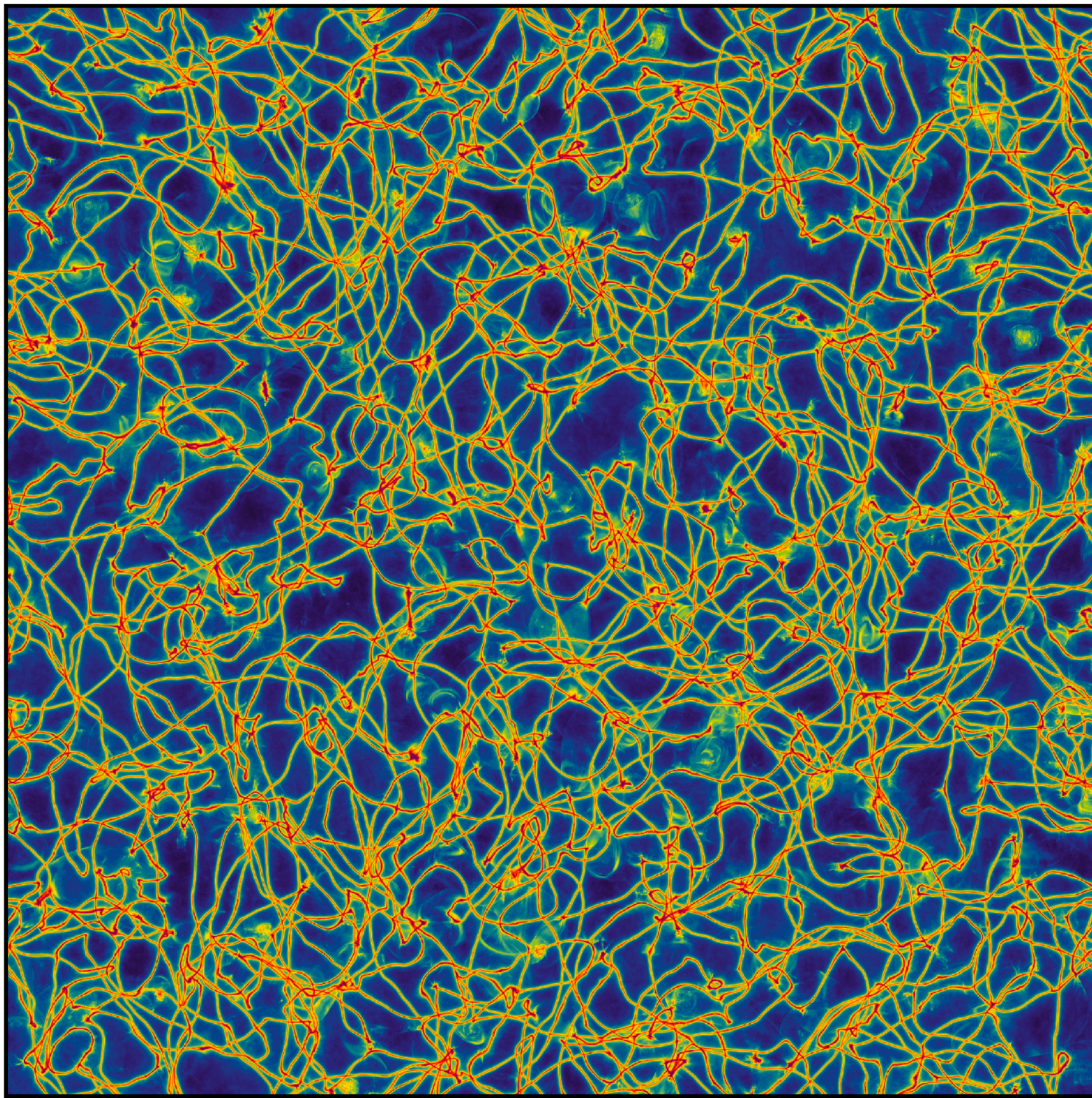
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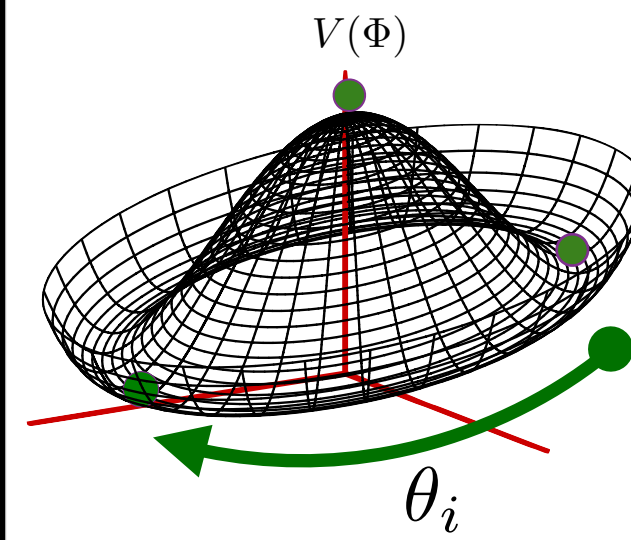
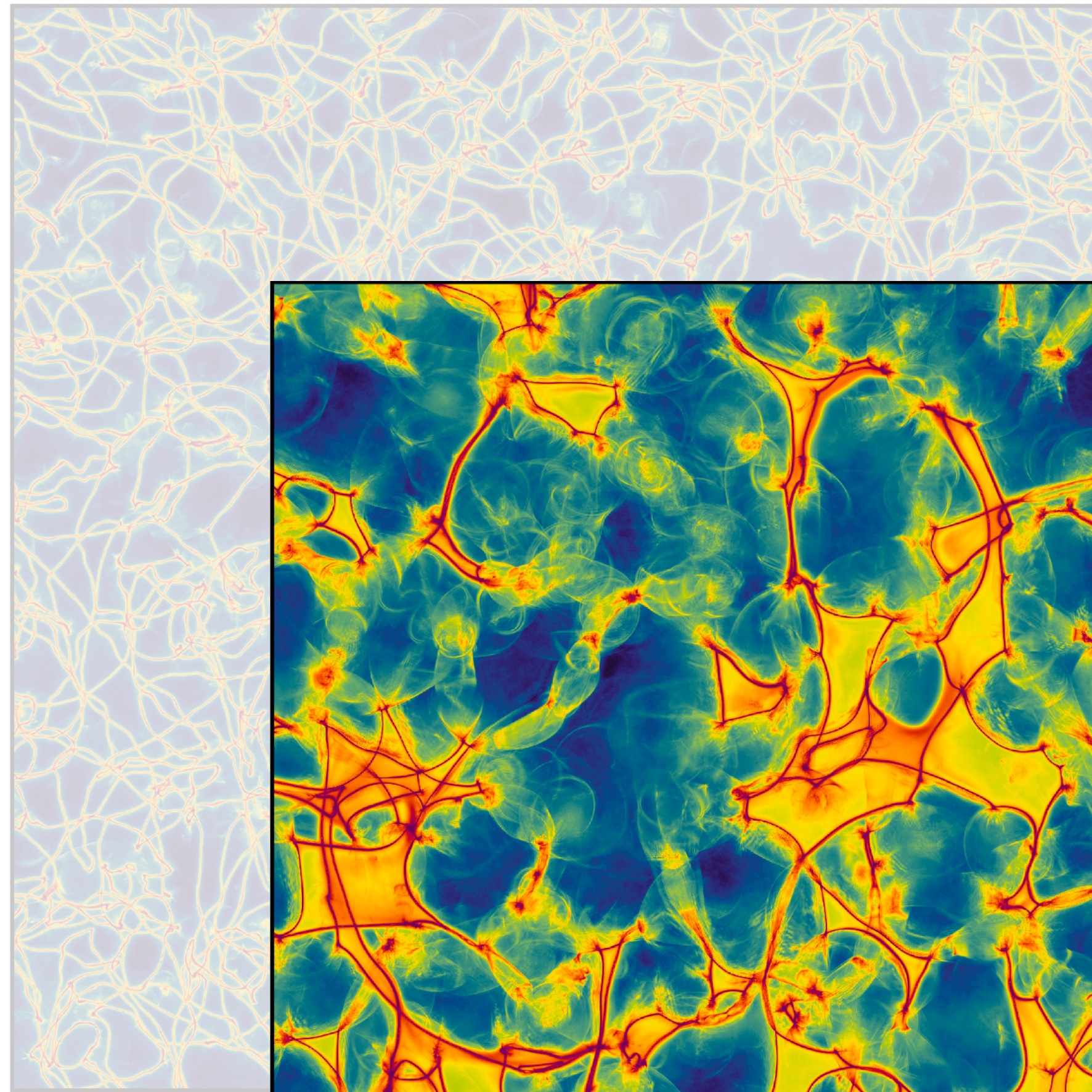


Evolution of the axion field in the post-inflationary scenario



String network scaling

Evolution of the axion field in the post-inflationary scenario



PQ

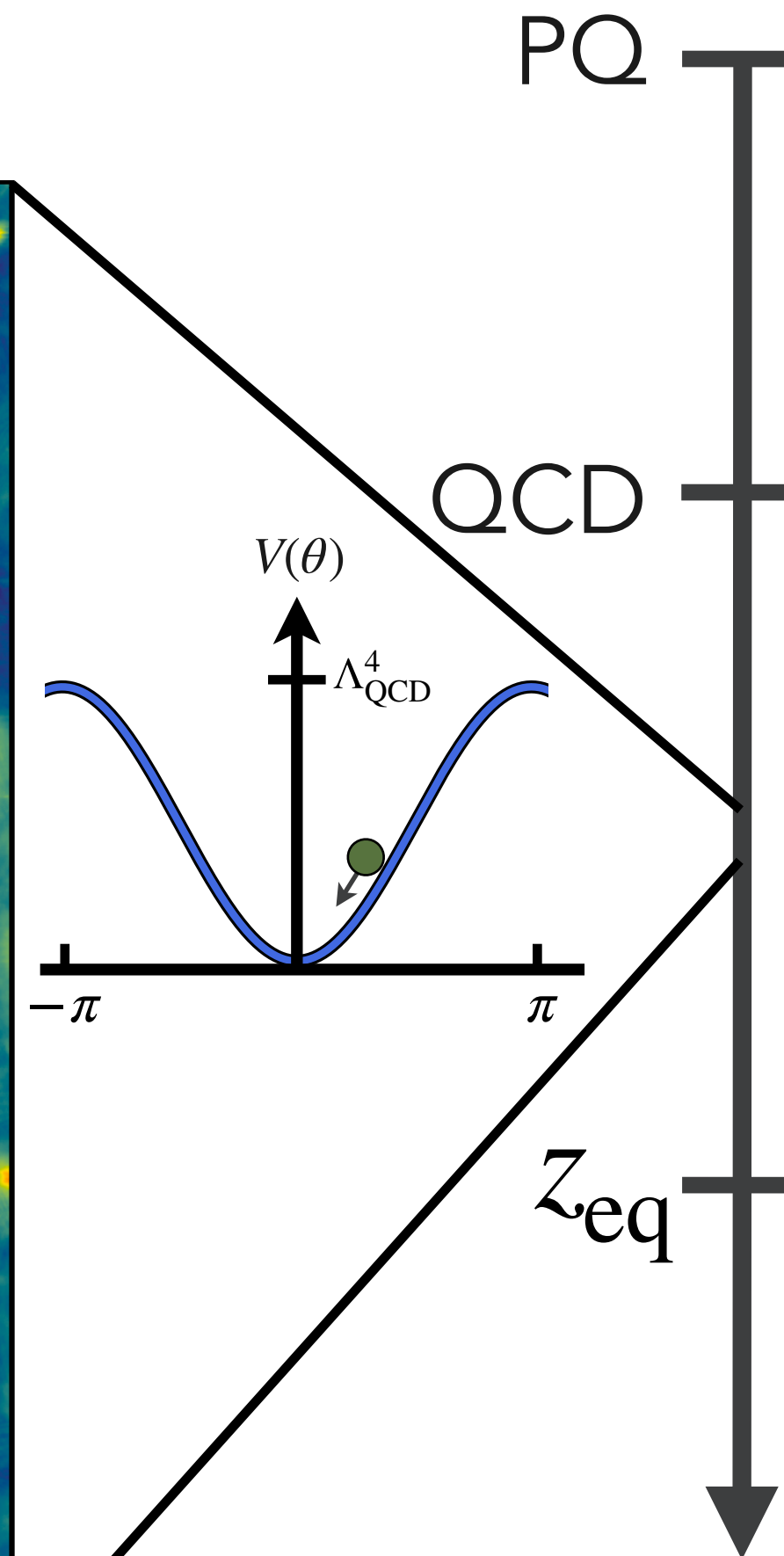
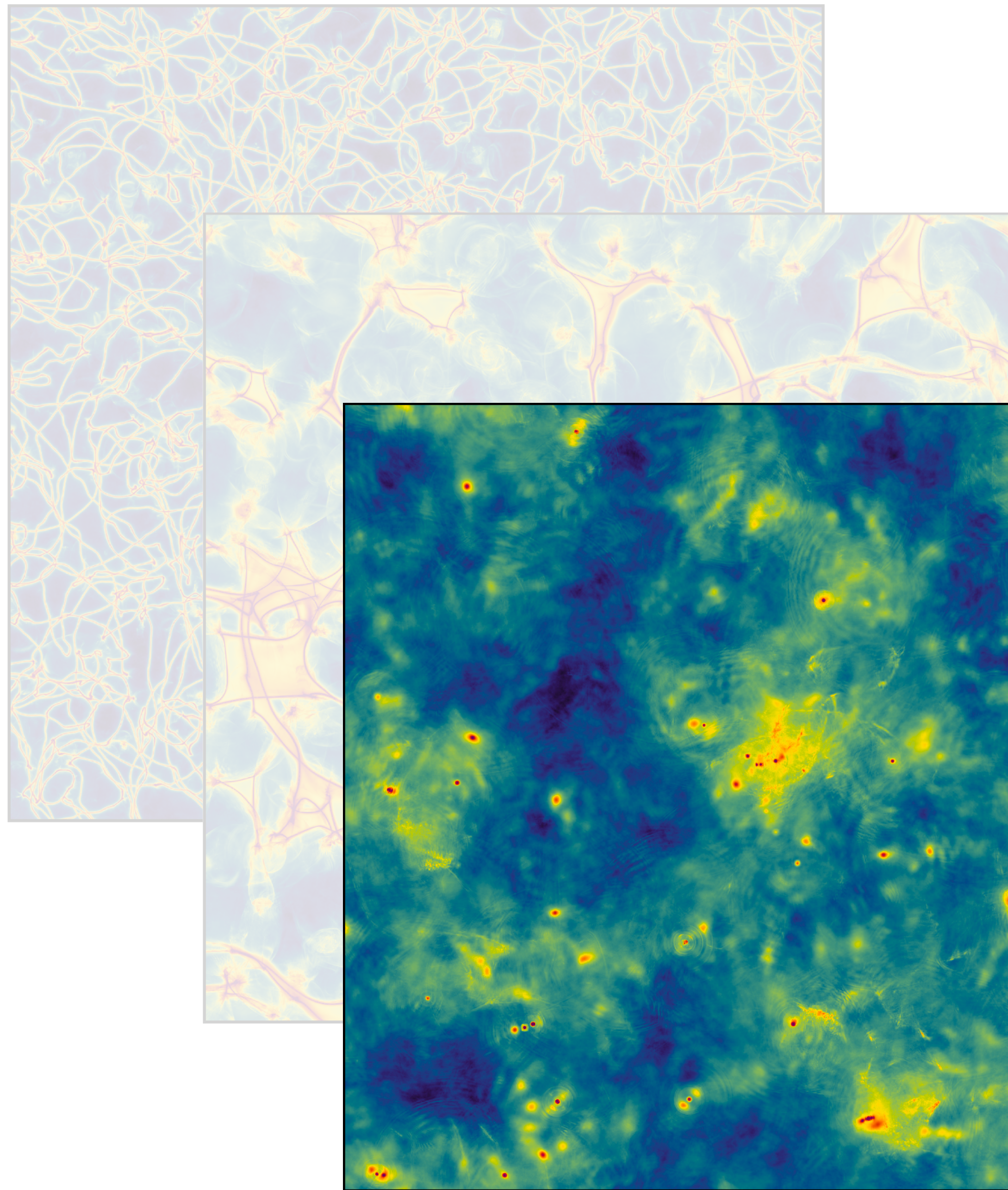
QCD

z_{eq}

String network scaling

Domain walls attached to strings
→ network collapses

Evolution of the axion field in the post-inflationary scenario



String network scaling

Domain walls attached to strings
→ network collapses

Inhomogeneous distribution of
axions free streams until non-
relativistic

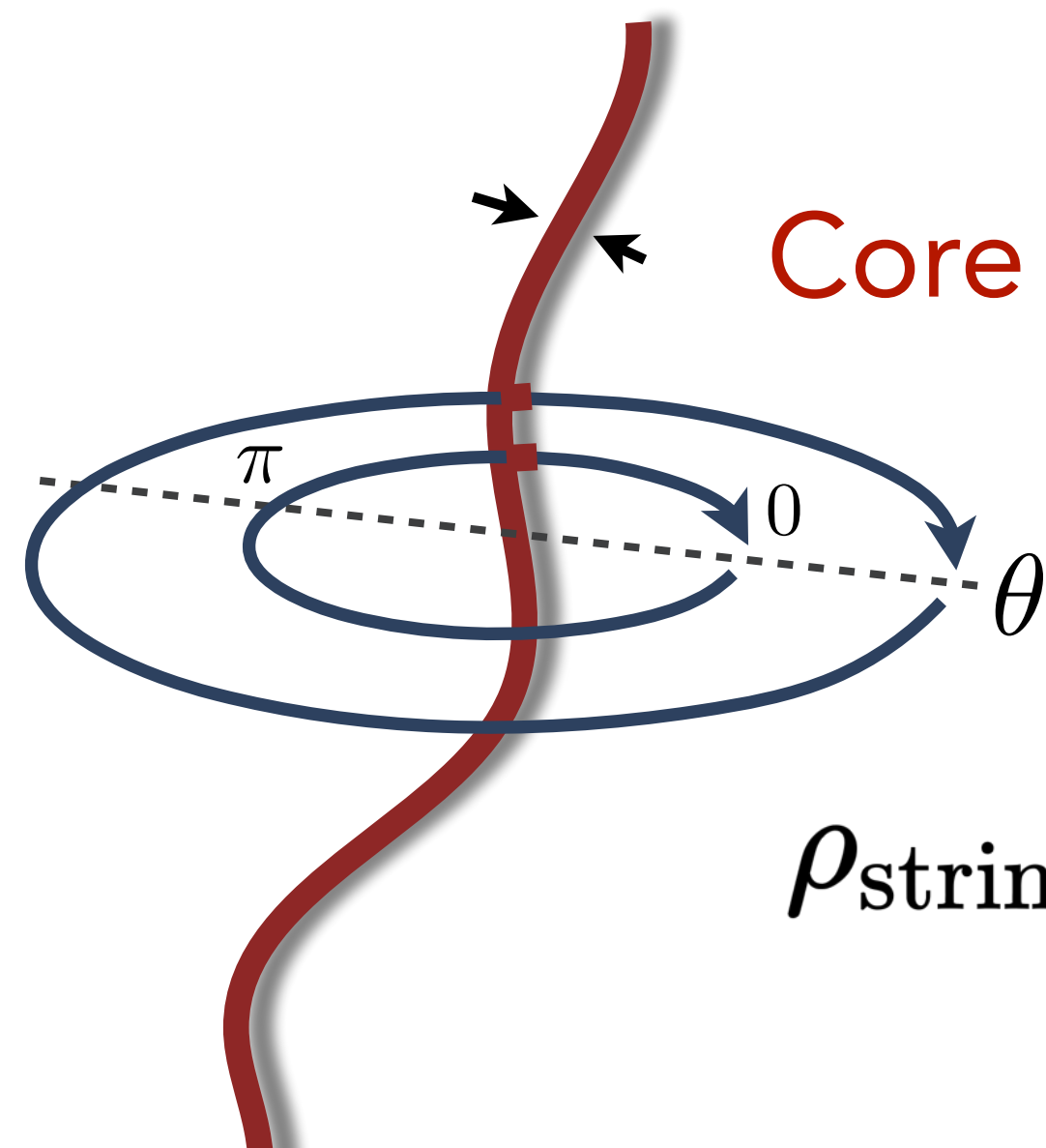
Seeds of structure
gravitationally collapse
into miniclusters and halos

Axion string network

M. Gorghetto

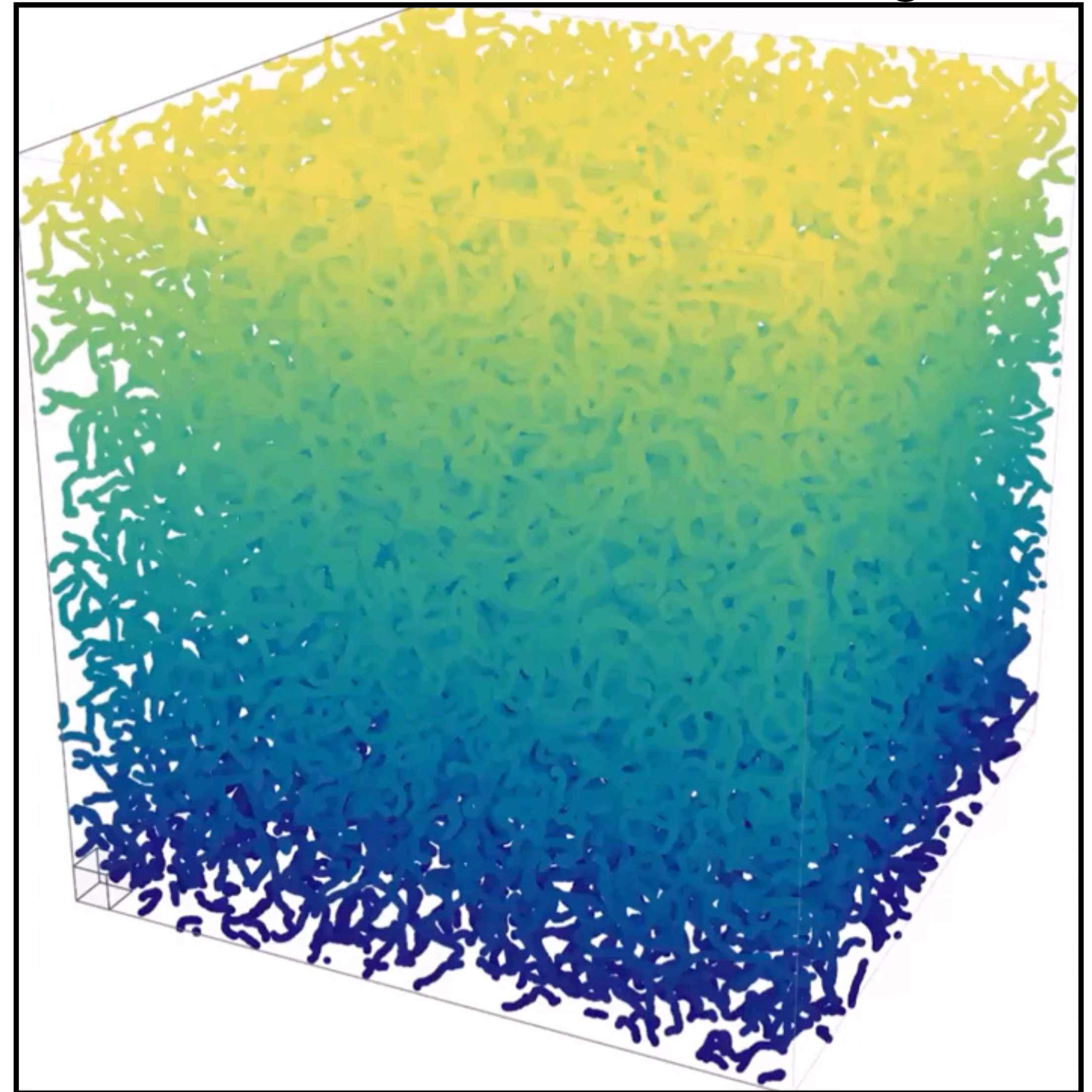
Network is expected to enter a “scaling solution” where strings straighten and loops collapse at roughly the same rate as the horizon grows

→ Number of strings per Hubble volume, $\xi(t) = \text{const.} + \text{log-violations}$



Core size set by $m_r \sim \sqrt{\lambda} f_a$

$$\rho_{\text{strings}} \sim \xi \times f_a^2 H^2 \log \frac{m_r}{H}$$

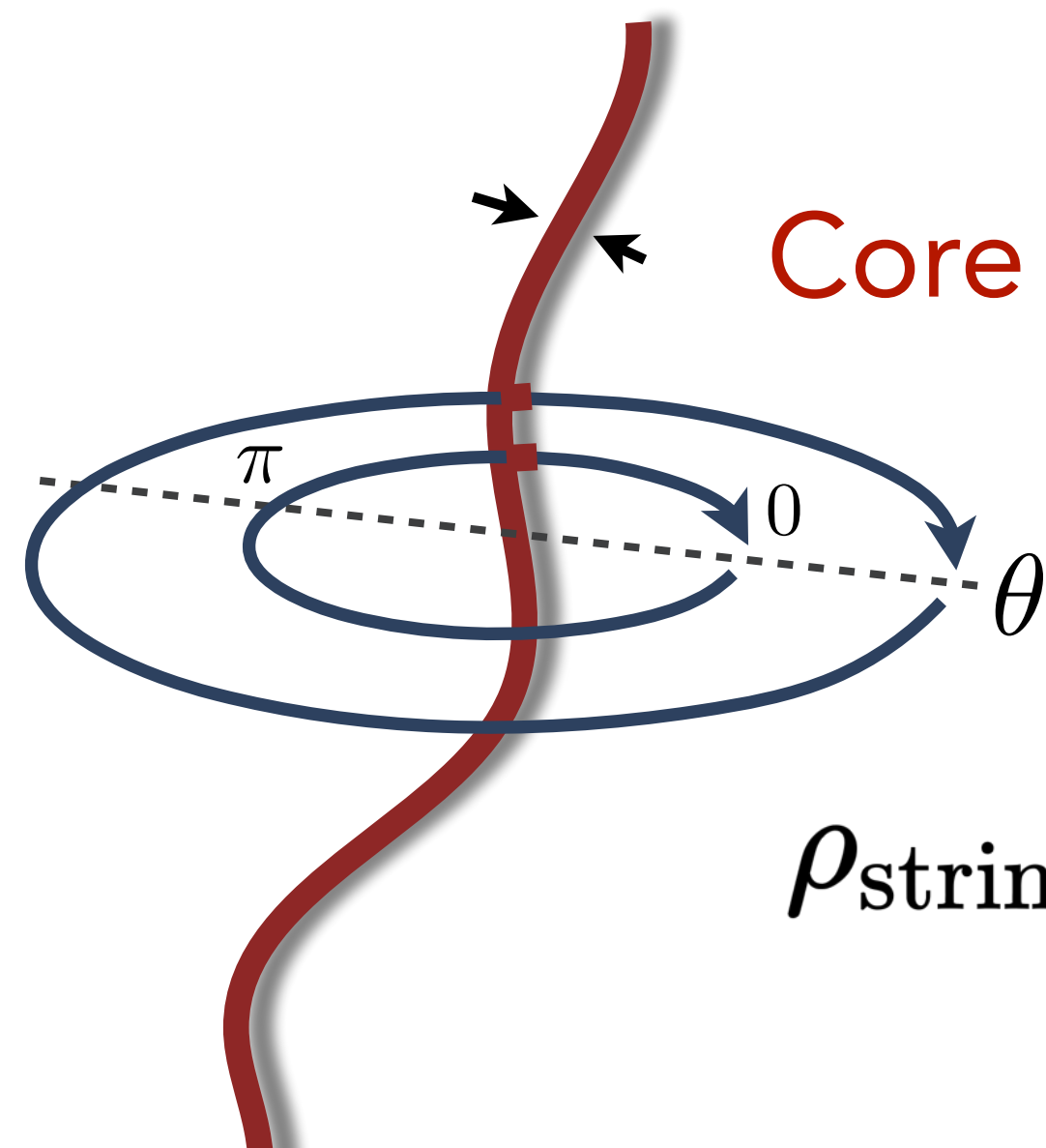


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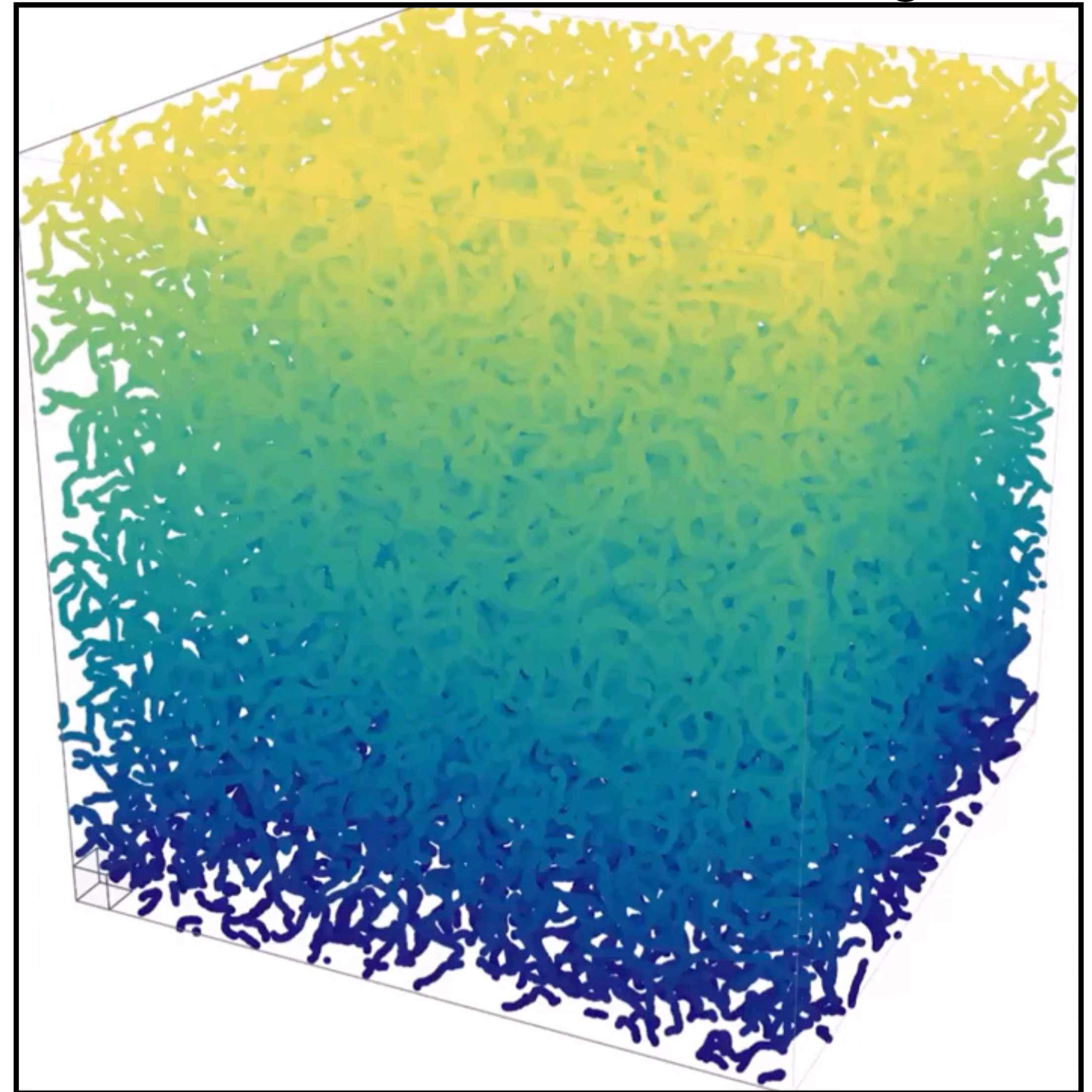
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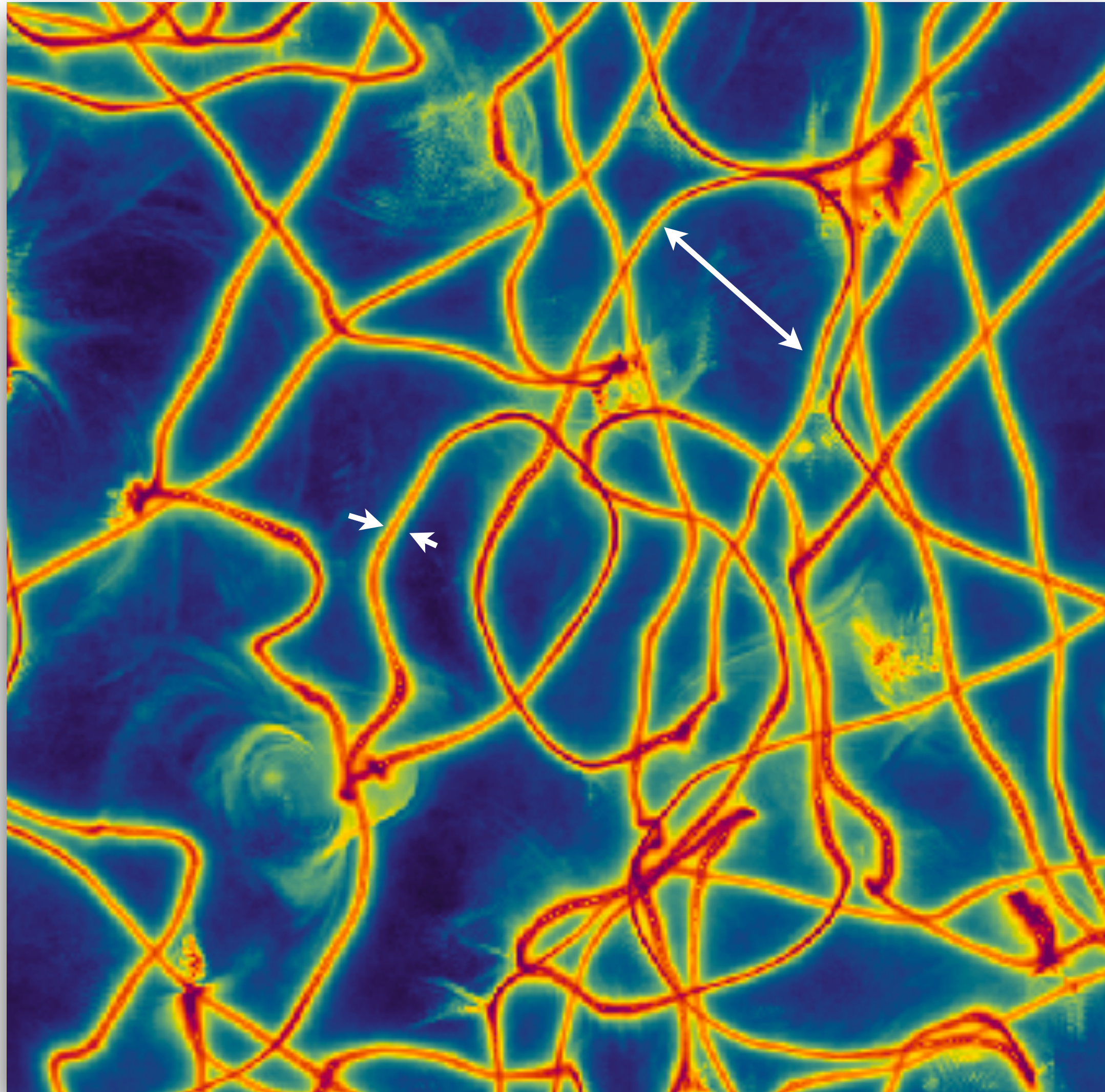


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Axion string simulation limitations



Required dynamical range:

Largest scale = Inter-string spacing $\sim H(t)^{-1}$

Smallest scale = string core size = $m_r^{-1} \sim f_a^{-1}$

- Dynamical range is a function of time, so m_r/H gets larger (strings shrink in co-moving coords)
- For physical QCD axion models need to get to $\log(m_r/H) \rightarrow \sim 70$
- With $\sim 8000^3$ lattice sites can do up to $\log(m_r/H) \sim 6-9$

Axion string radiation uncertainties

Currently not fully understood what will be the ultimate spectrum of axions radiated from strings

UV dominated ($q < 1$)

→ Most radiation comes from string cusps and loops (fewer axions)

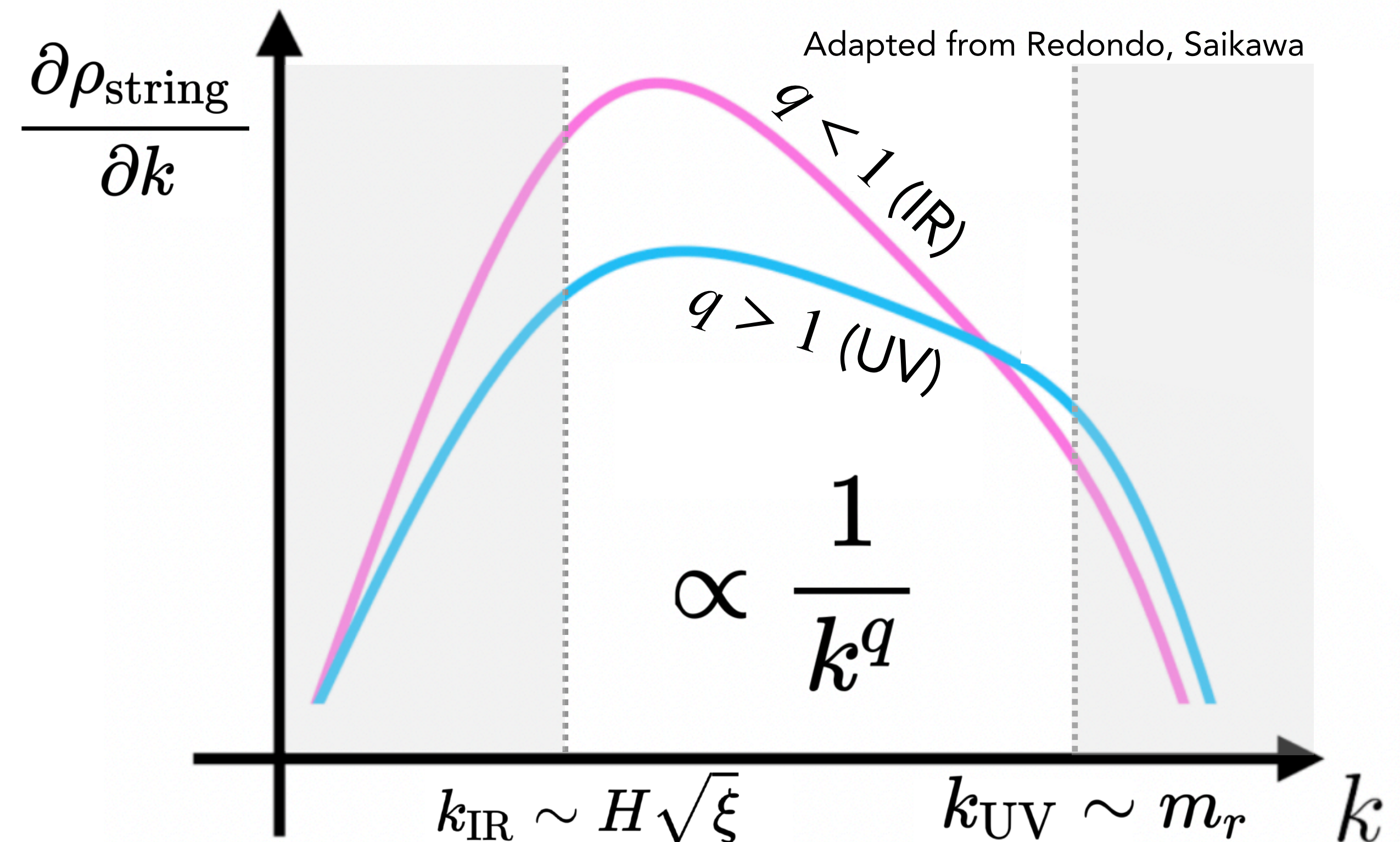
IR dominated ($q > 1$)

→ More radiation comes from straightening of long strings (more axions)

Scale invariant ($q=1$)? e.g. [2108.05368]

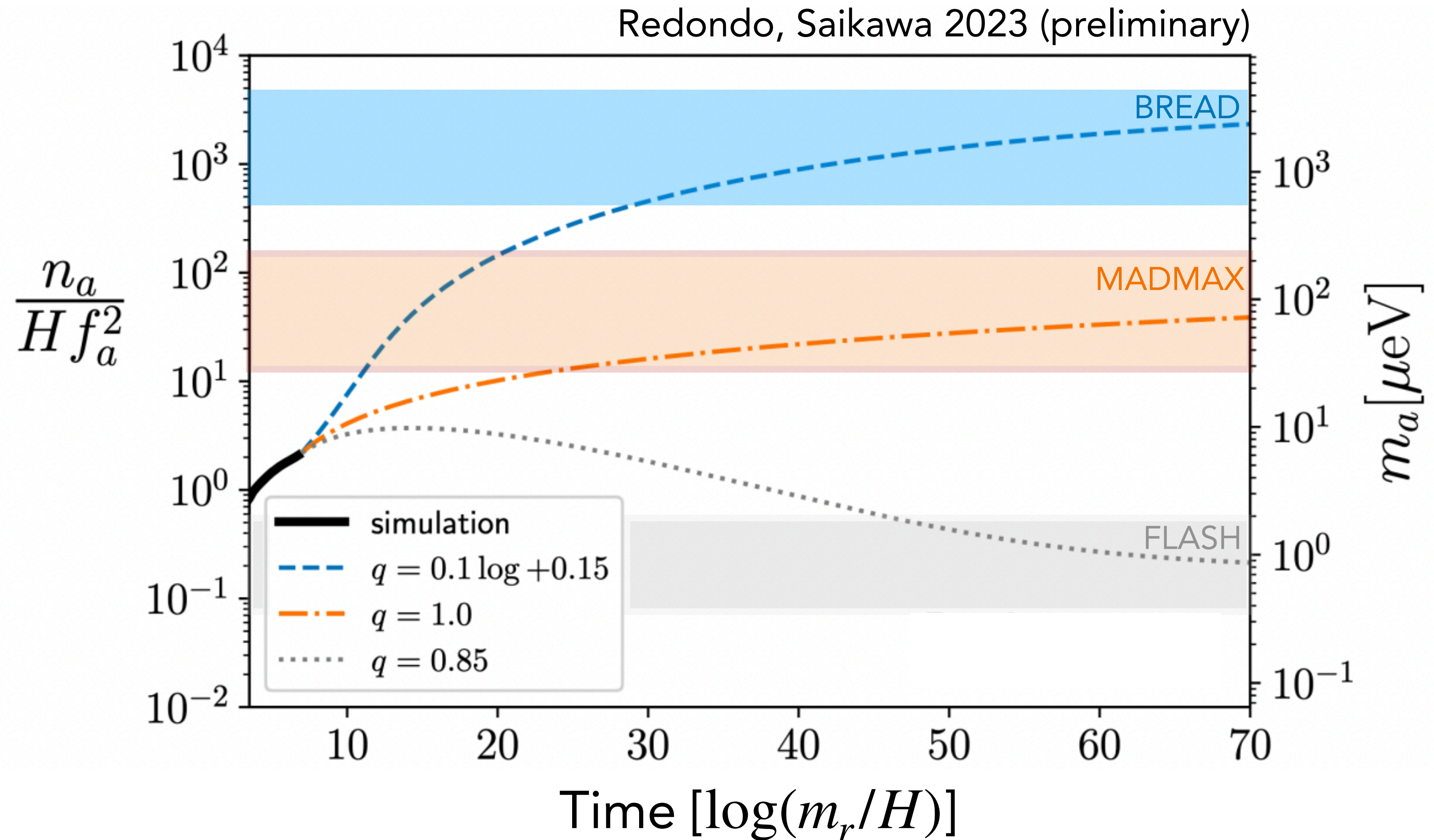
→ Equal axions from each decade in k

$$\text{Num. density of axions when they become DM} = \int_{k_{\text{IR}}}^{k_{\text{UV}}} \frac{dk}{k} \frac{\partial \rho_{\text{string}}}{\partial k}$$



Axion string radiation uncertainties

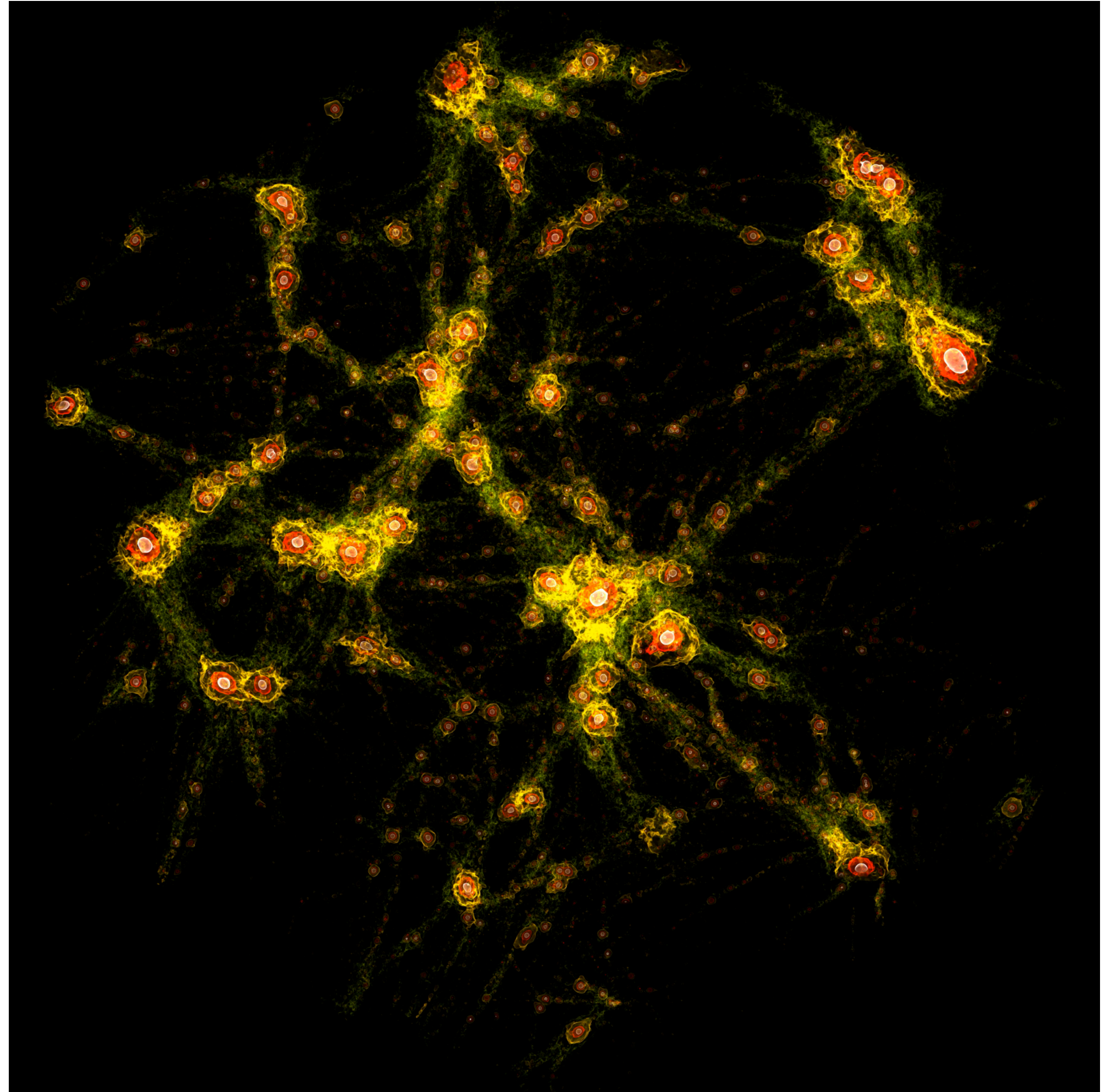
Extrapolating beyond the end of the simulations could be treacherous and has large consequences for axion mass prediction



After t_{QCD} axion field forms
quasi-stable solitons that lay
down small-scale perturbations

These eventually form AU—mpc
gravitationally bound clumps of
axions with masses
 $M \in [10^{-15}, 10^{-9}] M_{\odot}$

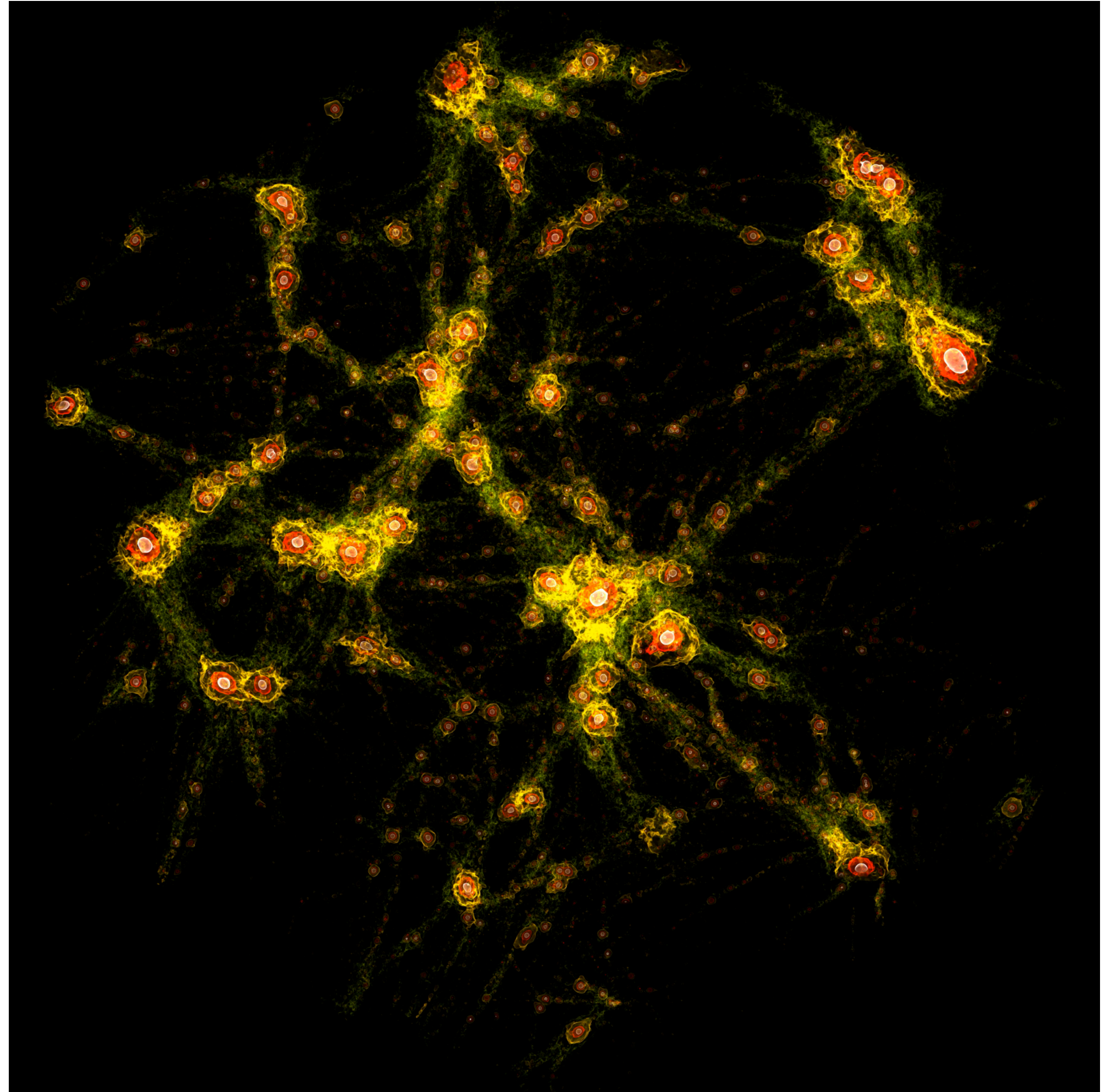
→ **axion miniclusters**



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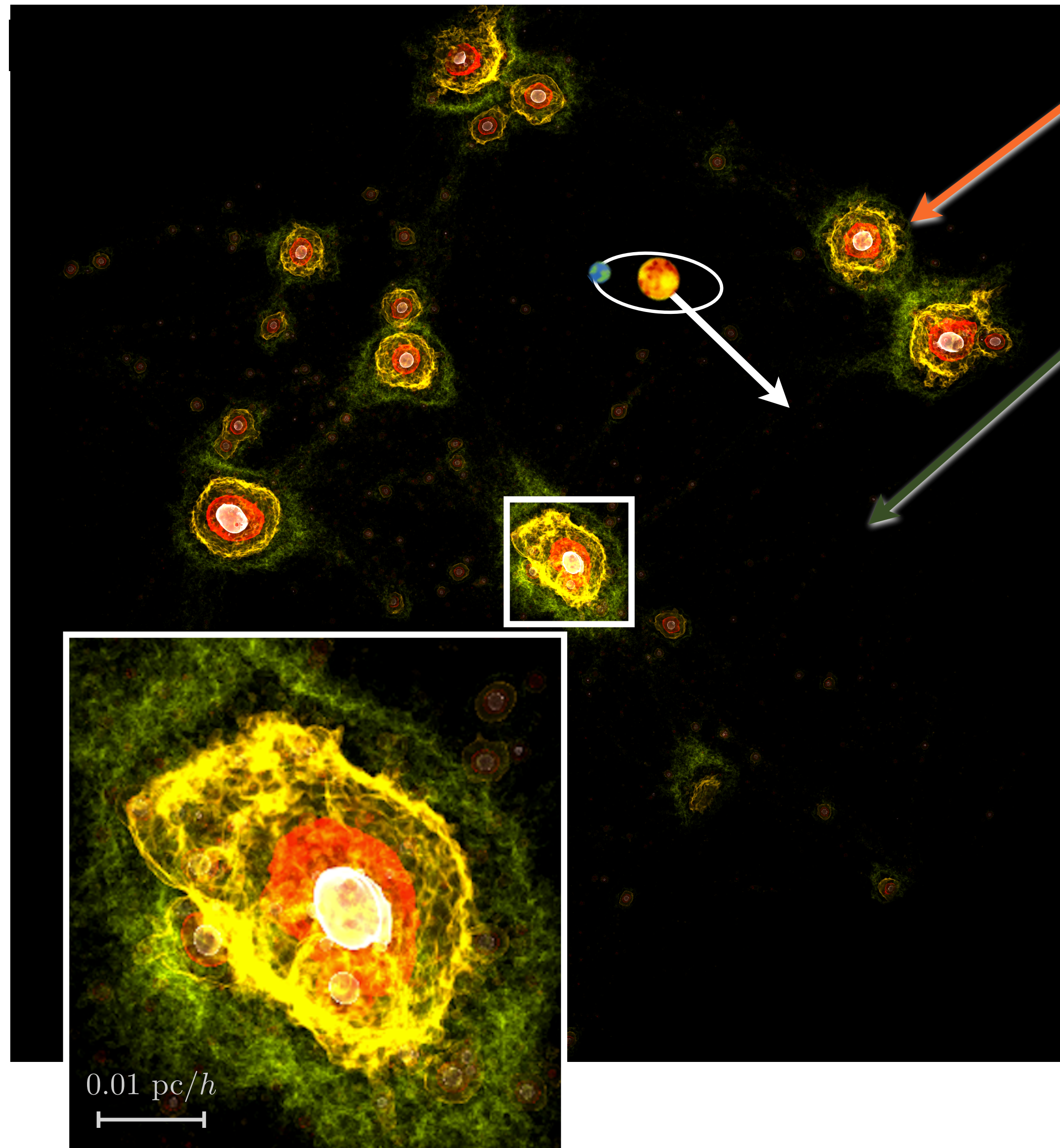


Miniclusters

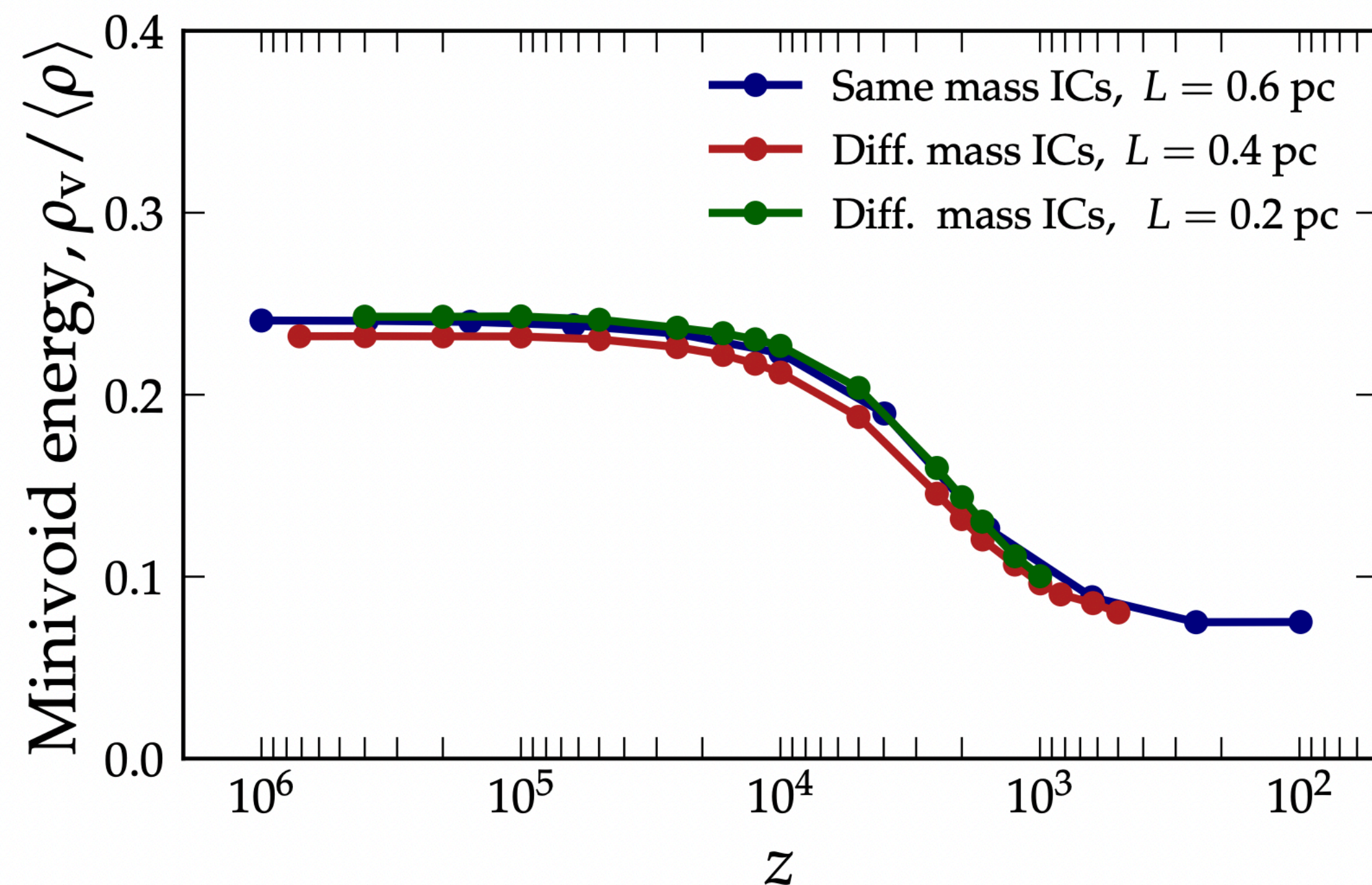
Minivoids

Miniclusters contain $>80\%$ of the axions but make up $<1\%$ of the volume

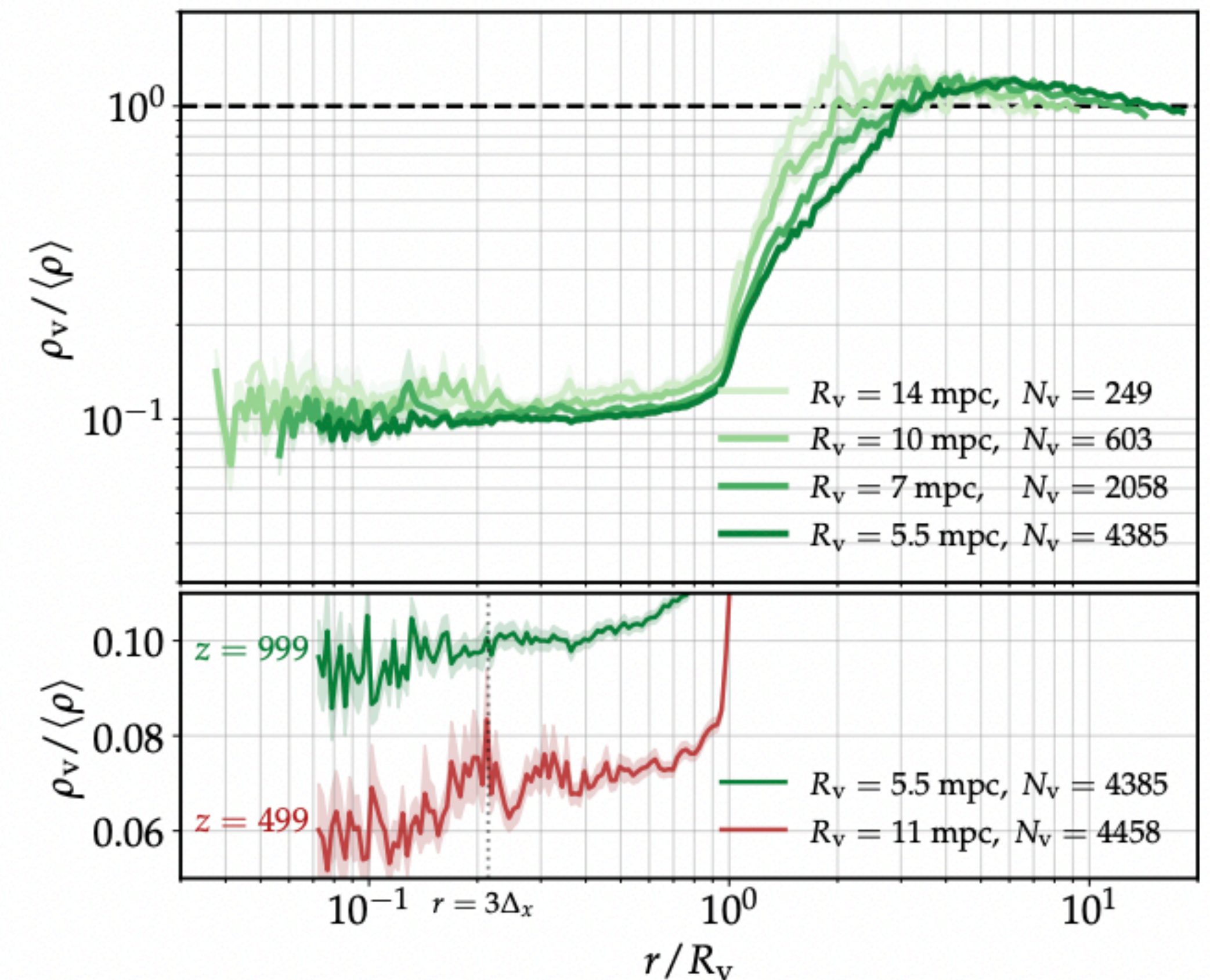
Earth travels through galaxy at about 0.2 mpc per year, so experiments are much more likely to sample the *minivoids* than the *miniclusters*



Minivoids are mostly stable by final simulation time ($z \sim 100$)



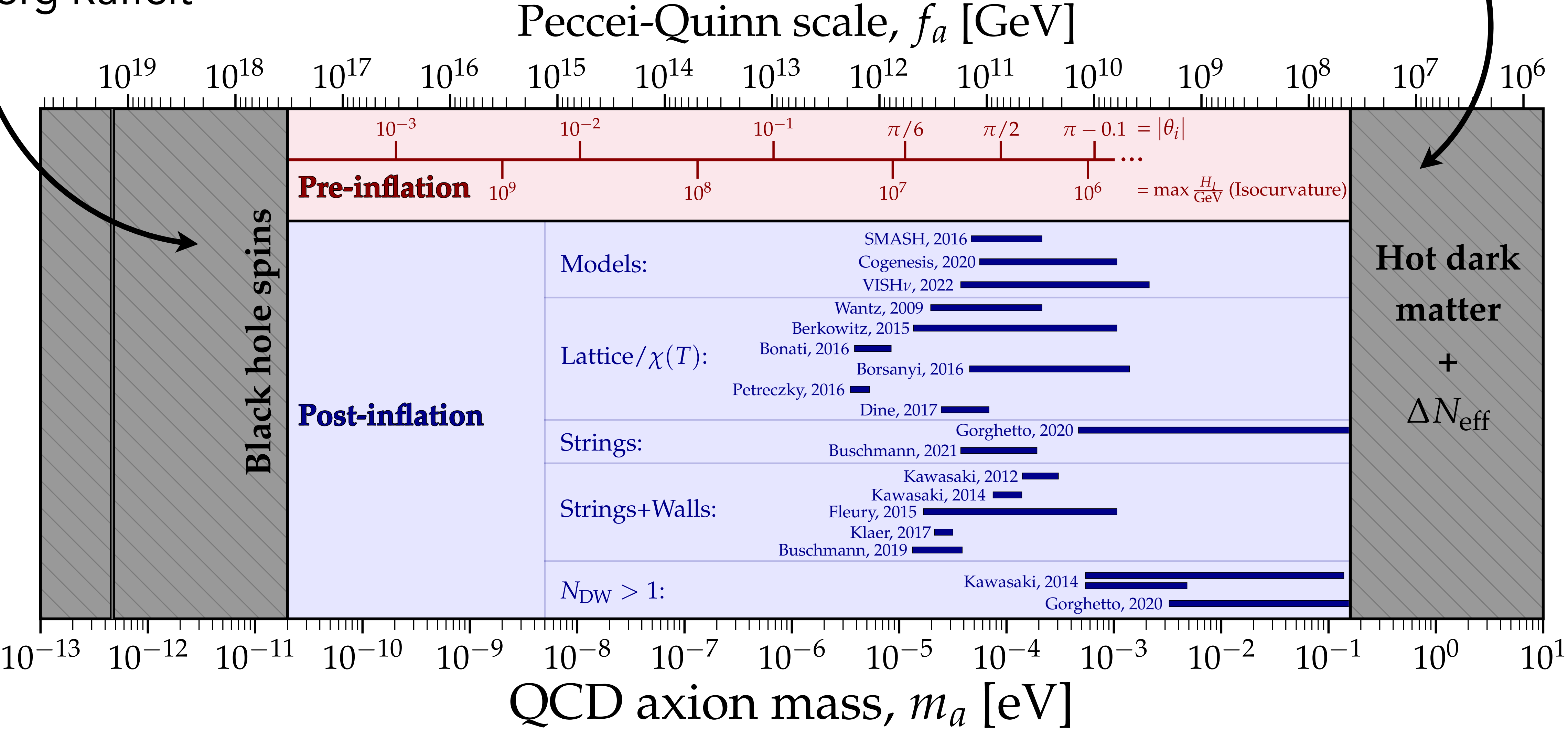
Typical “worst case scenario” density would be inside the minivoids
 $\sim 10\%$ of large-scale average density

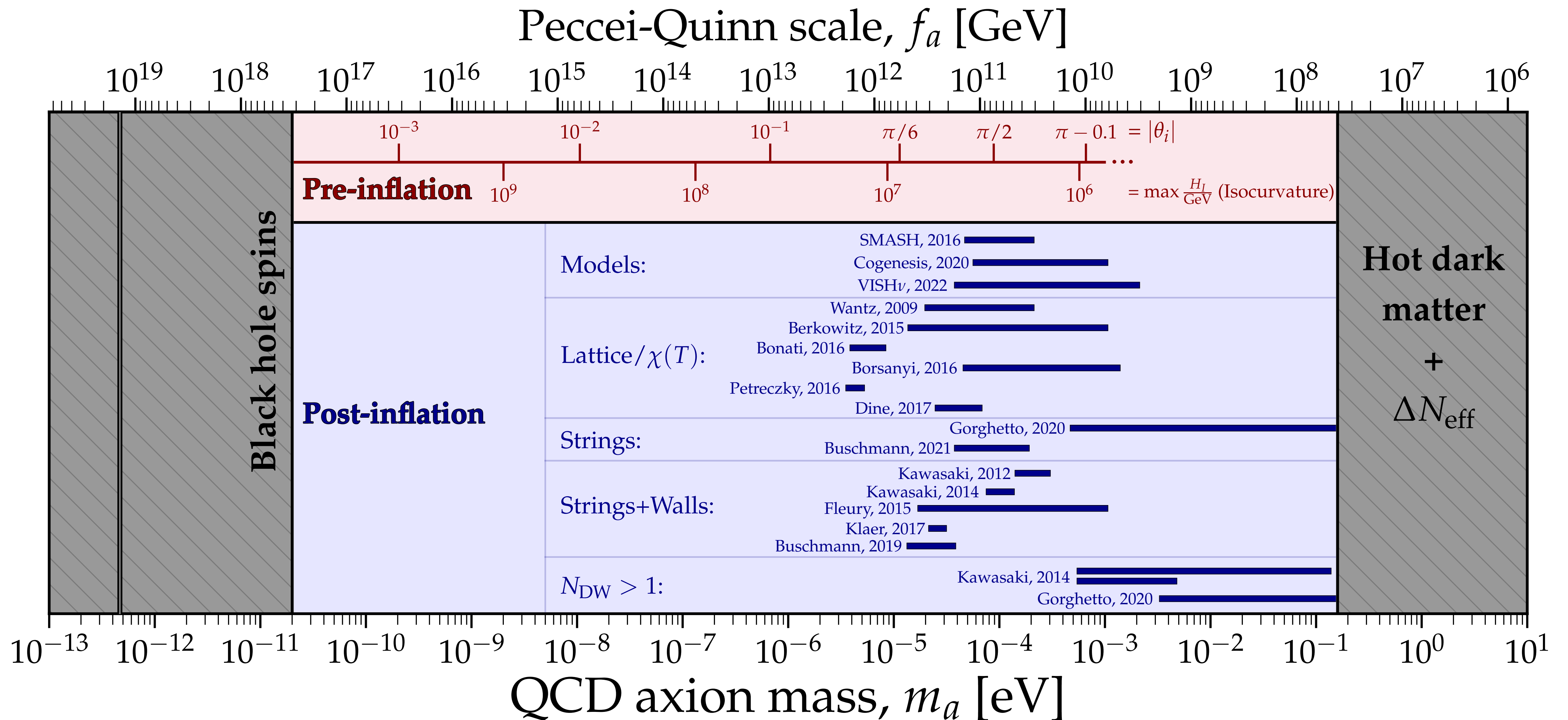


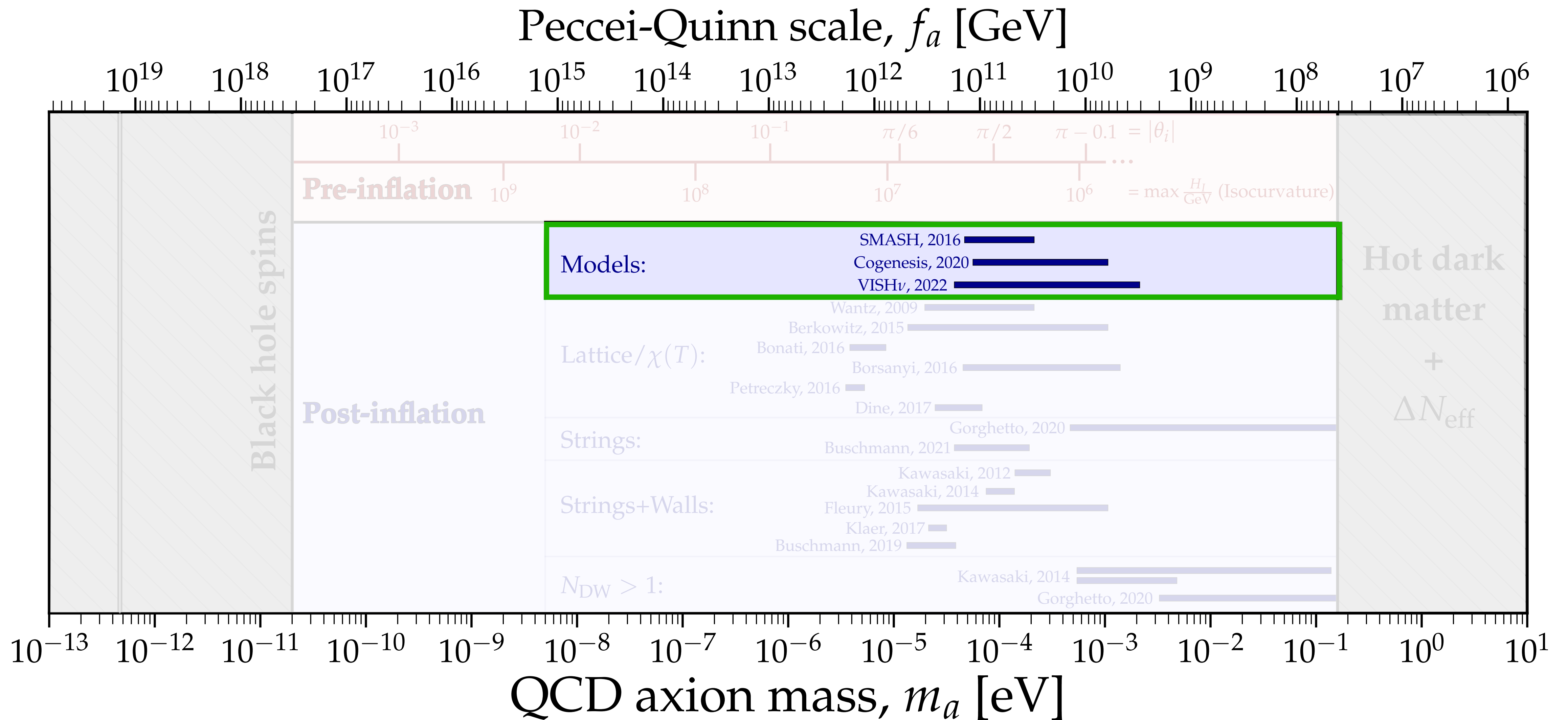
Summary of axion mass post-dictions

See lecture by
Georg Raffelt

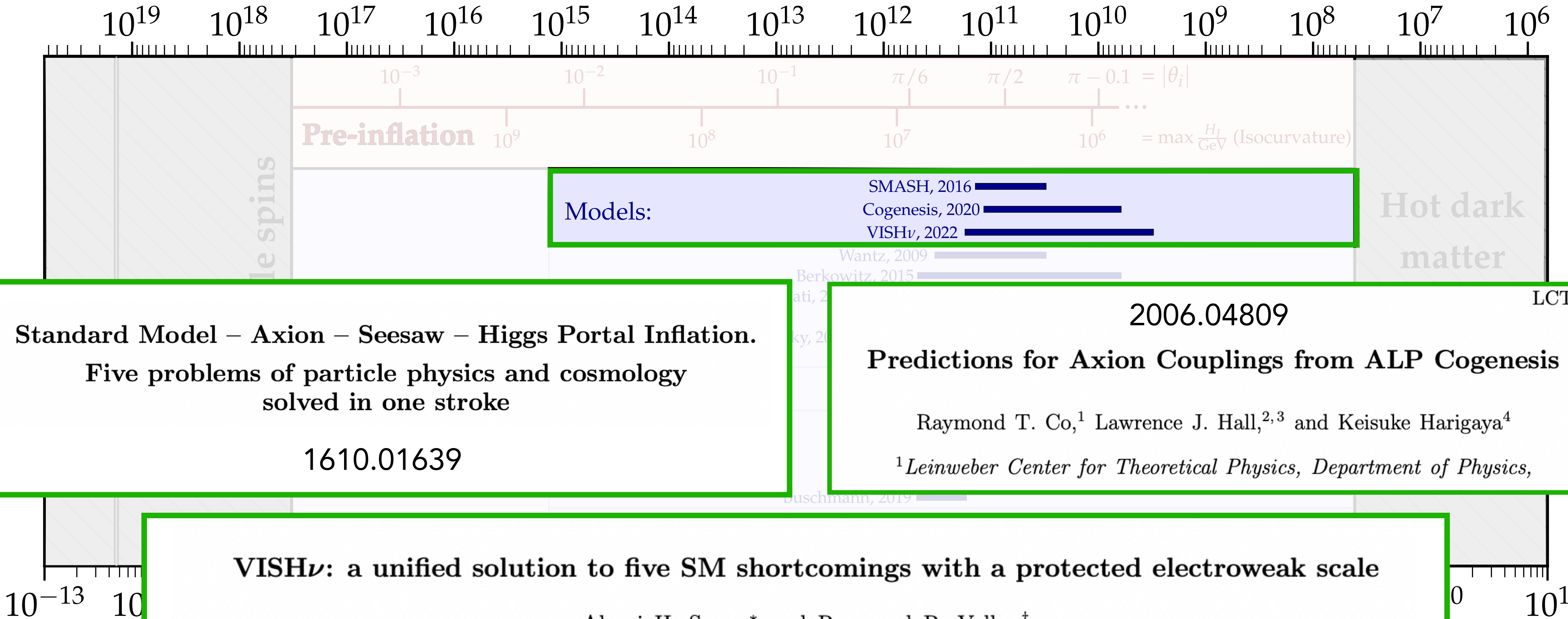
Next lecture

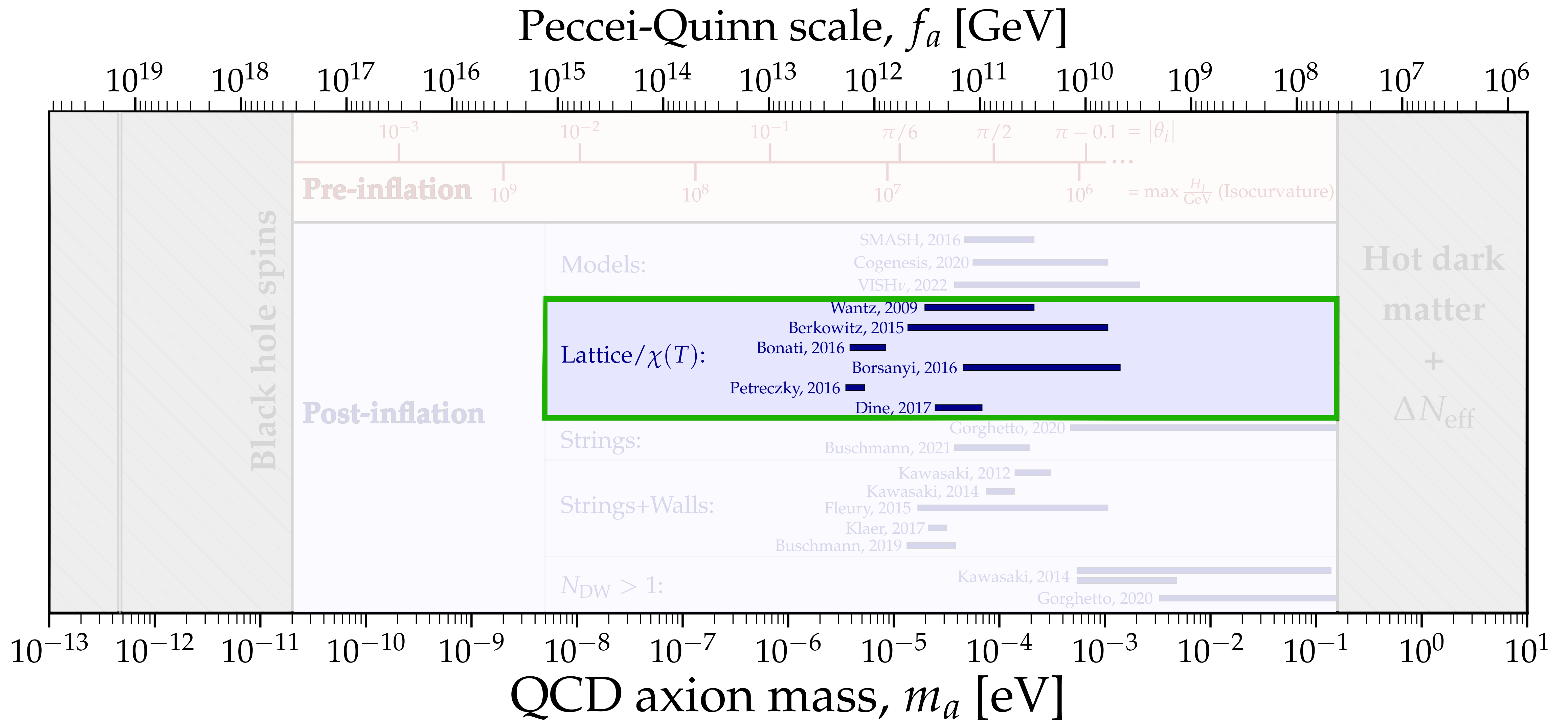






Peccei-Quinn scale, f_a [GeV]





Borsanyi+ [1606.07494]

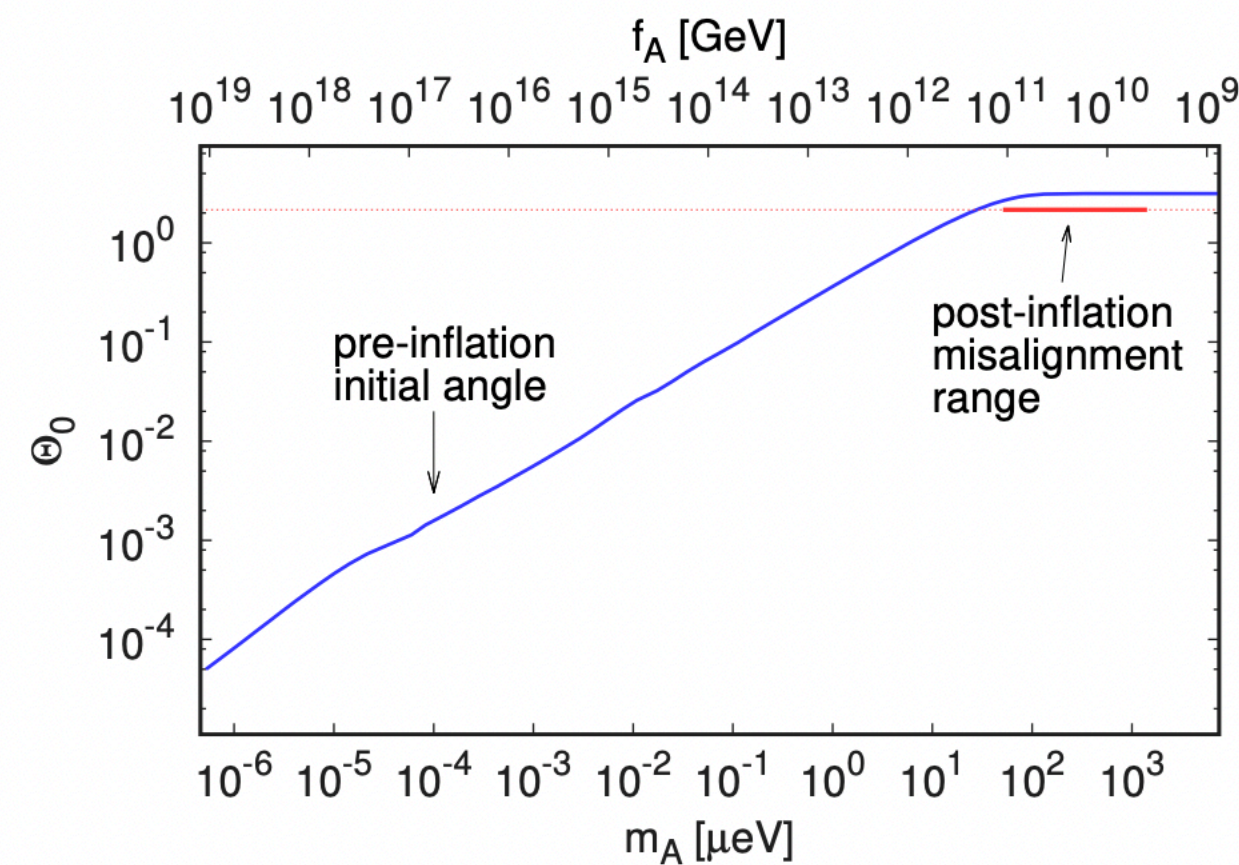


Figure 3: Relation between the axion's mass and the initial angle θ_0 in the pre-inflation scenario. The post-inflation scenario corresponds to $\theta_0 = 2.155$ with a strict lower bound on the axion's mass of $m_A = 28(2)\mu\text{eV}$. The thick red line shows our result on the axion's mass for the post-inflation case.

ei-Quinn scale, f_a [GeV]

10^{14} 10^{13} 10^{12} 10^{11} 10^{10} 10^9 10^8 10^7 10^6

10^{-1} $\pi/6$ $\pi/2$ $\pi - 0.1 = |\theta_i|$...
 10^8 10^7 10^6 $= \max \frac{H_I}{\text{GeV}}$ (Isocurvature)

Models:
SMASH, 2016
Cogeneration, 2020
VISHν, 2022

Lattice/ $\chi(T)$:

Wantz, 2009
Berkowitz, 2015
Bonati, 2016
Borsanyi, 2016
Petreczky, 2016
Dine, 2017

Strings:

Gorghetto, 2020
Buschmann, 2021

Strings+Walls:

Kawasaki, 2012
Kawasaki, 2014
Fleury, 2015
Klaer, 2017
Buschmann, 2019

$N_{\text{DW}} > 1$:

Kawasaki, 2014
Gorghetto, 2020

Hot dark
matter

+

ΔN_{eff}

10^{-13} 10^{-12} 10^{-11} 10^{-10} 10^{-9} 10^{-8} 10^{-7} 10^{-6} 10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 10^0 10^1

QCD axion mass, m_a [eV]

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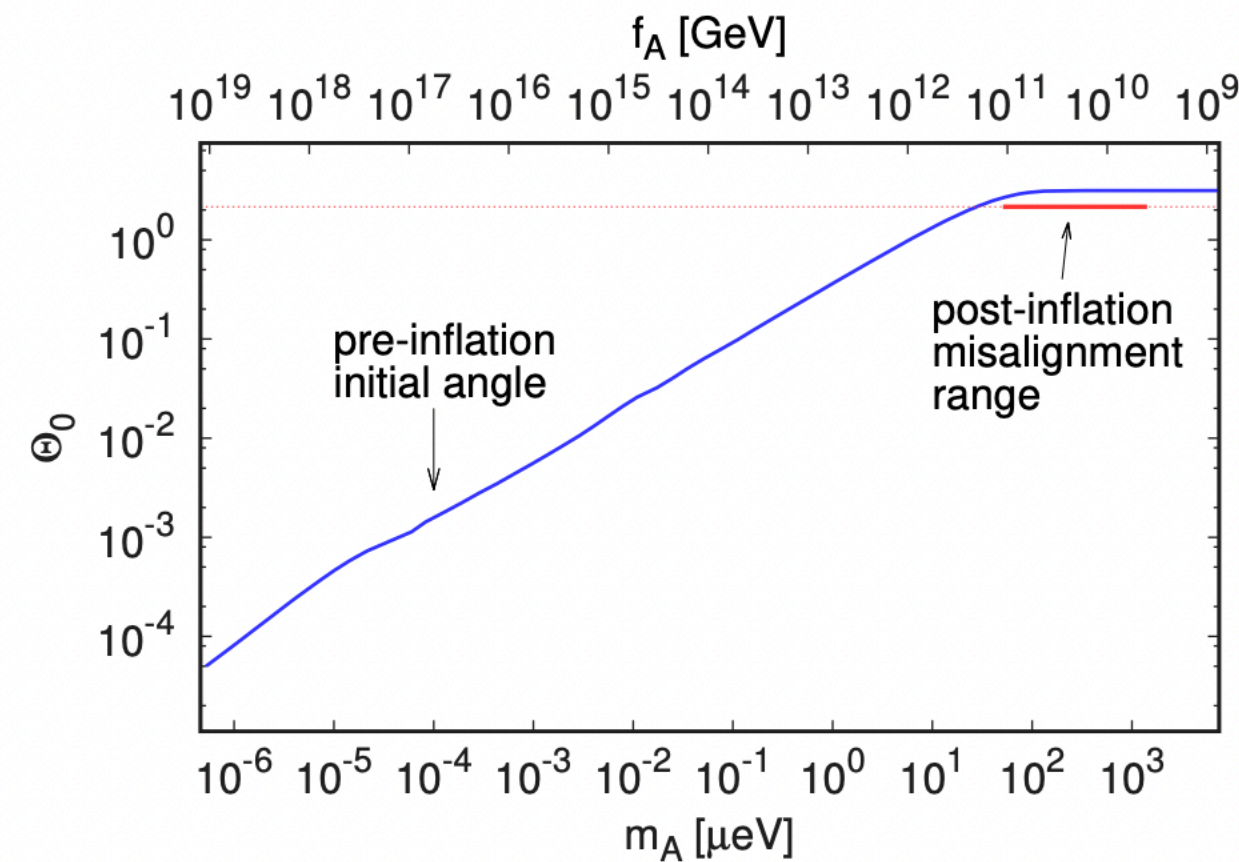
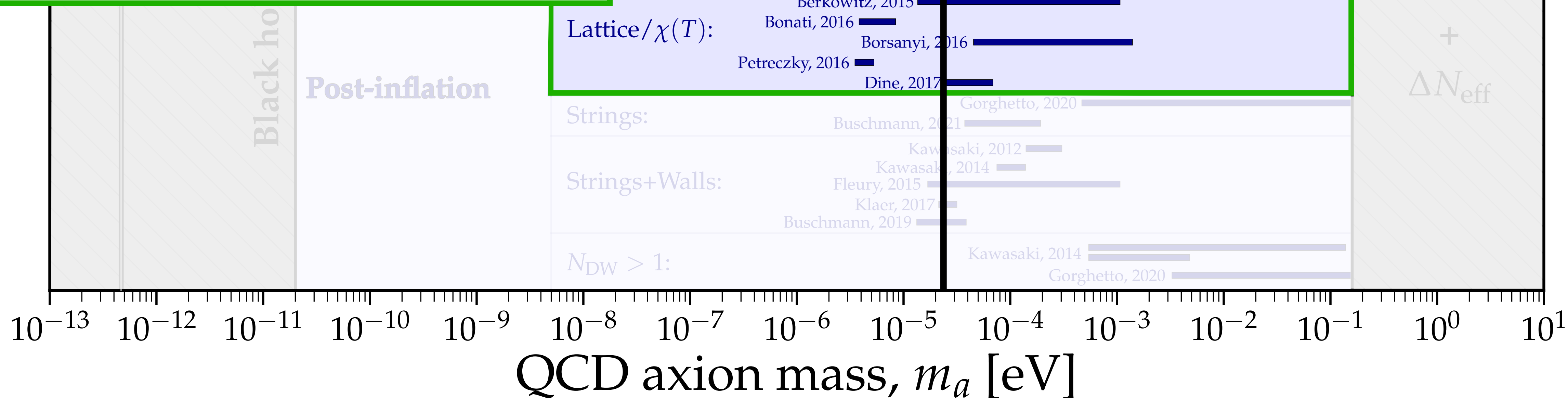
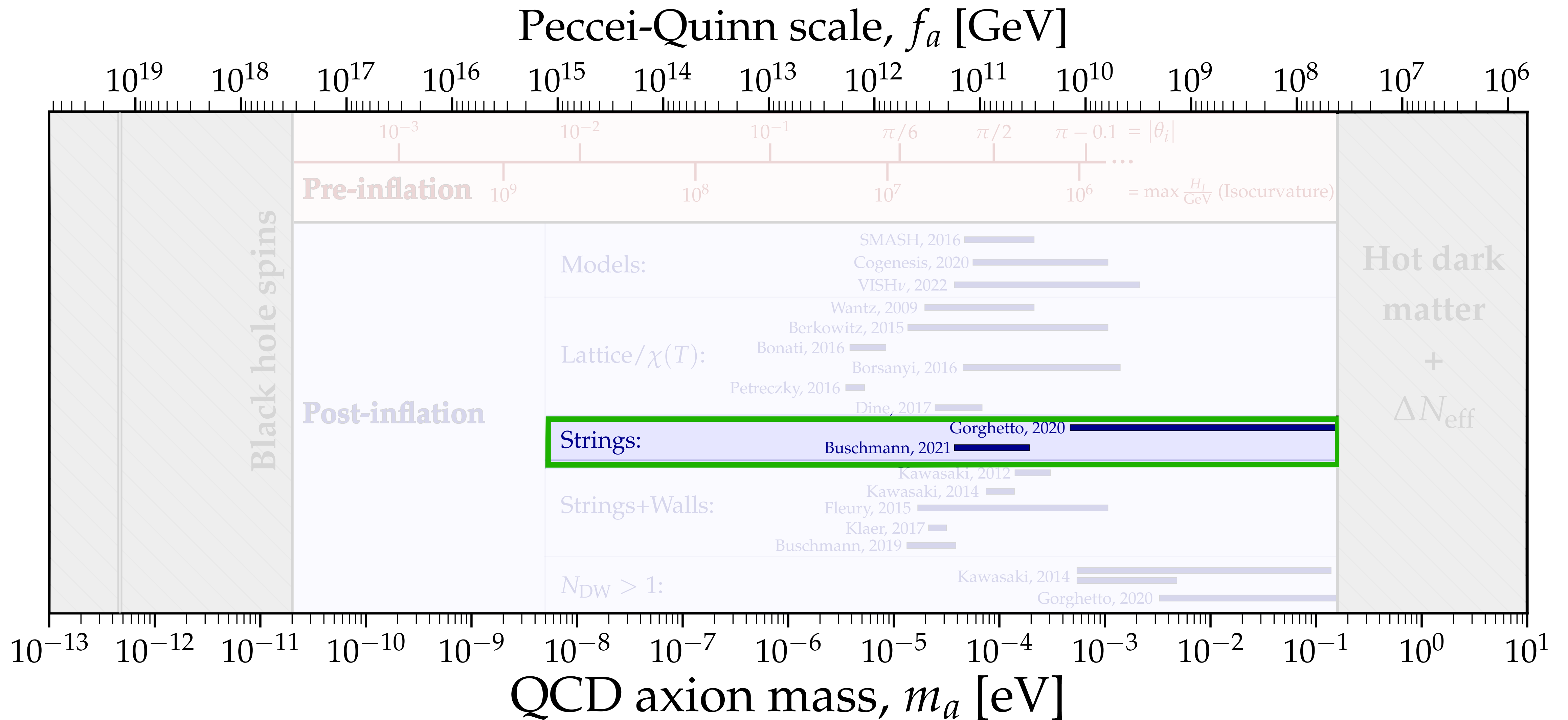


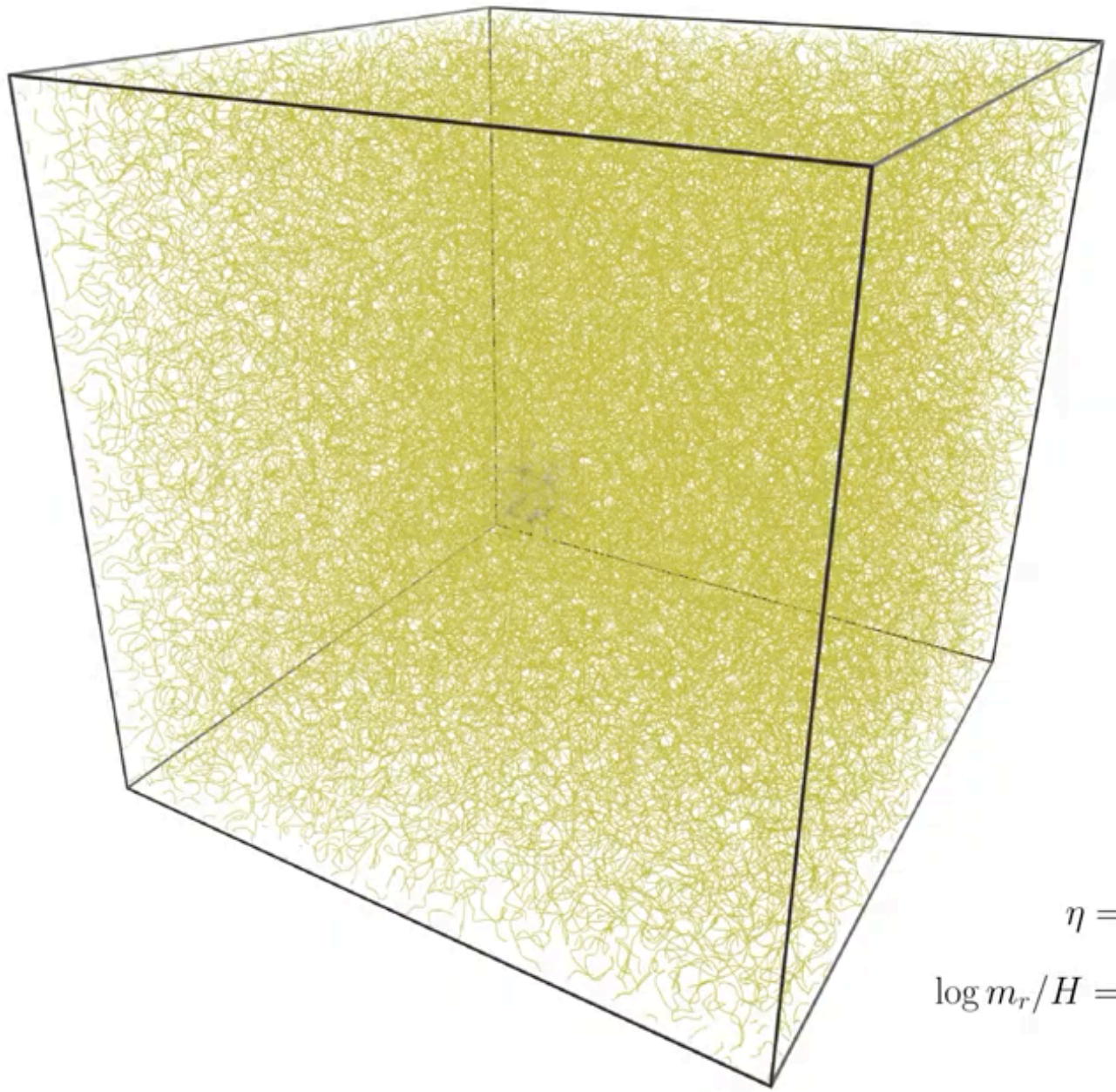
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Minimum mass for post-inflationary scenario.
Strings add more axions
→ drive m_a prediction down





M. Buschmann



$\eta = 0.1$

$\log m_r/H = -3.58$

Peccei-Quinn scale, f_a [GeV]

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Hot dark matter

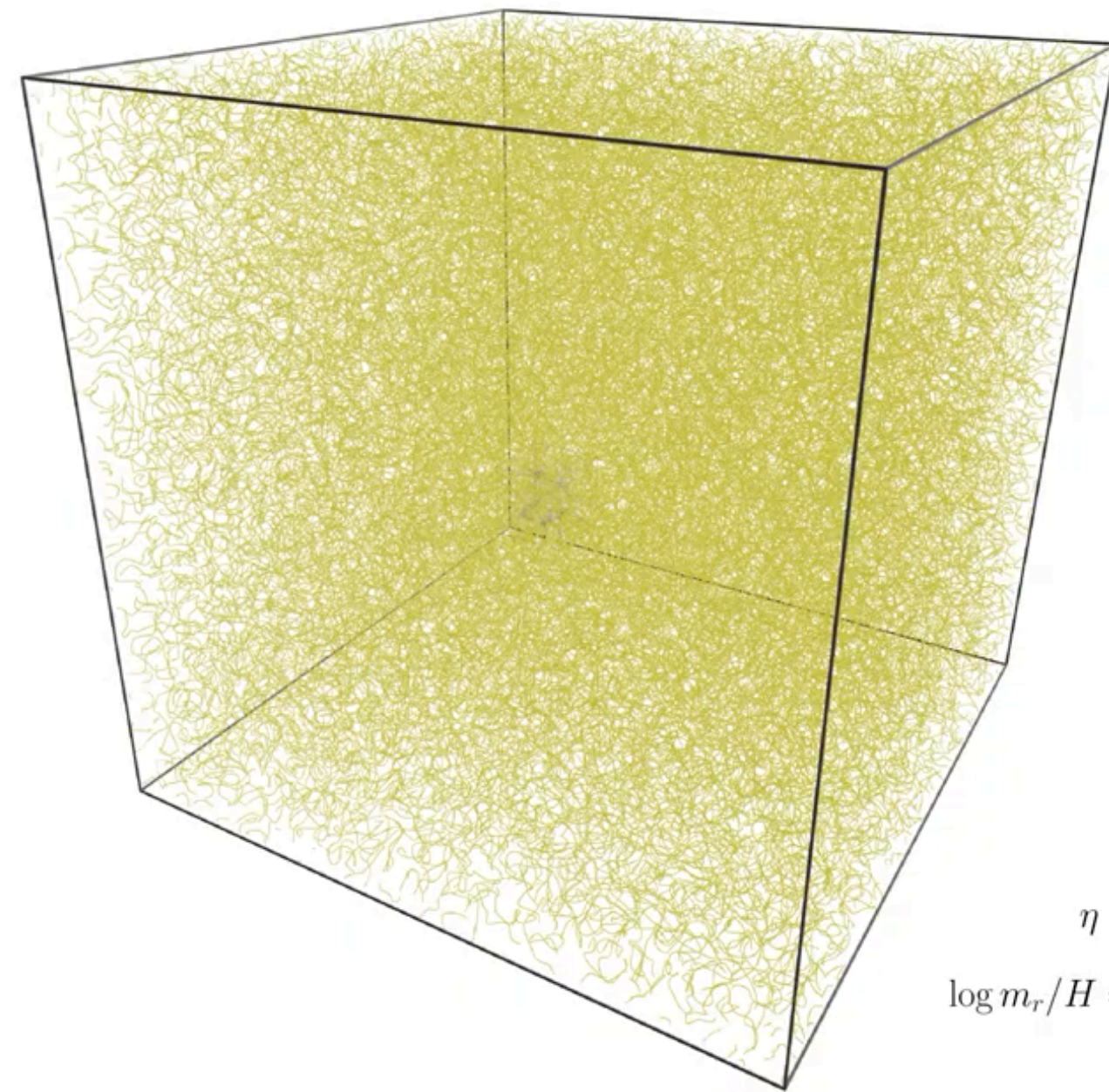
+

ΔN_{eff}

QCD axion mass, m_a [eV]

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M. Buschmann



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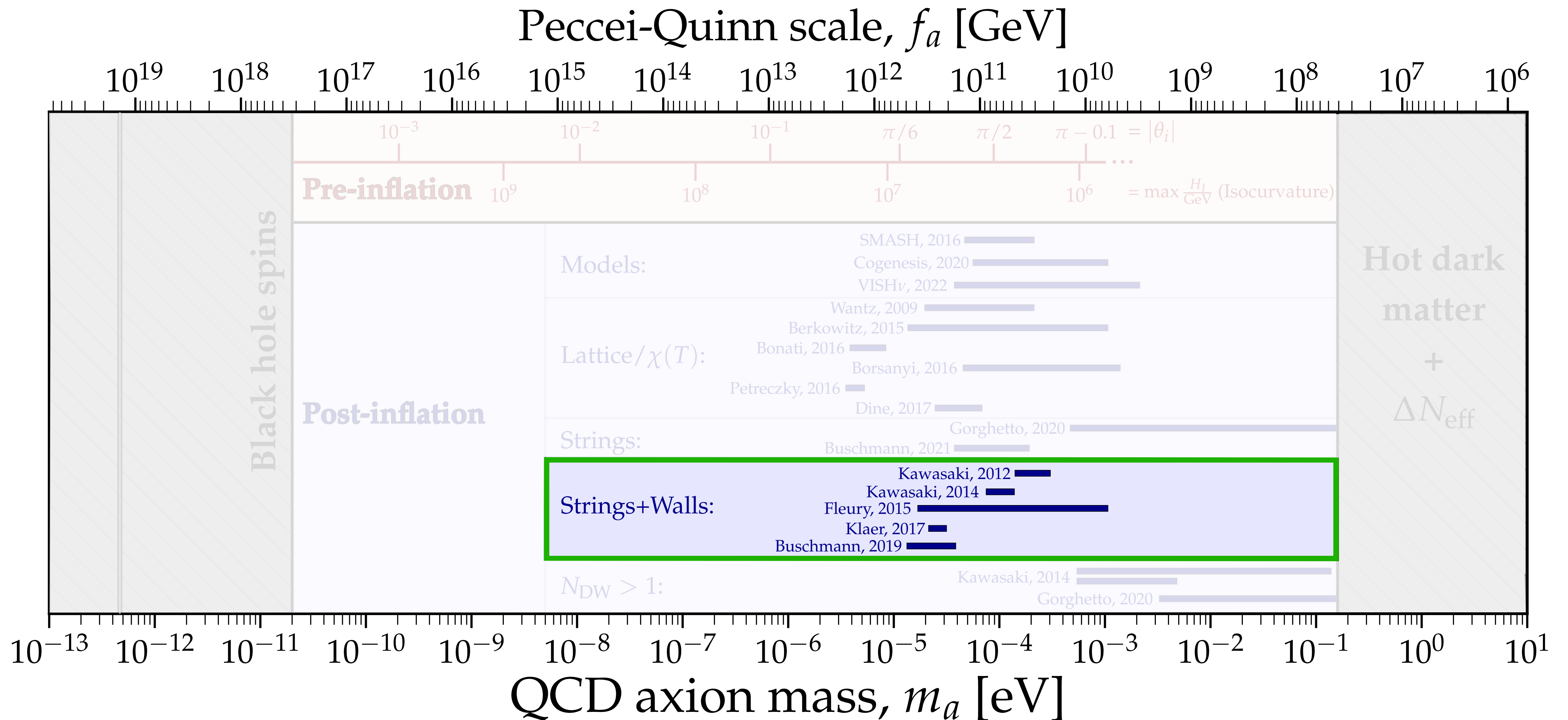
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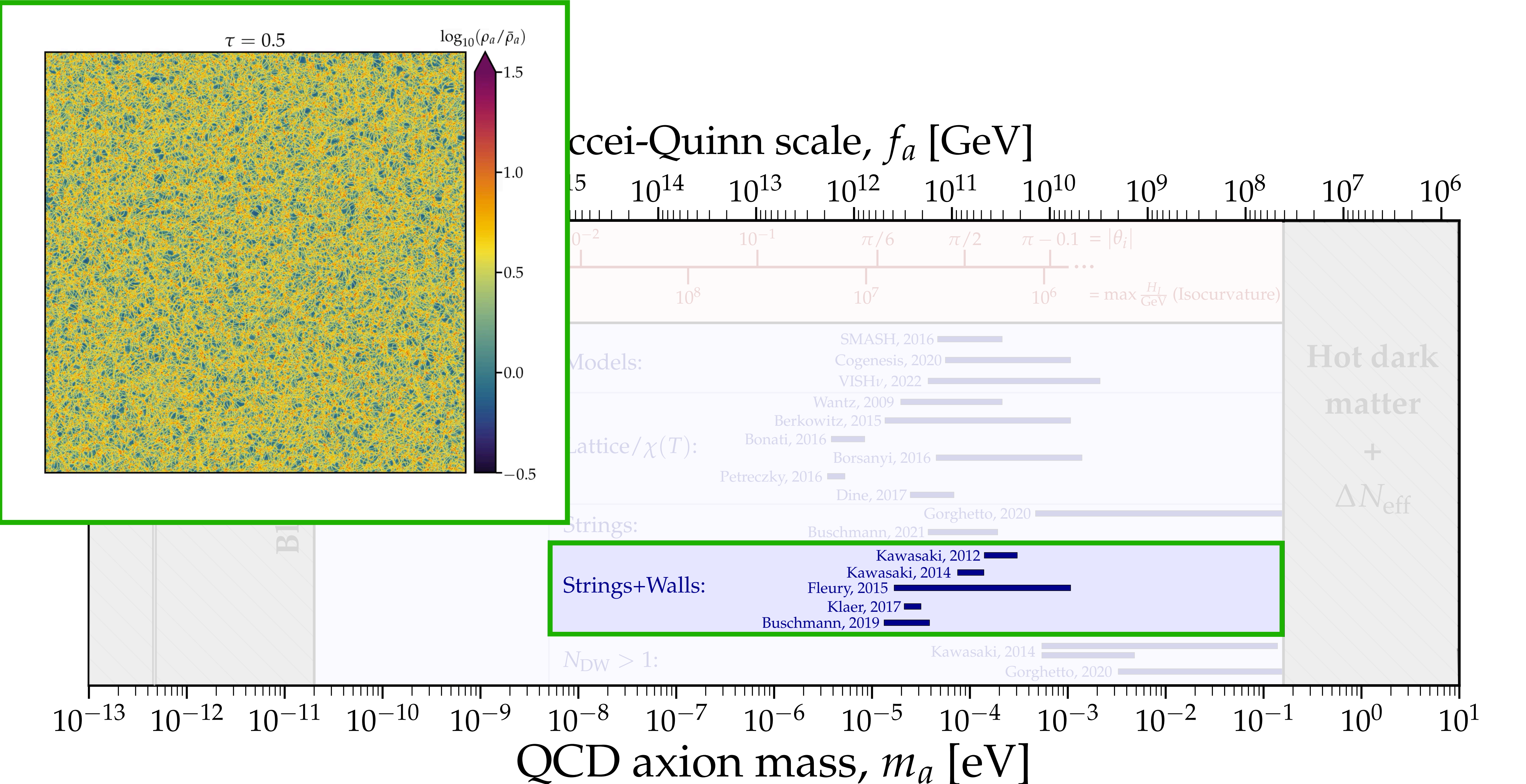
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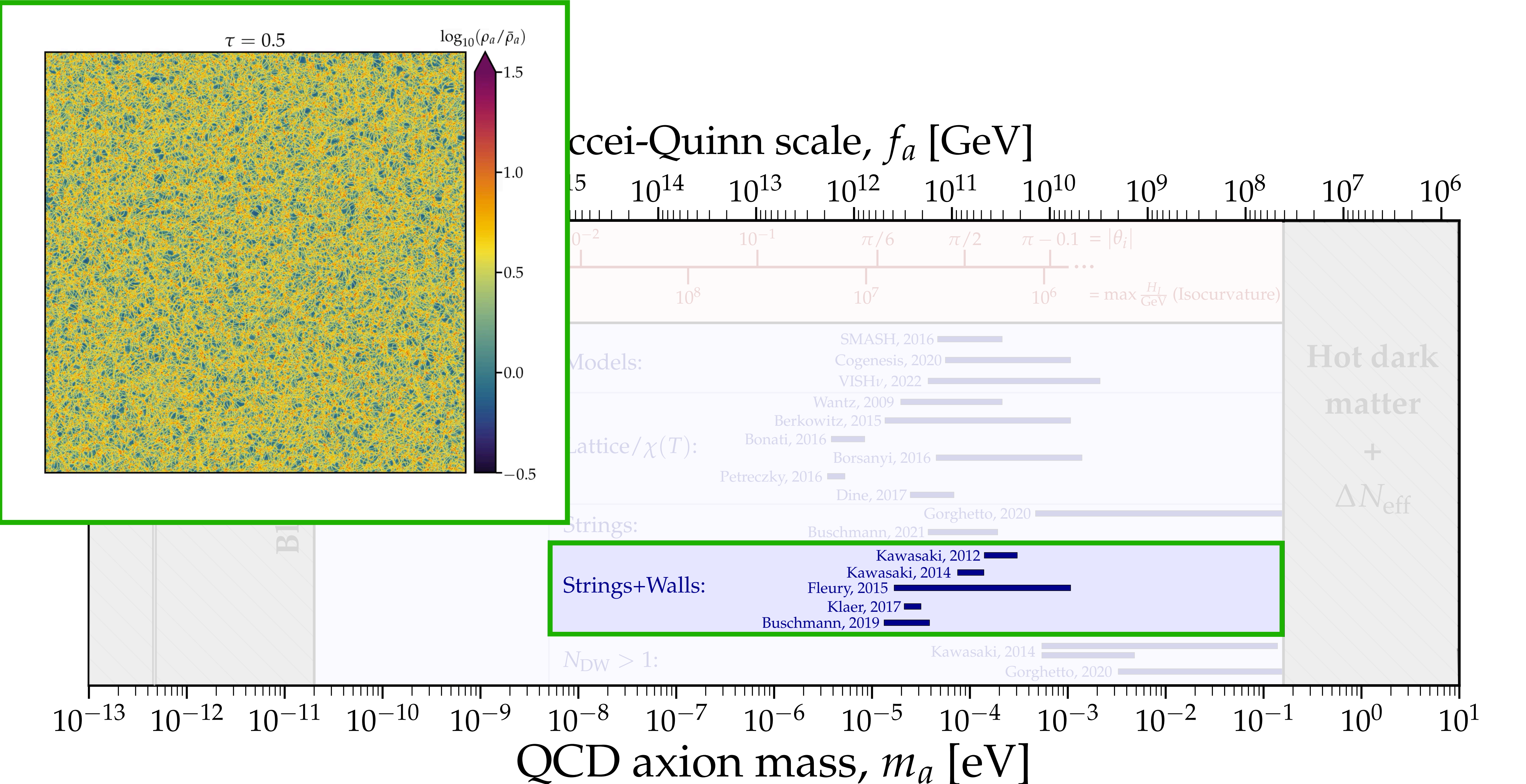
ΔN_{eff}

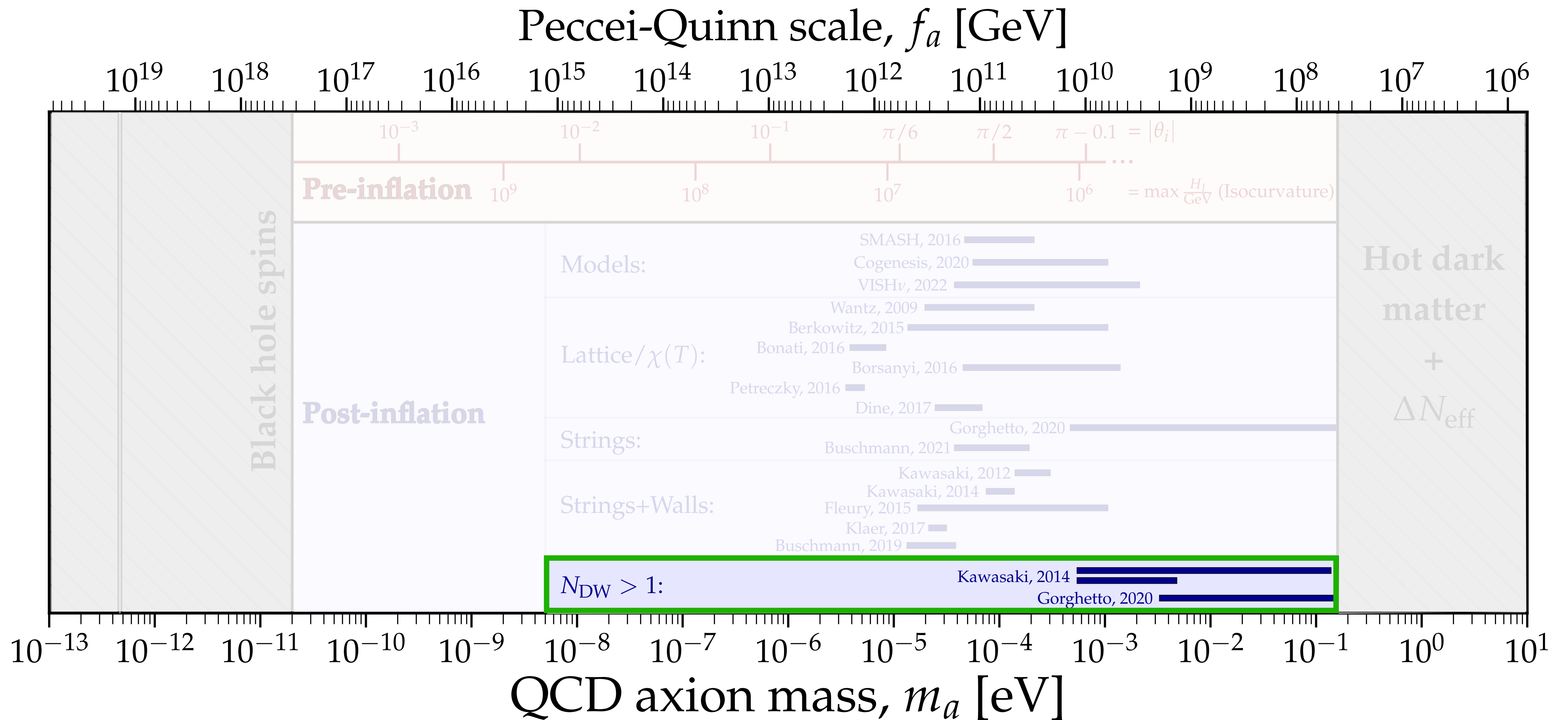
QCD axion mass, m_a [eV]

10^{-13} 10^{-12} 10^{-11} 10^{-10} 10^{-9} 10^{-8} 10^{-7} 10^{-6} 10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 10^0 10^1

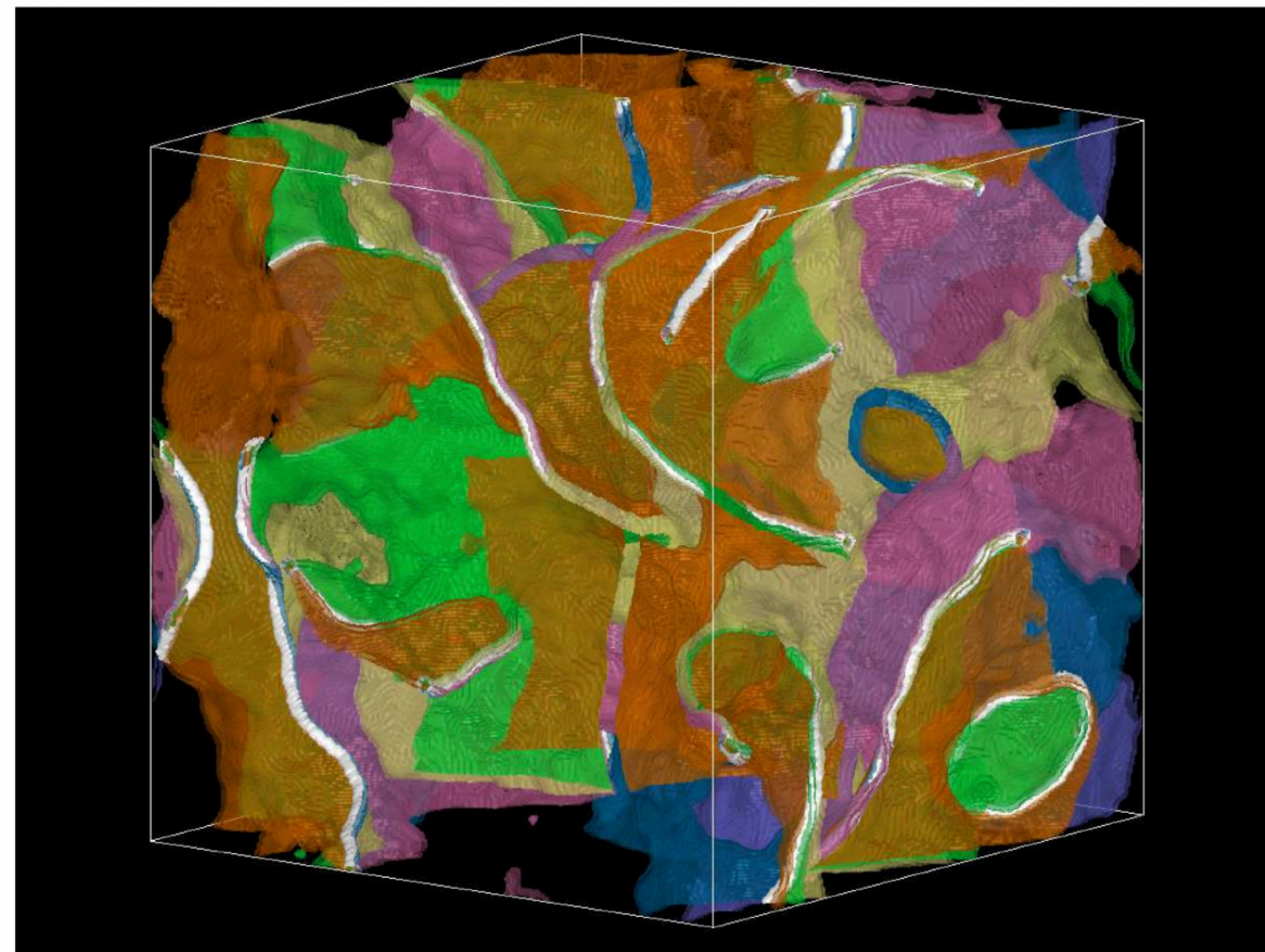








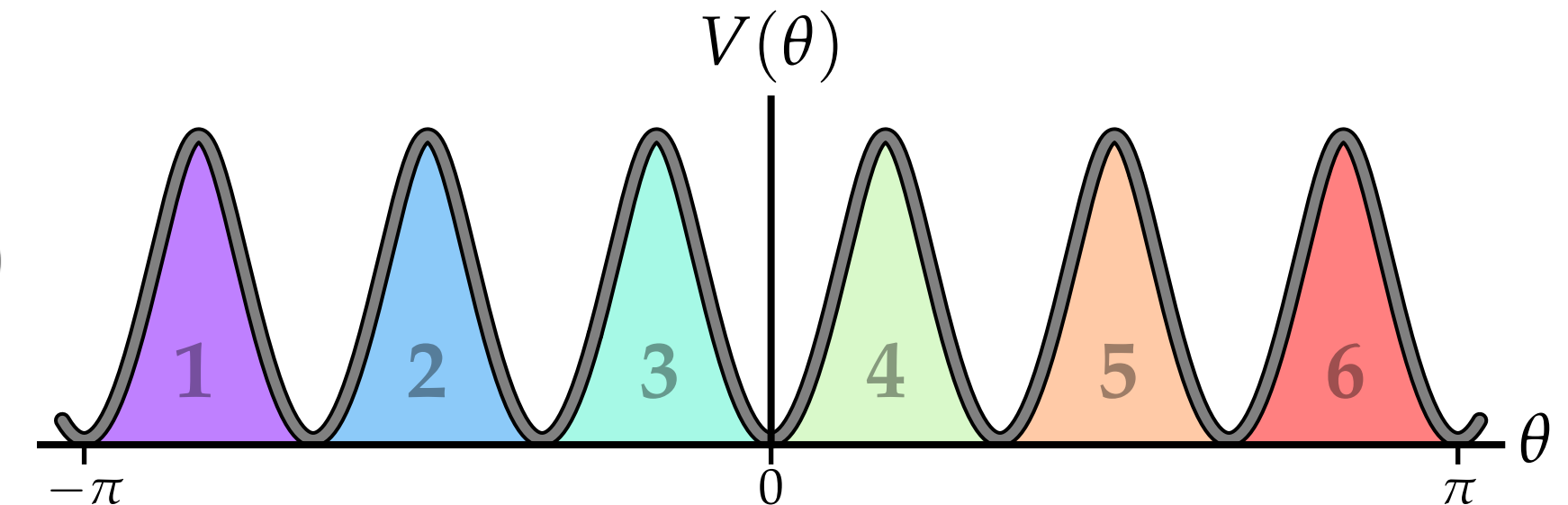
Axion models with domain wall number > 1



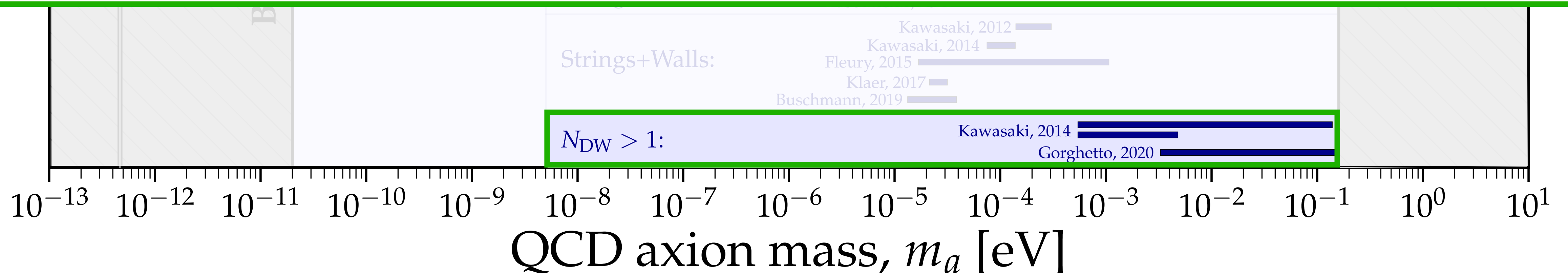
(e) $N_{\text{DW}} = 6$

Hiramatsu+ [1207.3166]

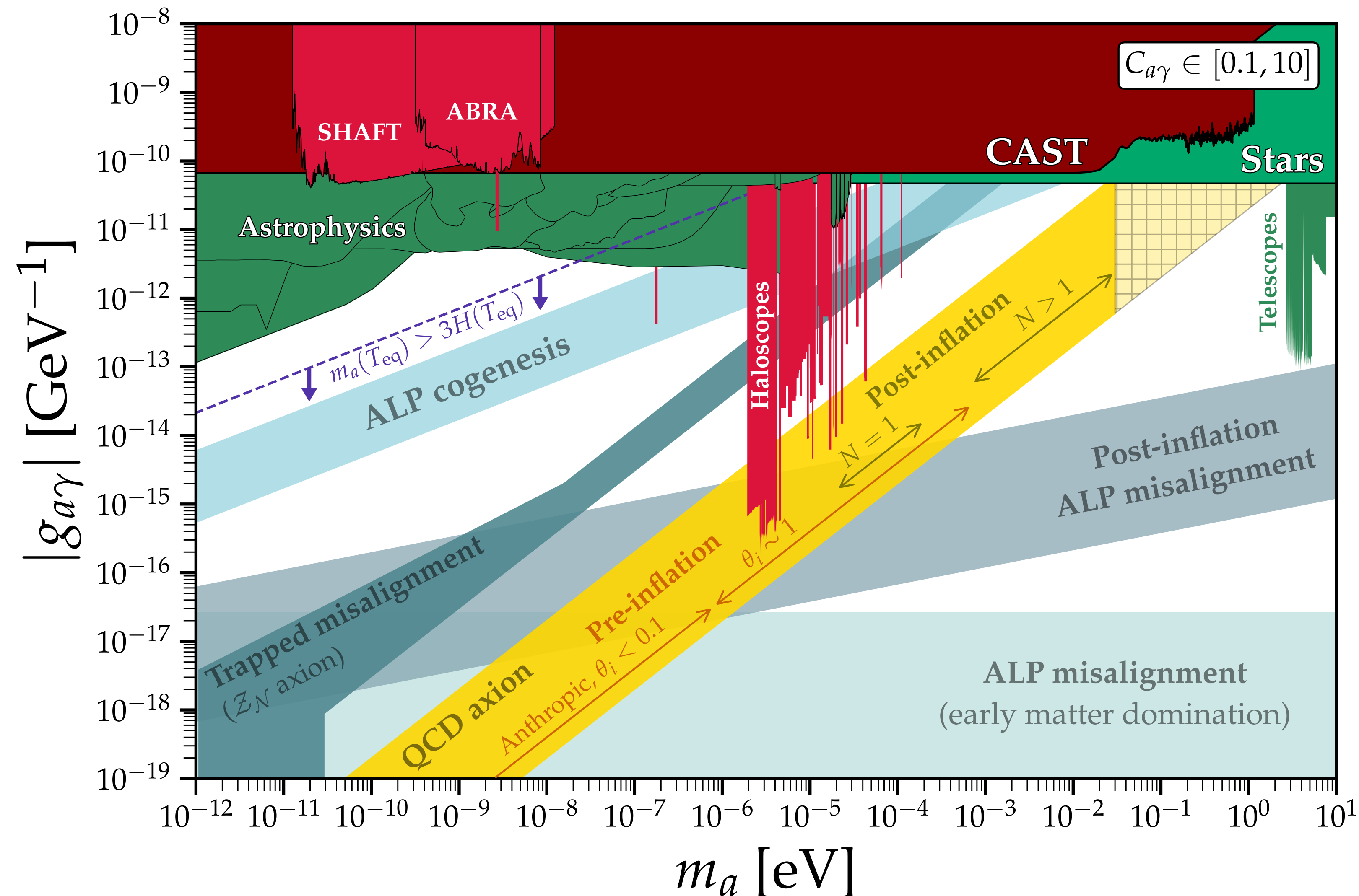
$$V(\theta) \approx -\chi(T) \cos(N_{\text{DW}}\theta)$$



- Axion cycles around N_{DW} times between $(-\pi, \pi)$
- In general get more axions from wall decay, so preferred m_a is higher
- On their own $N_{\text{DW}} > 1$ models are not allowed. Domain wall network gets stuck and overwhelms the cosmic energy density.
- Must have some "bias" to tip the scales to preferring one minimum

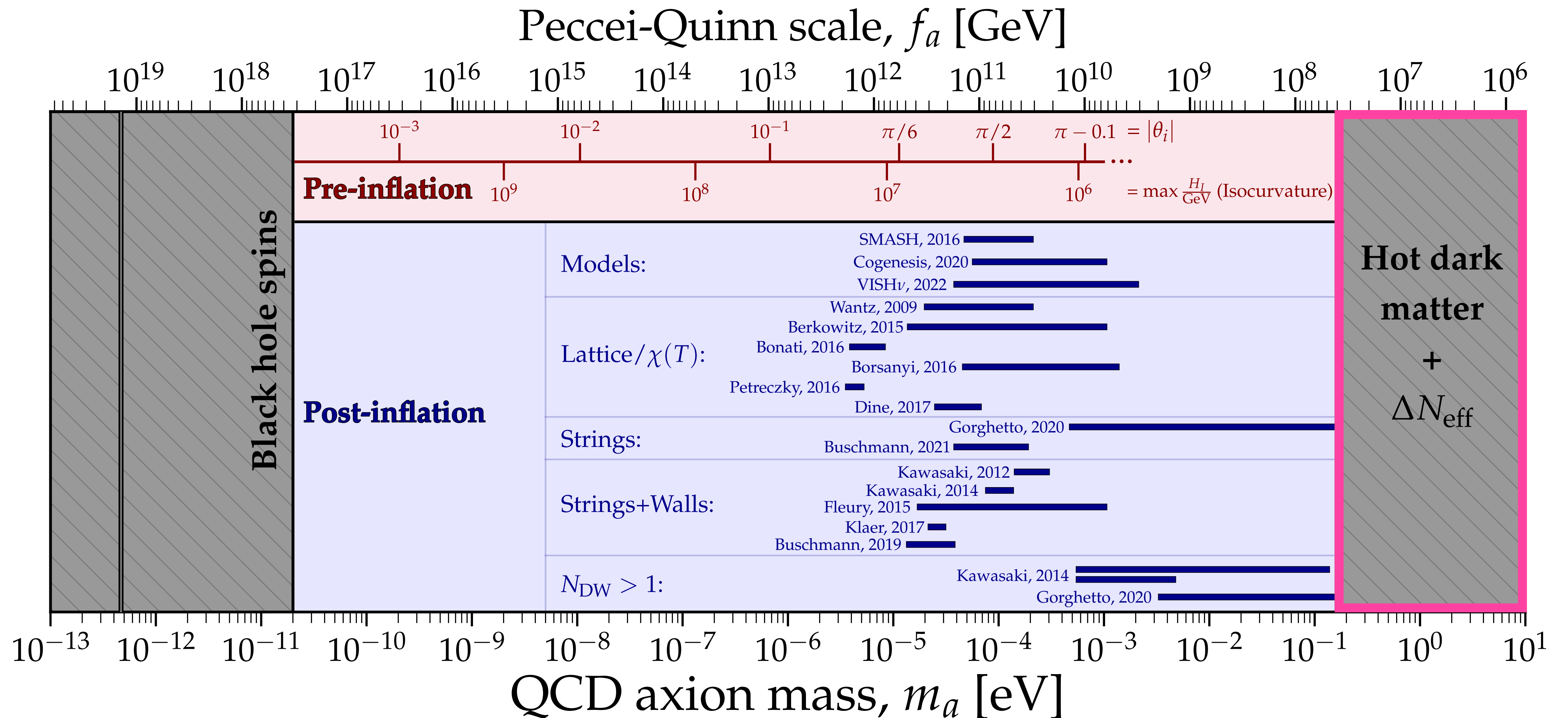


Non-standard axion/ALP post-dictions



- QCD axion with a non-standard mass,
→ e.g. [2102.01082]
- ALP with a temperature-independent mass
→ e.g. [2112.05117]
- Axions in non-standard cosmologies
→ e.g. [1911.07853]
- Variants of misalignment mechanism: axion rotations, parametric resonance etc.
→ e.g. [2006.04809]

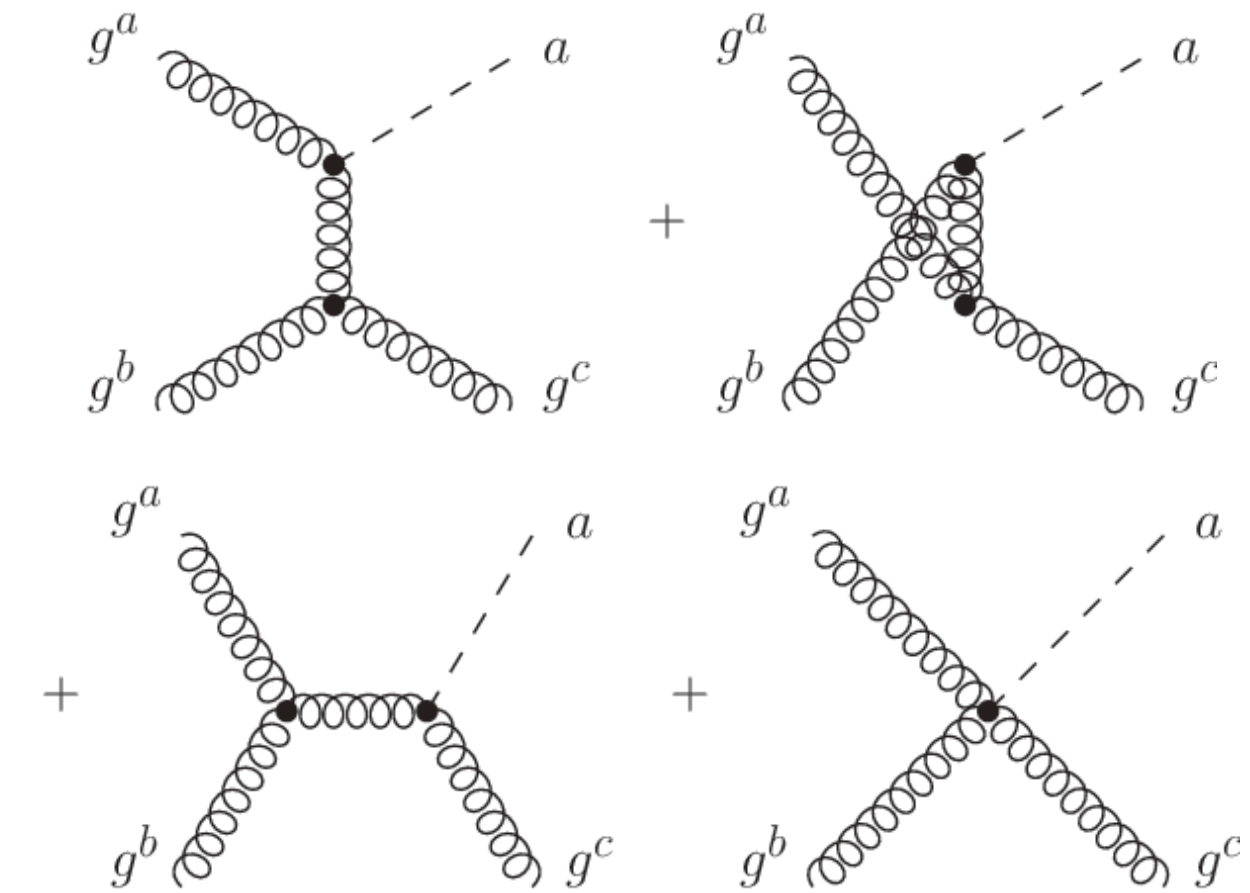
How cosmology can bound the axion parameter space



Thermal production of axions

At very high temperatures even axions can be in thermal equilibrium with SM bath

Via e.g. $g + g \leftrightarrow g + a$ (high T) or $\pi + \pi \leftrightarrow \pi + a$ (low T)

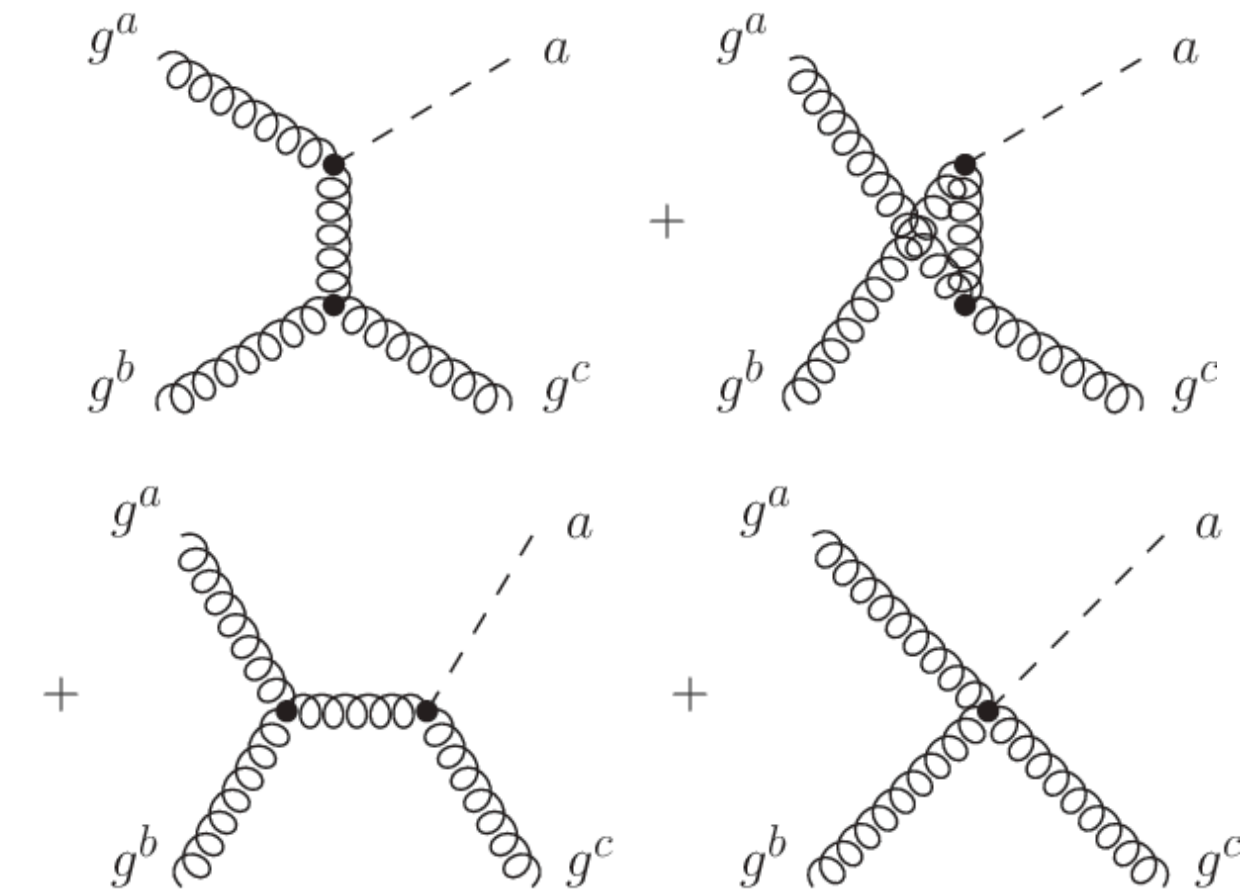


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Production rate: $\Gamma \sim n \langle \sigma v \rangle \sim 10^{-4} \frac{T^3}{f_a^2}$



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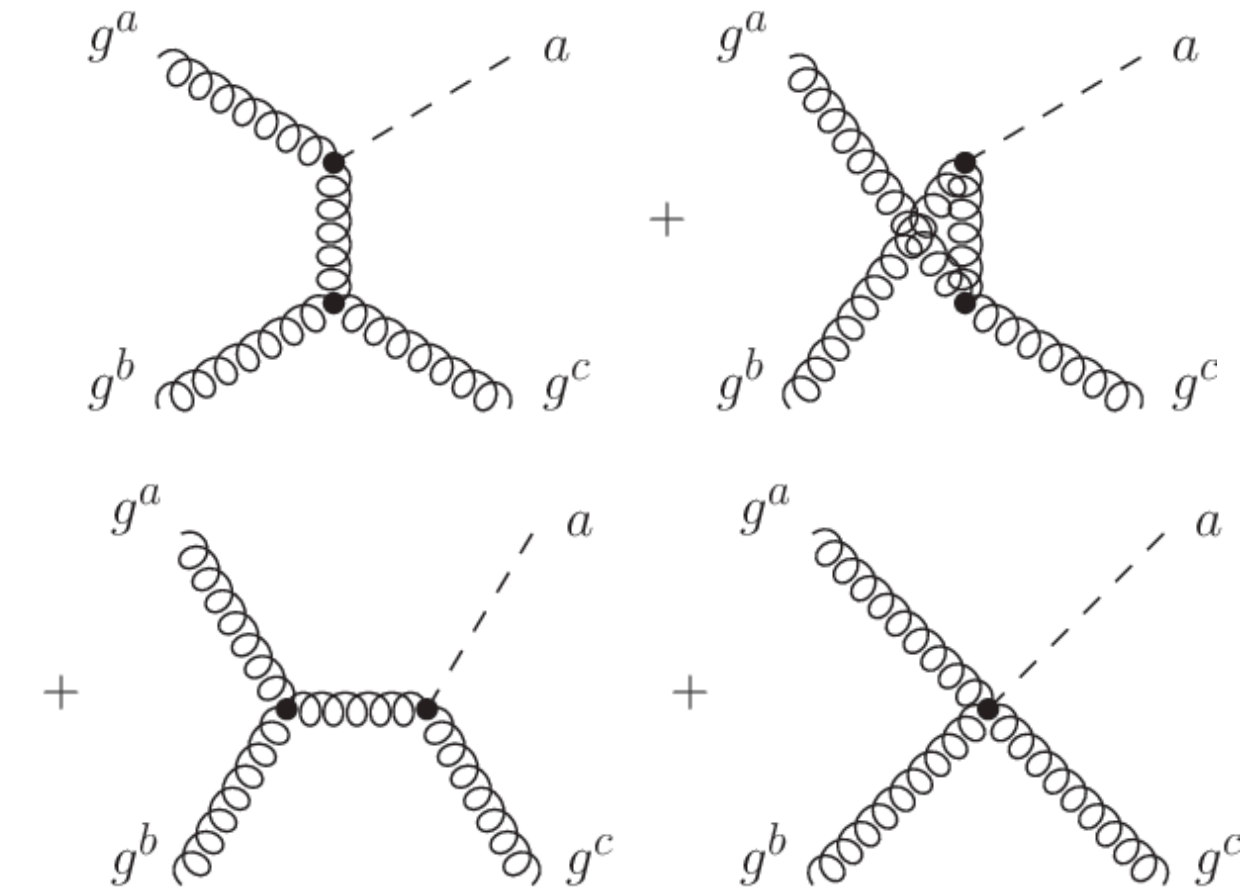
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Axions "freeze-out" when

$$\Gamma < H(t) \sim T^2/M_{\text{Pl}}$$



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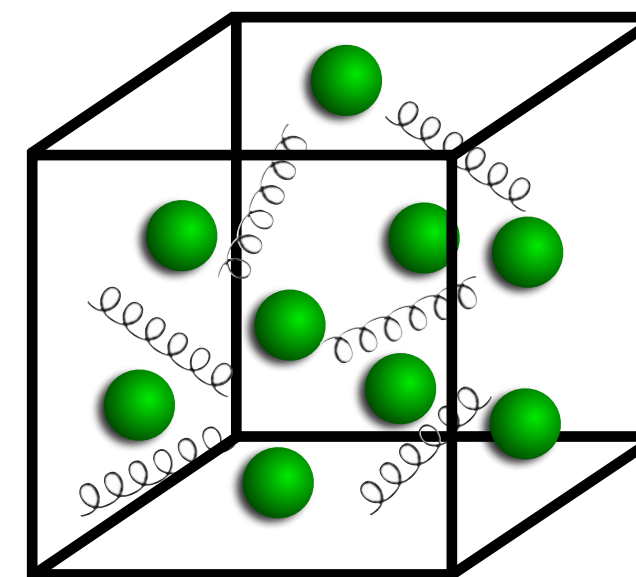
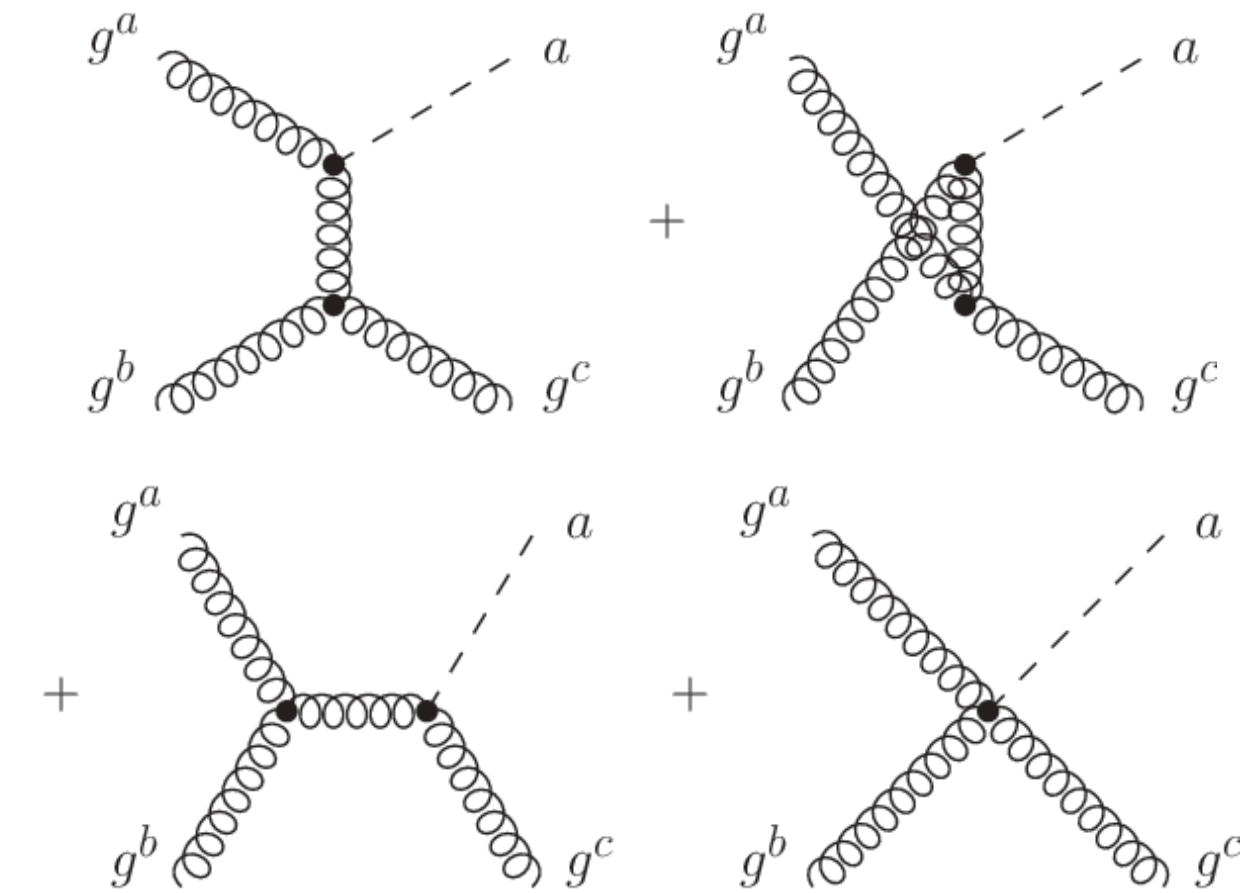
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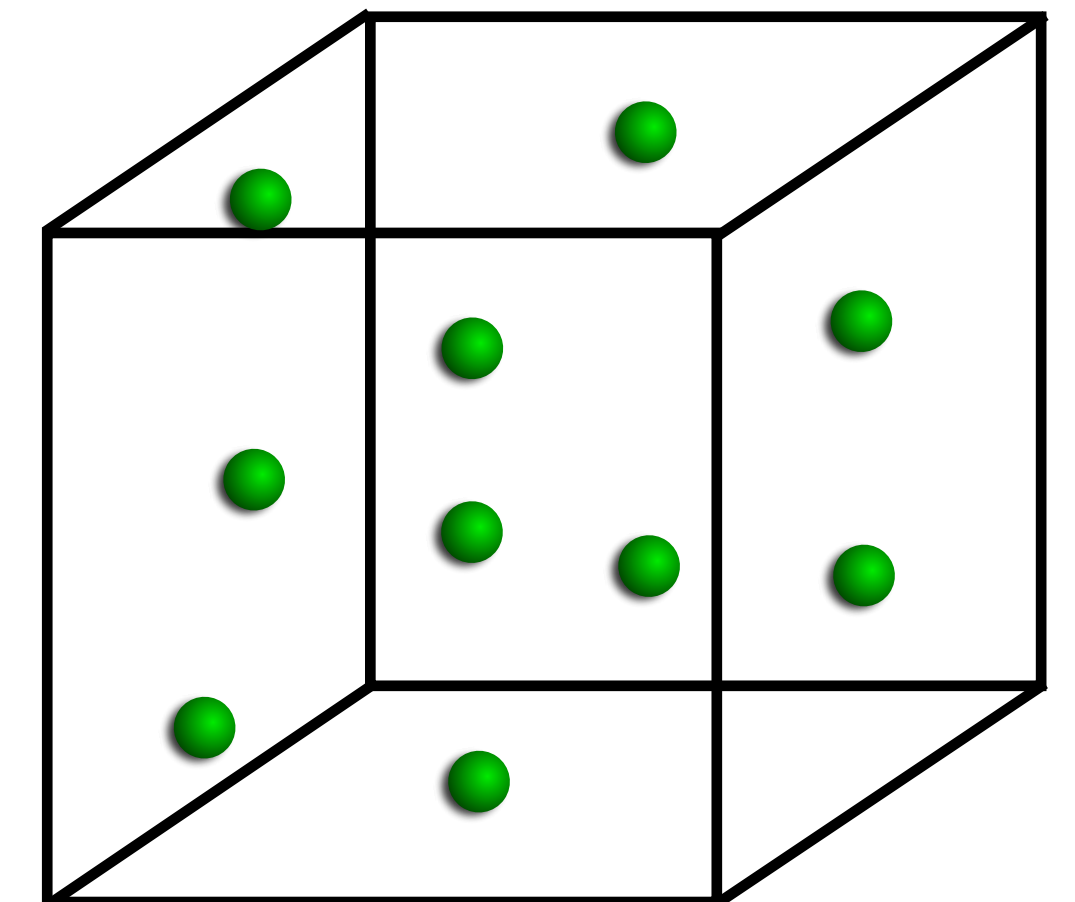
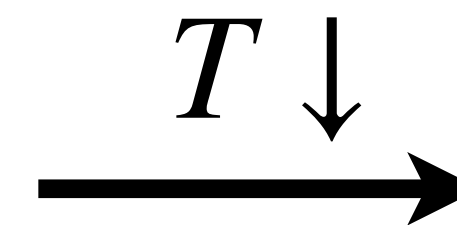


$$T_d \sim 10^8 \text{ GeV} \left(\frac{f_a}{10^{11} \text{ GeV}} \right)^2$$



$$\Gamma > H$$

thermal eq. maintained

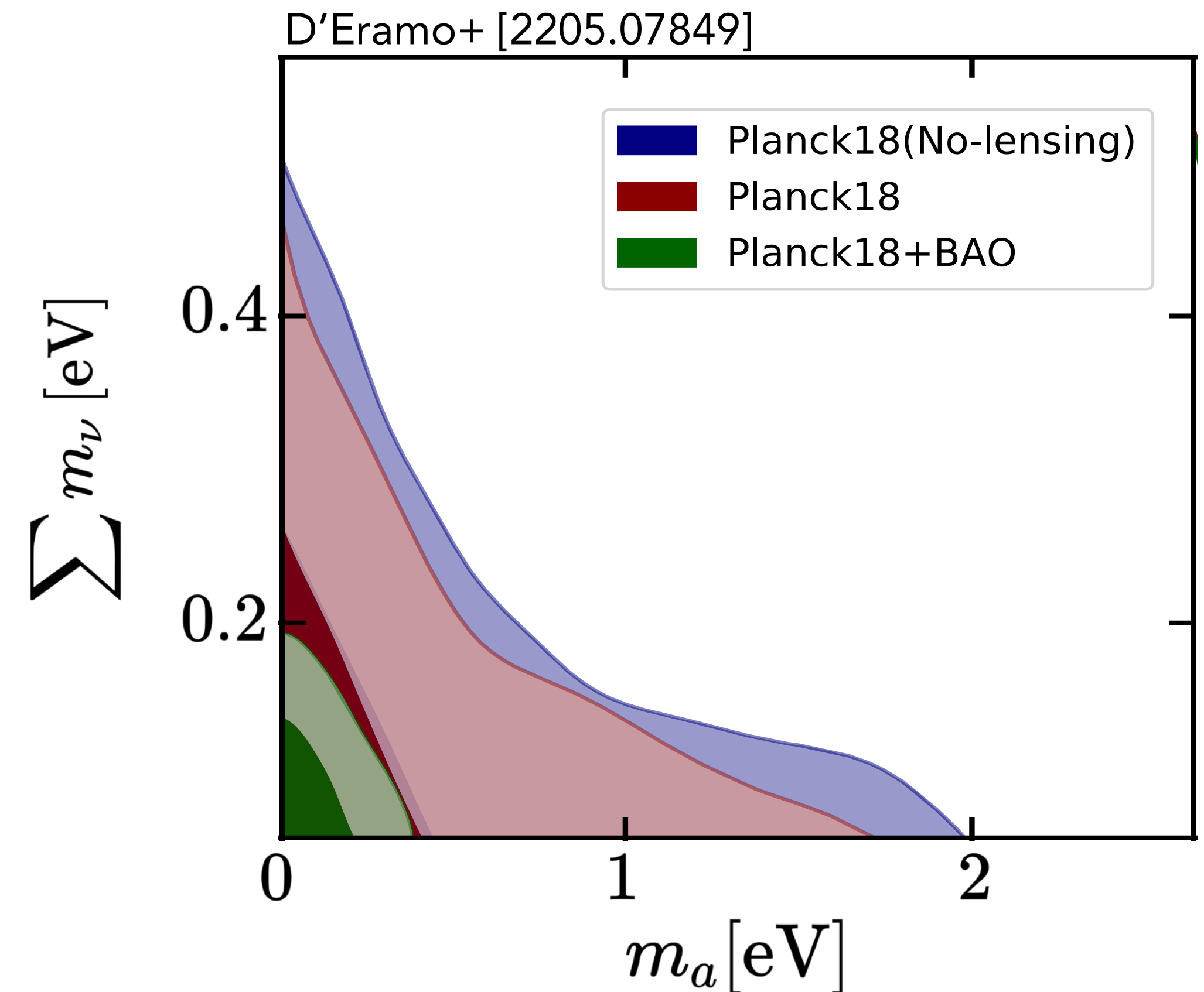
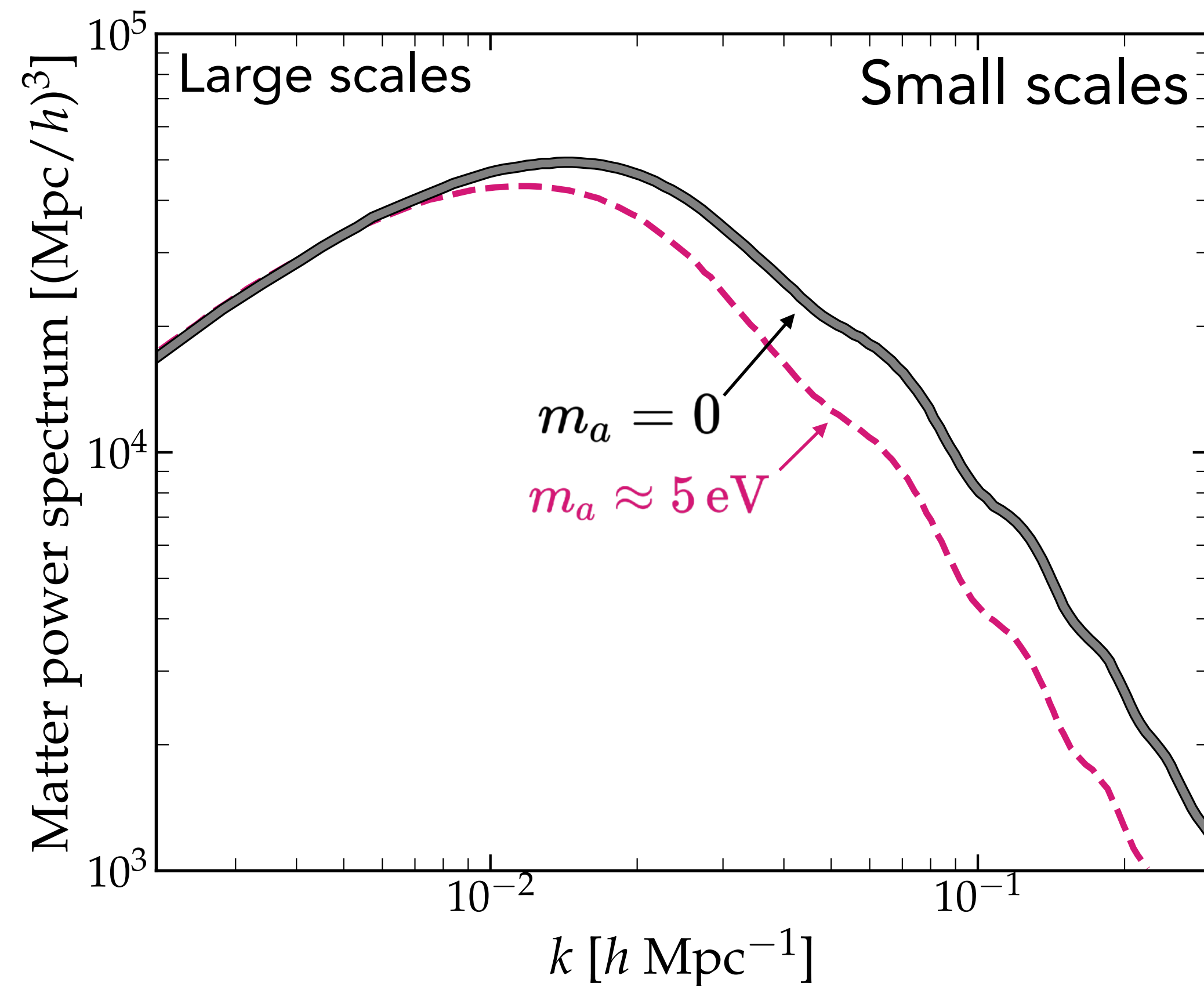


$$\Gamma < H$$

Production can't keep up with expansion

Hot dark matter axions

If they are heavy enough then the thermal axion background contributes a form of **hot dark matter** (similar to neutrinos) which is heavily constrained by LSS+CMB



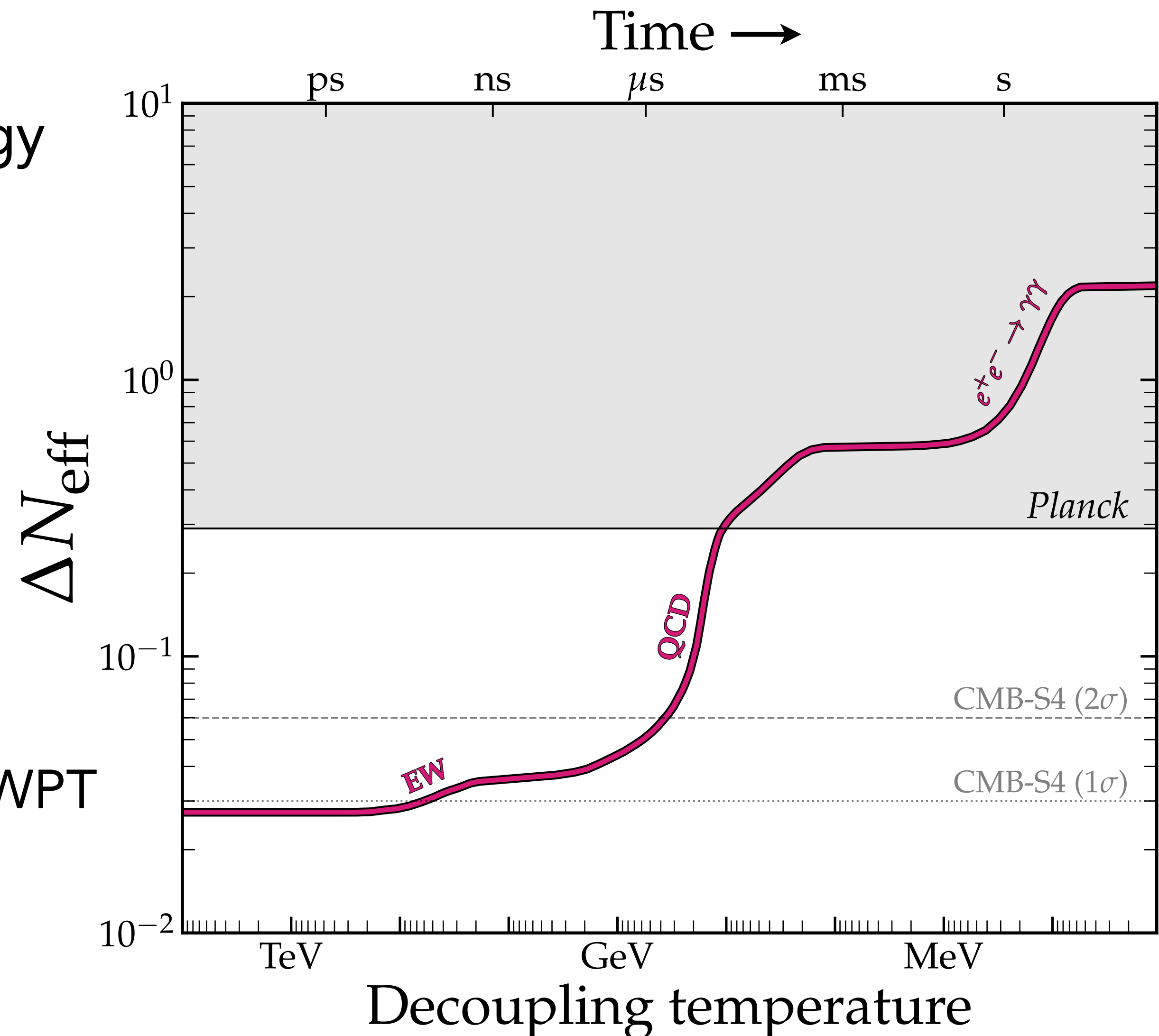
Thermal axions and N_{eff}

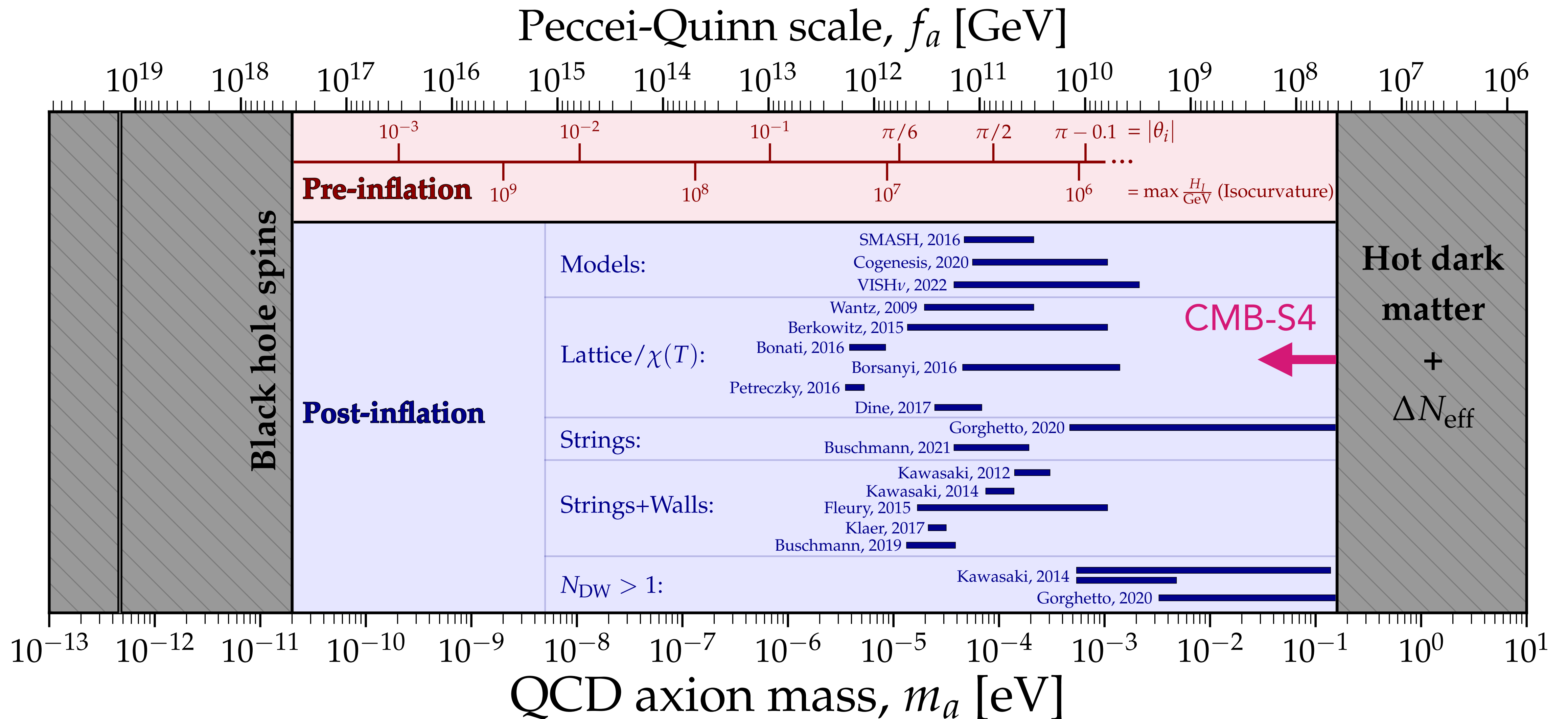
Quantify effects of new relativistic species on early-Universe expansion rate via energy density in units of a single neutrino (ΔN_{eff})

$$\rho_r = \left[1 + \frac{7}{8} \left(\frac{4}{11} \right)^{4/3} (N_{\text{eff}}^{\text{SM}} + \Delta N_{\text{eff}}) \right] \rho_\gamma$$

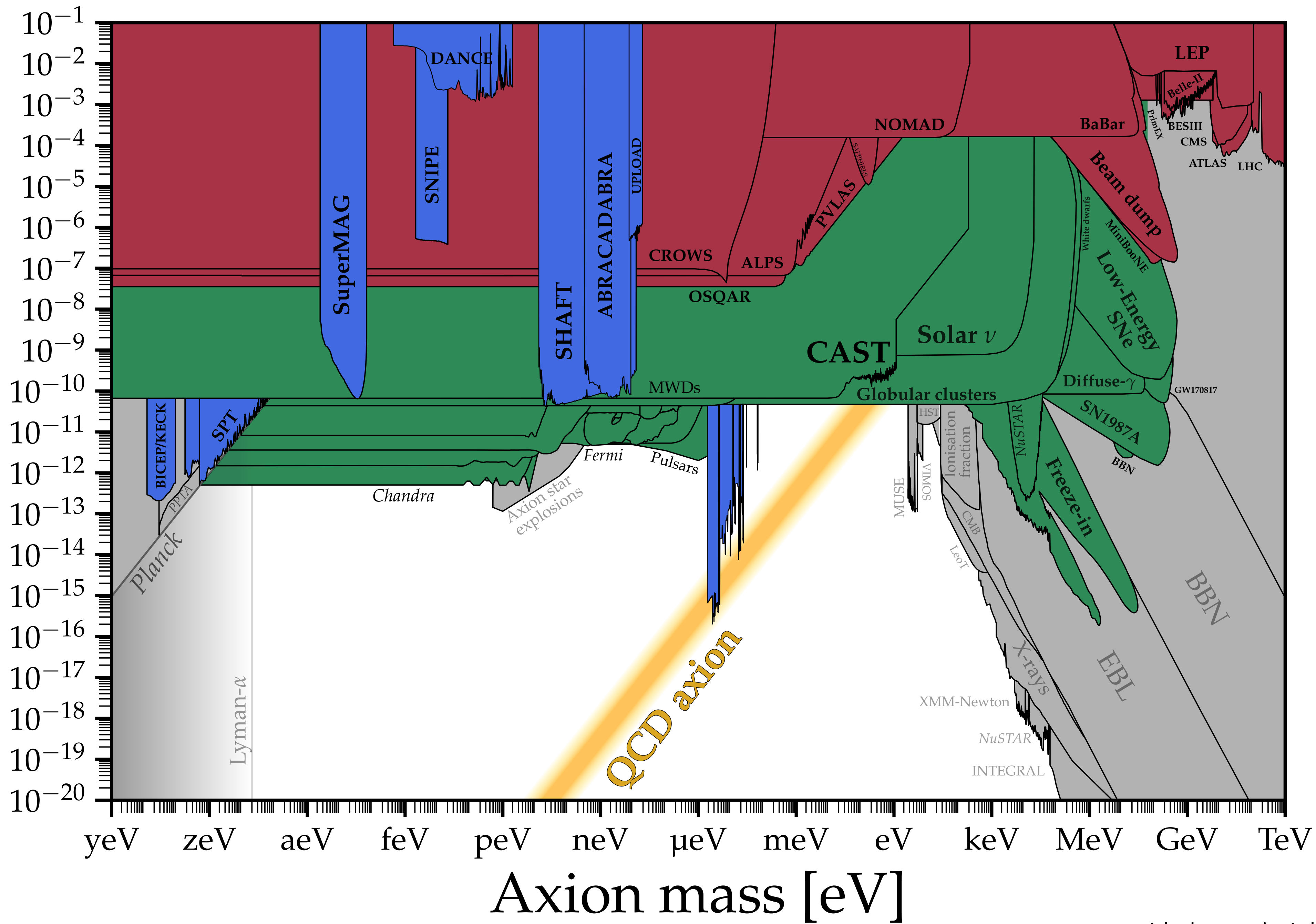
$\uparrow$
 $N_{\text{eff}}^{\text{SM}} = 3.044$

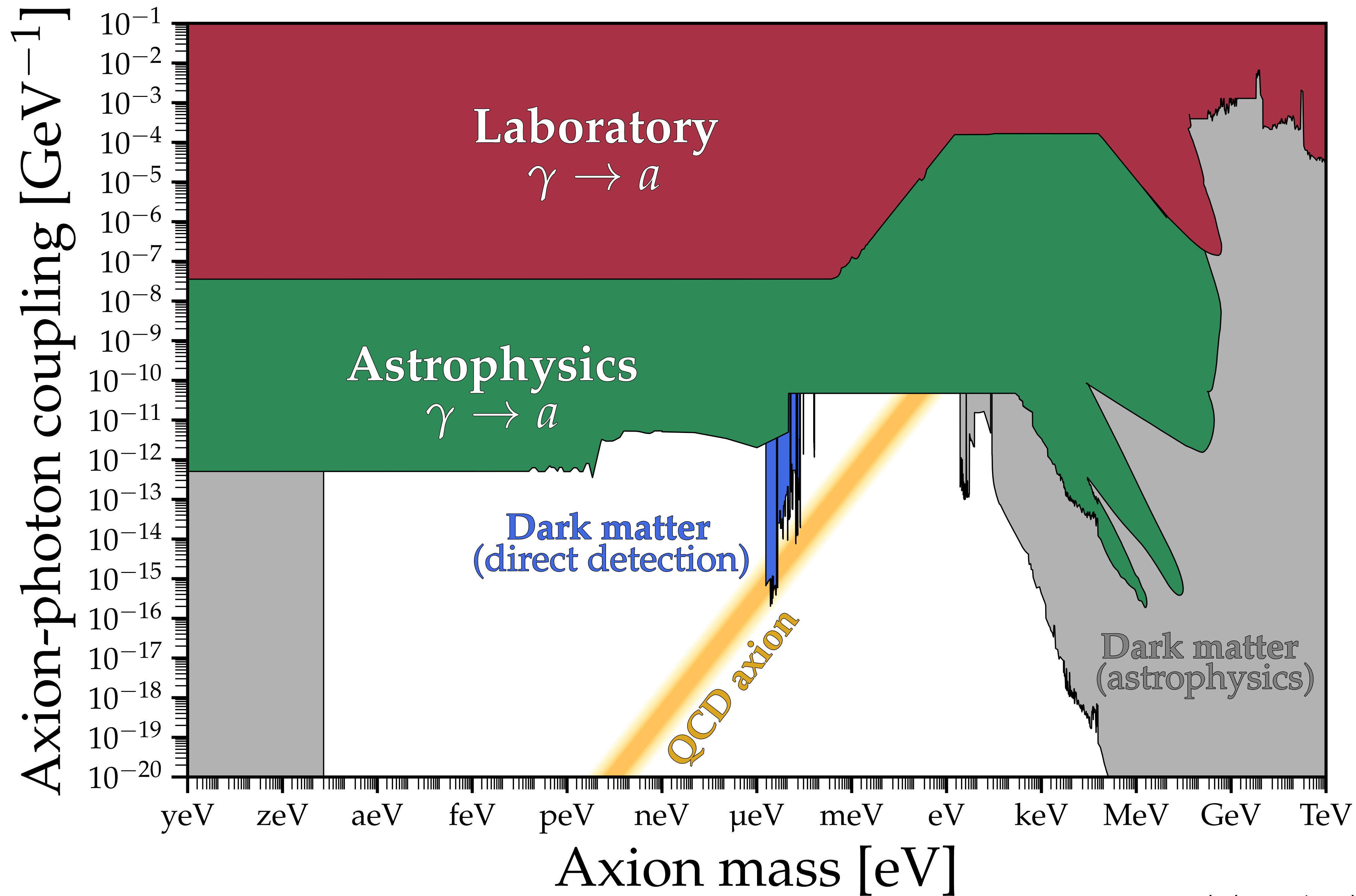
- CMB-Stage 4 is targeting $\Delta N_{\text{eff}} = 0.03$
- Could constrain axions decoupling before EWPT
- Quite plausible we could detect a relic population of thermal axions

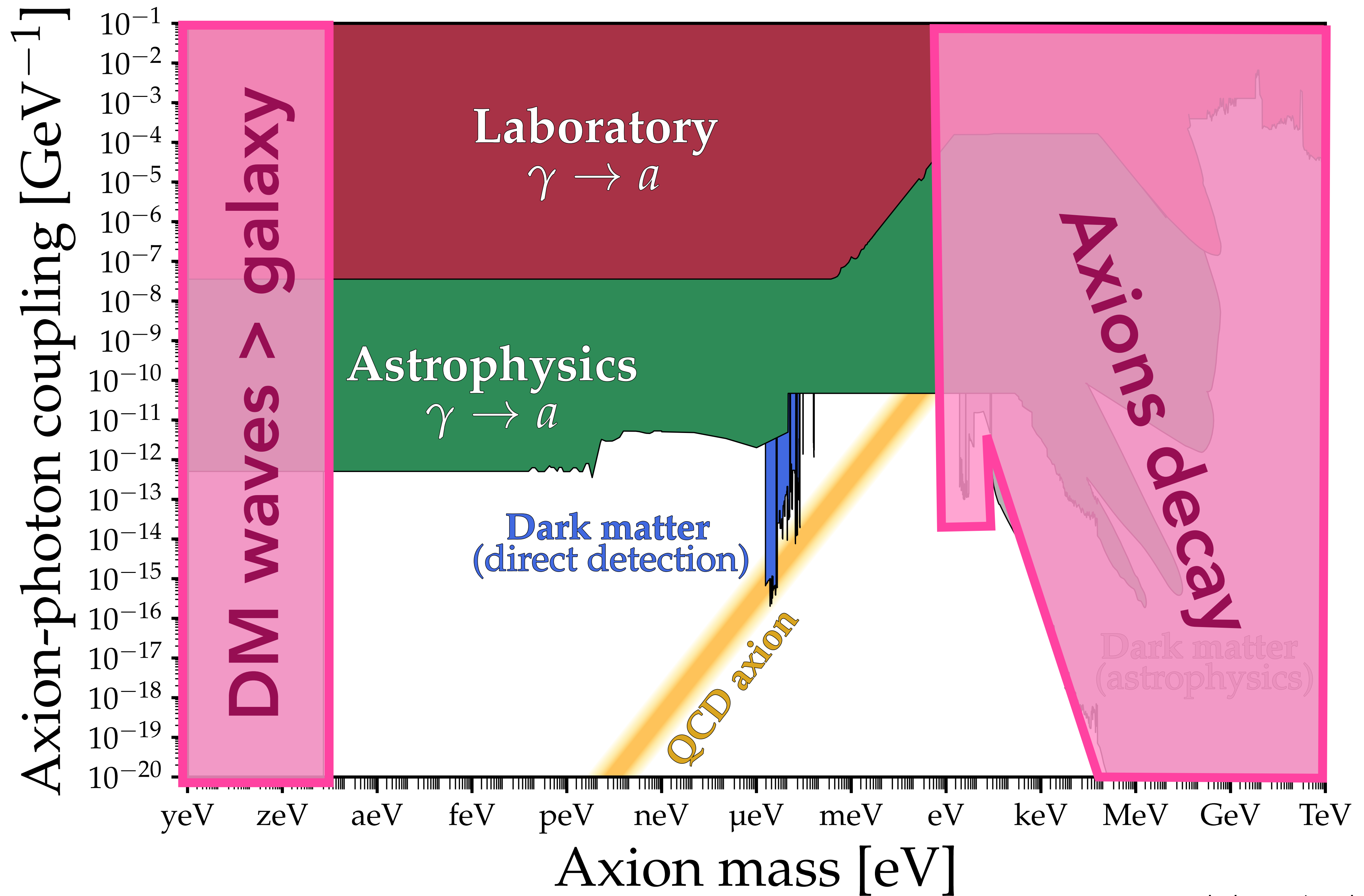




Axion-photon coupling $[\text{GeV}^{-1}]$



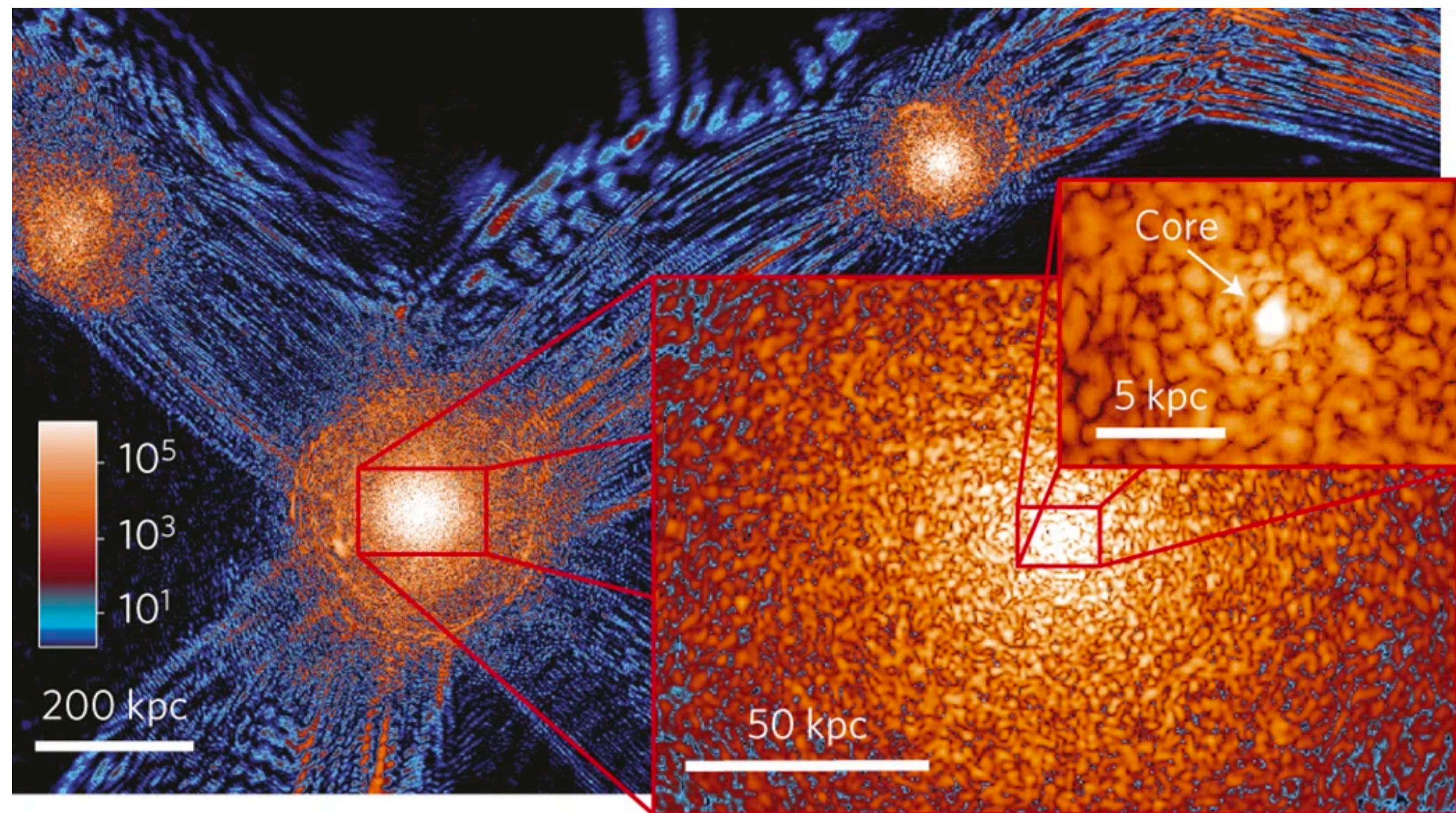




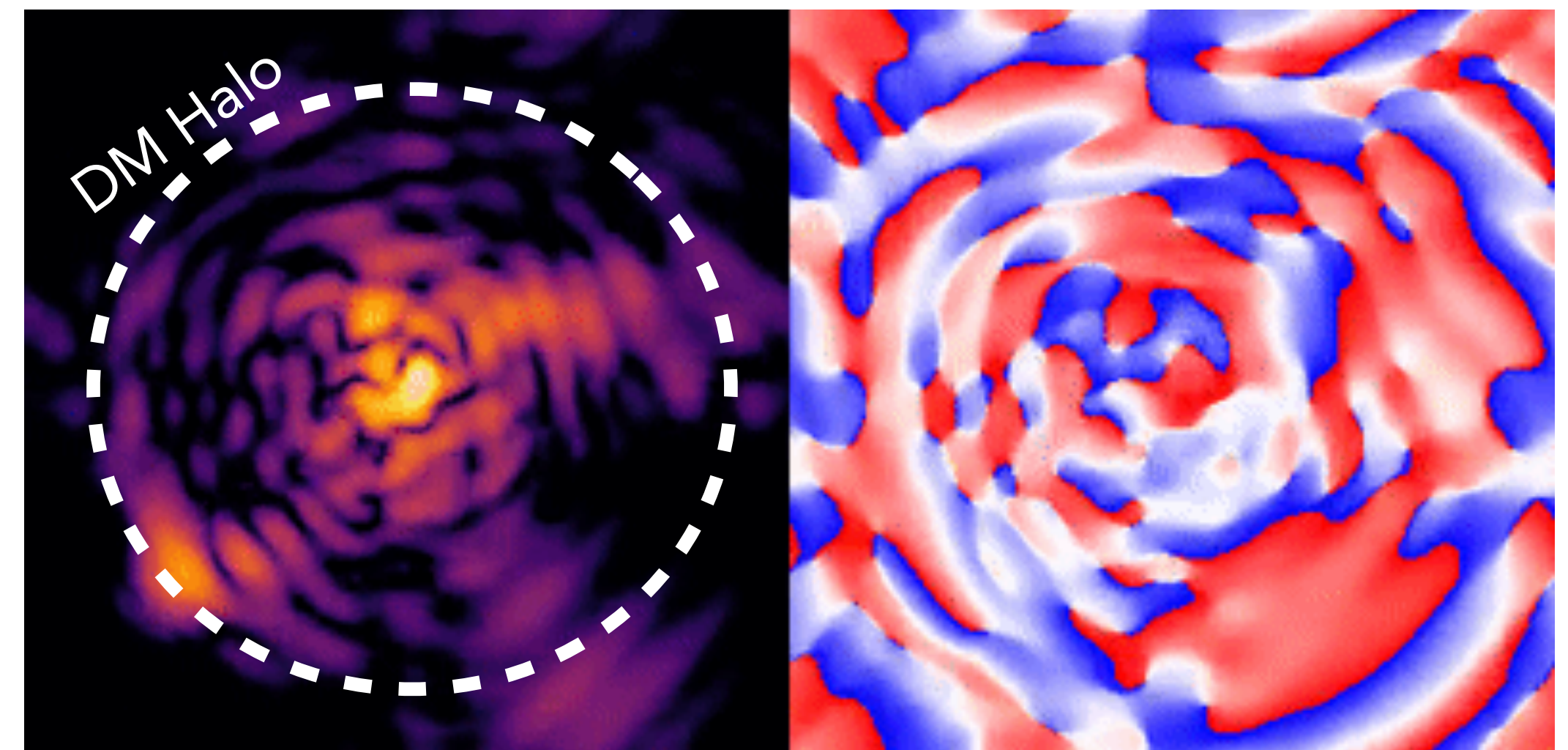
Lower bound on the axion mass: fuzzy dark matter

- Wave-like DM has a finite “incompressible” size given by its wave coherence length $\lambda = 1/m_\phi v$.
- Perturbations below an effective Jeans scale ($1/k_J$) cannot grow due to wave effects
- For very small m_ϕ this could become of comparable size to DM halos that we know exist

Schive+ [1406.6586]



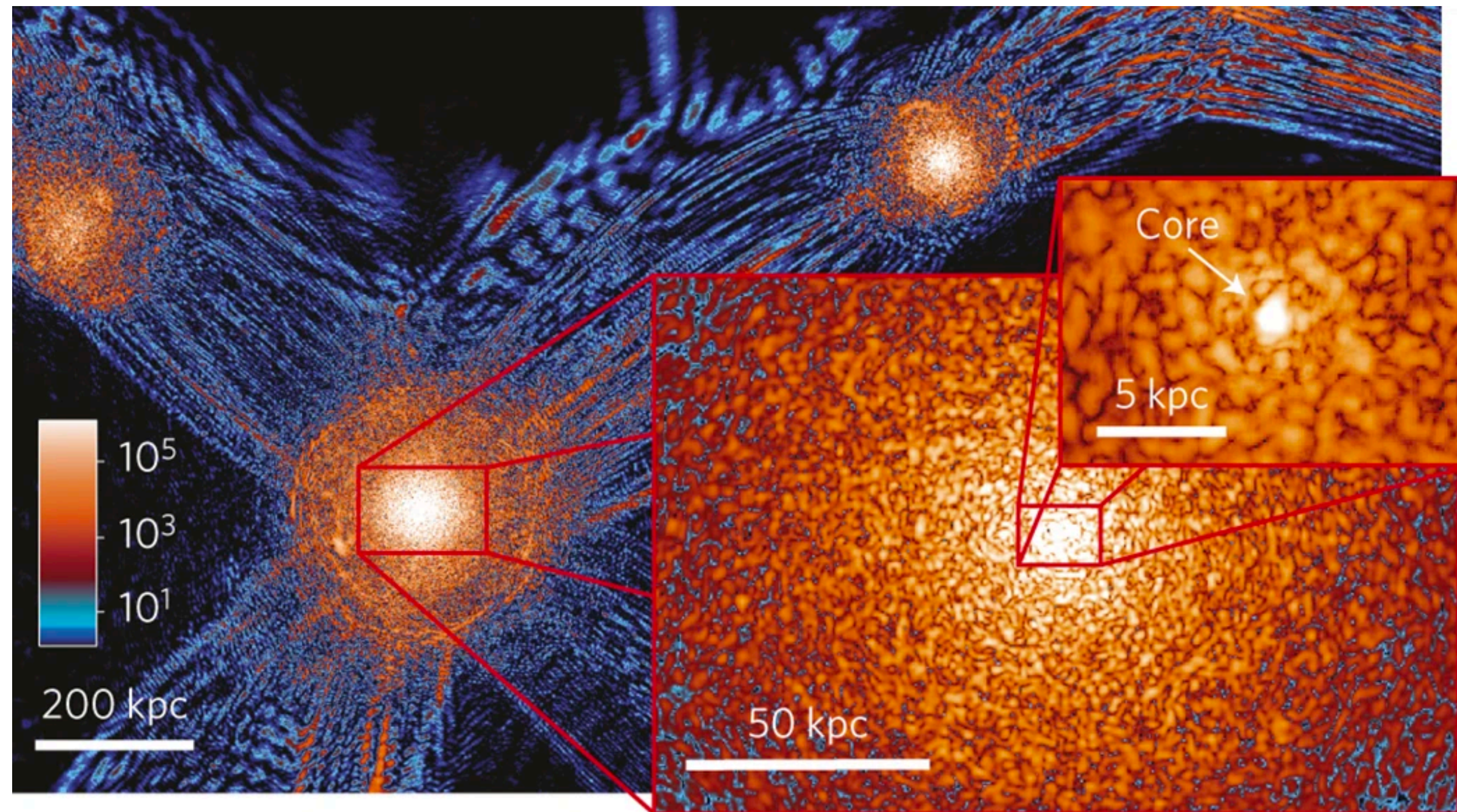
$$k_{J,\text{eq}} = 8.7 \text{ Mpc}^{-1} \left(\frac{m_\phi}{10^{-22} \text{ eV}} \right)^{1/2}$$



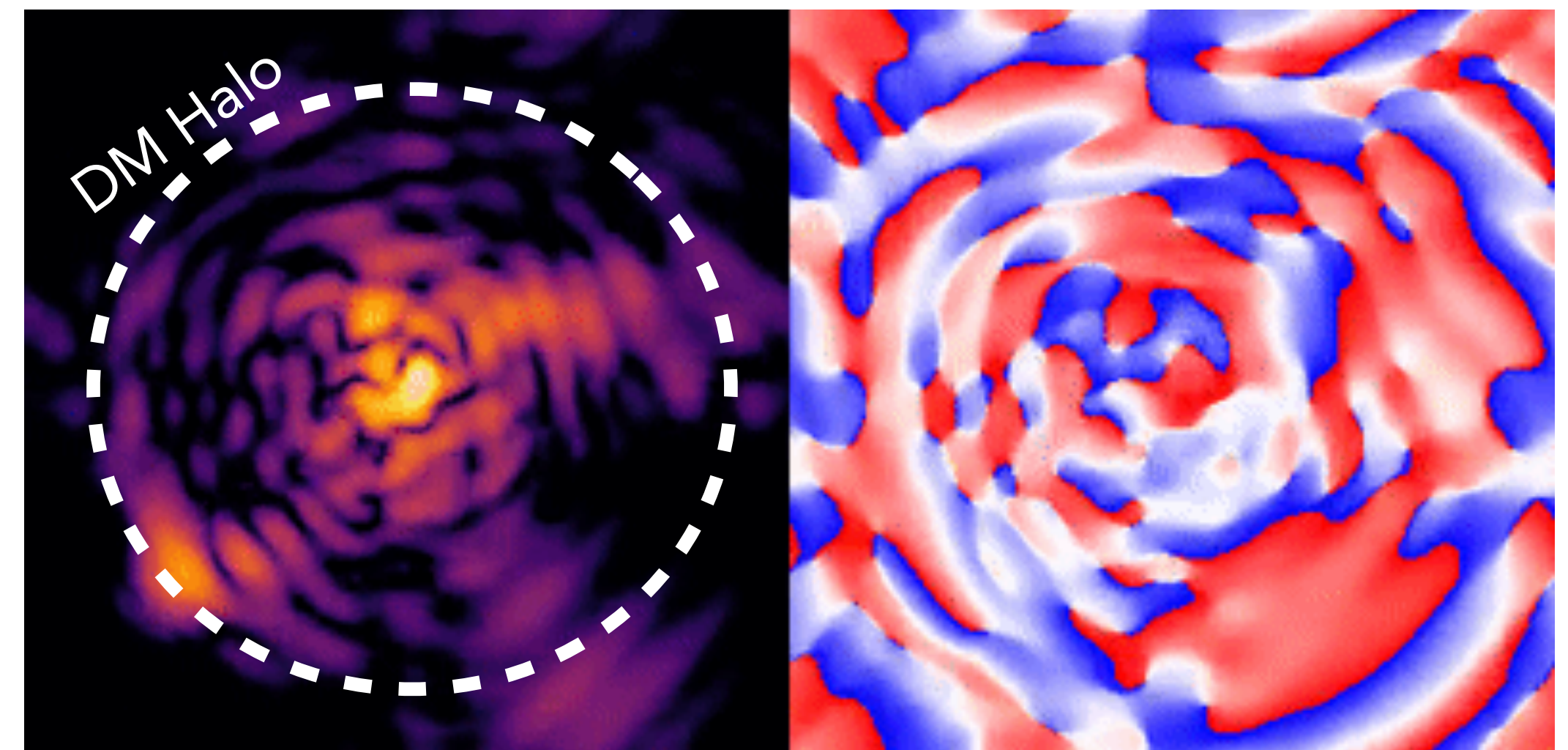
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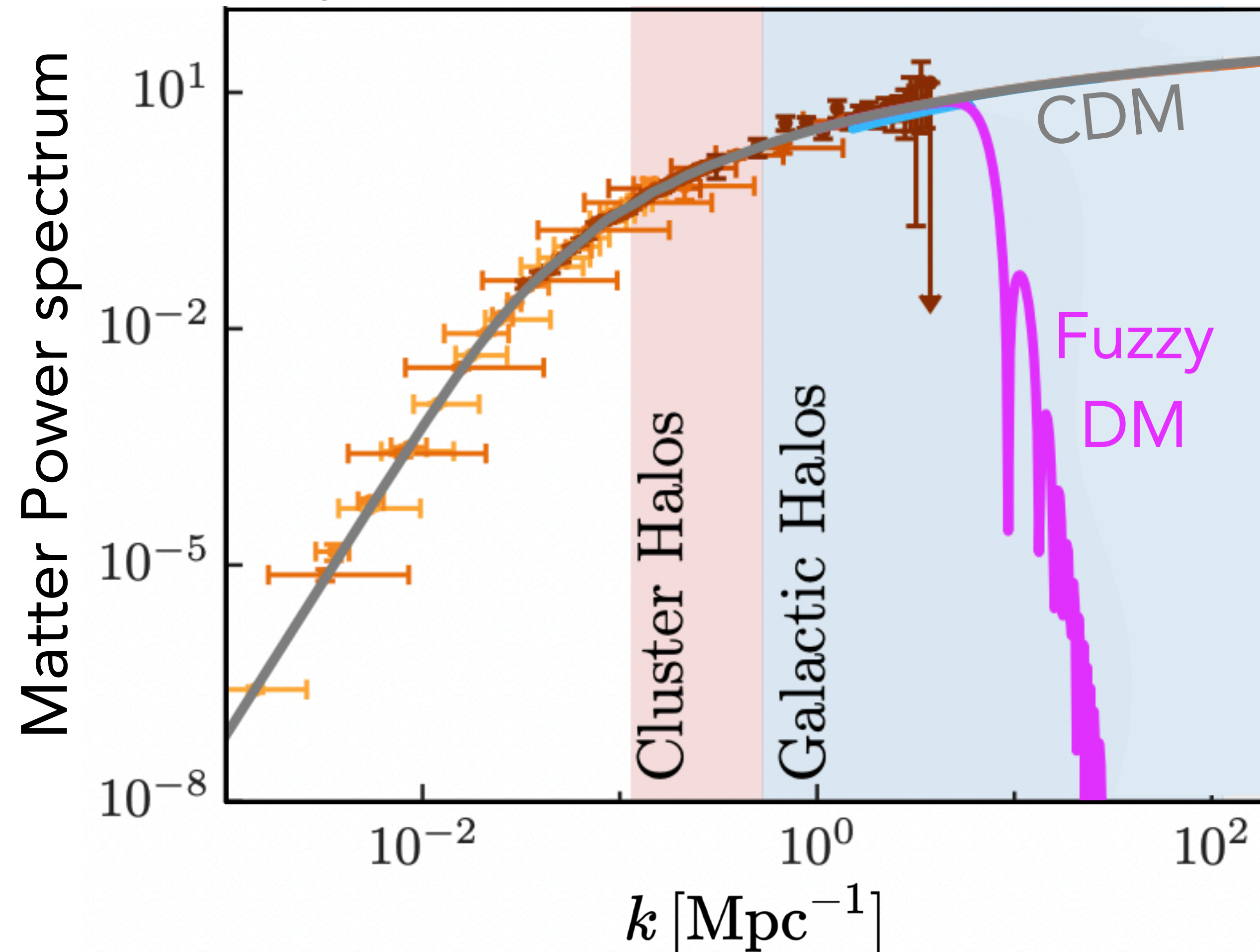


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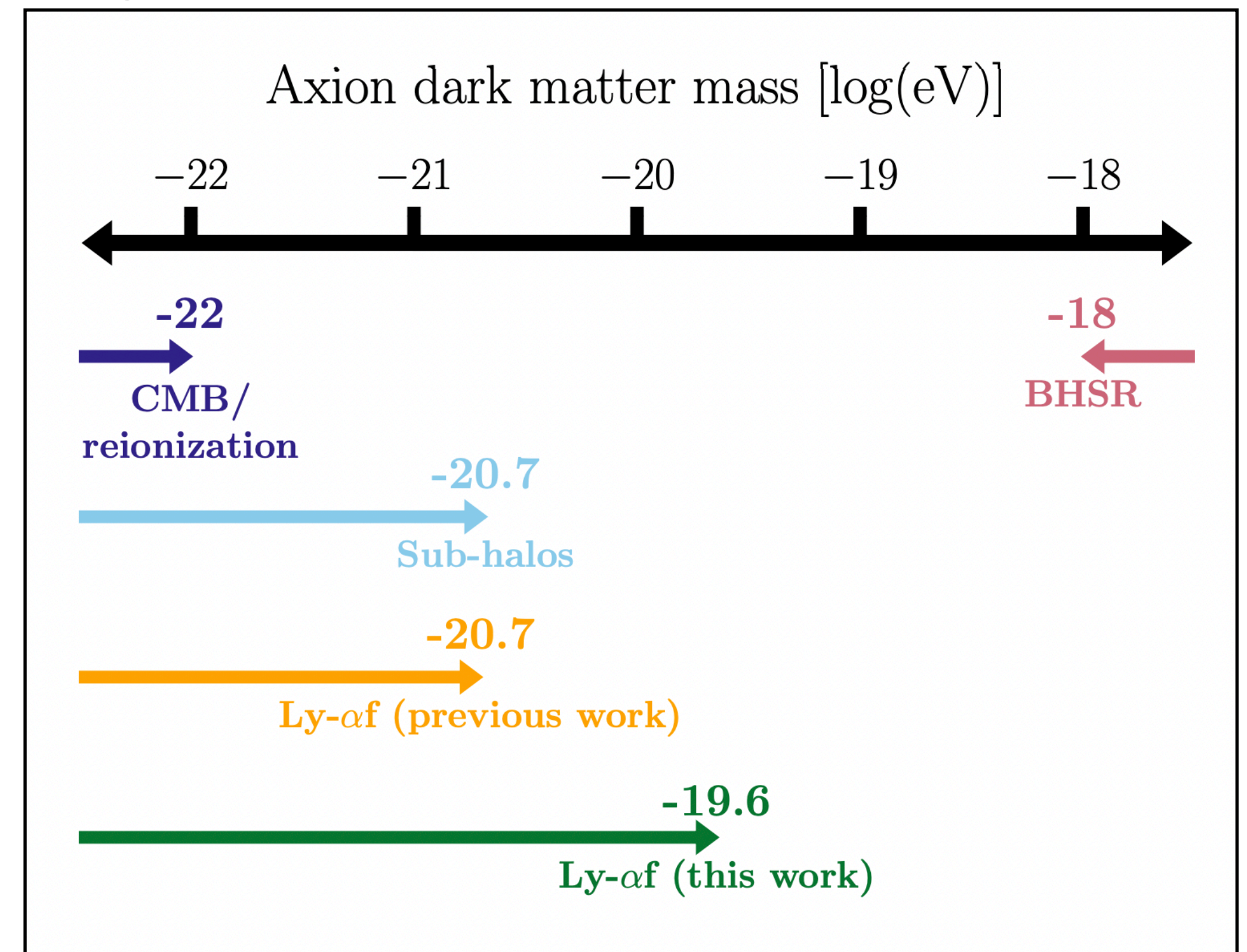


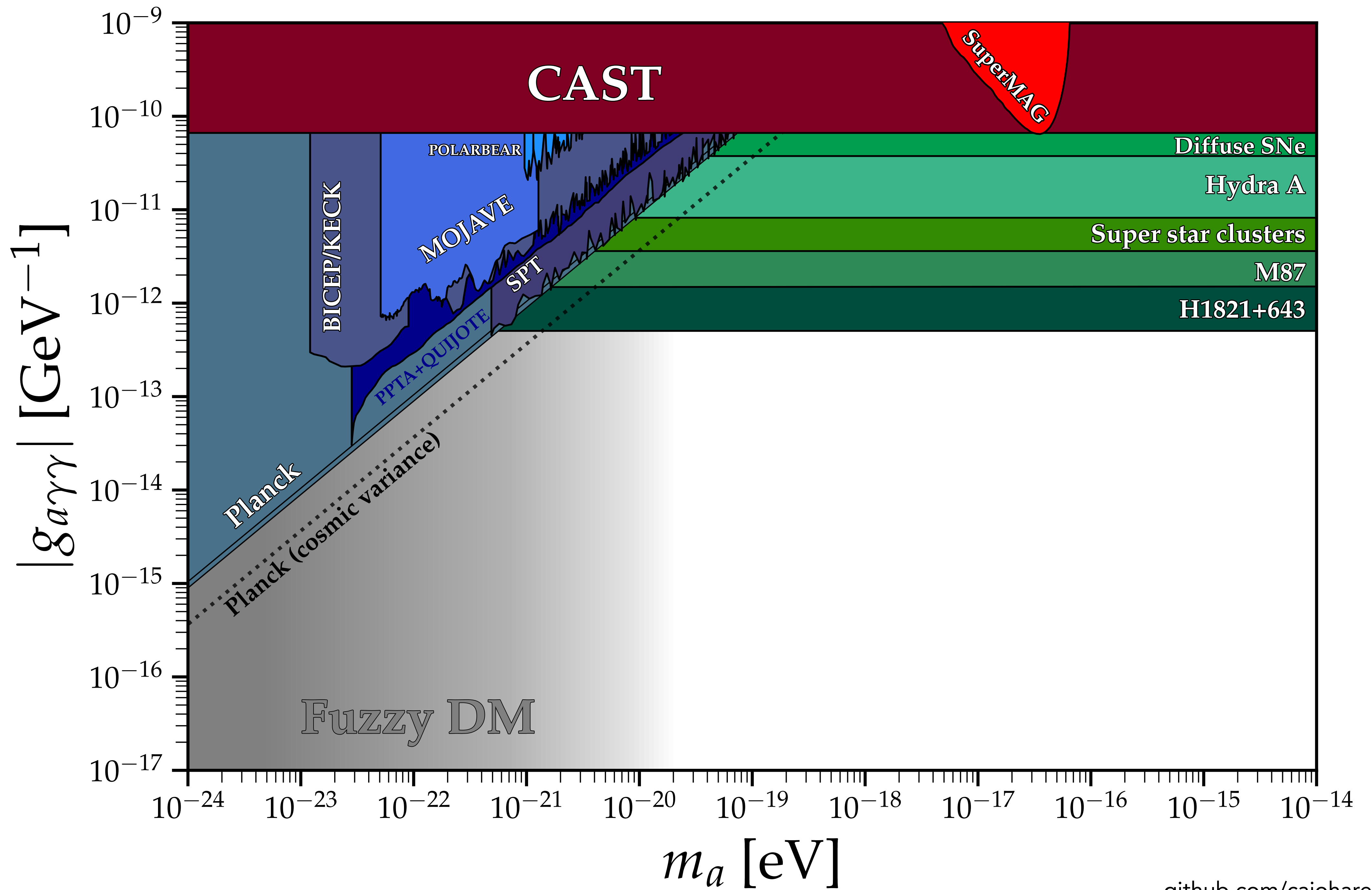
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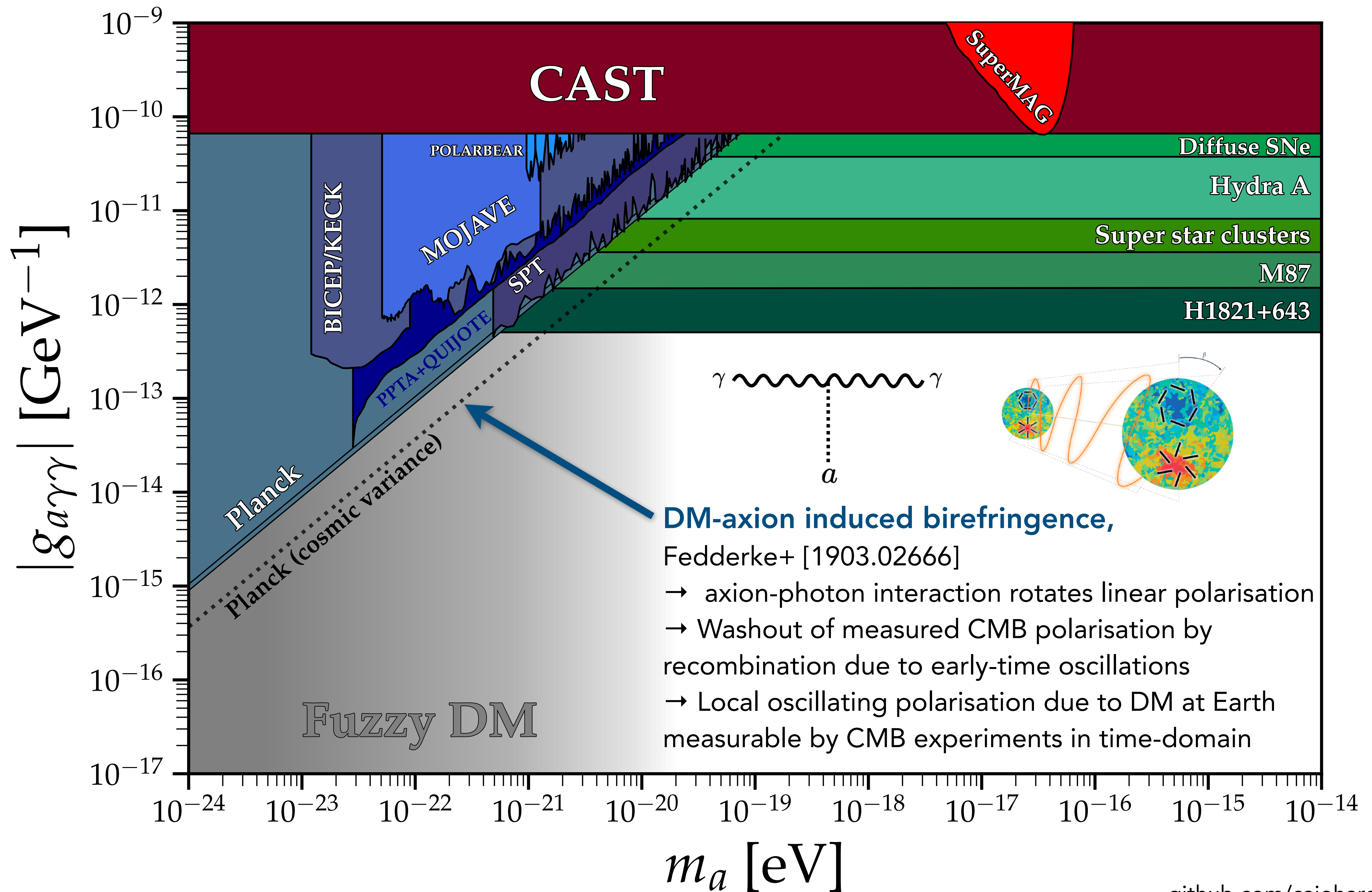
Adapted from [2203.07354]

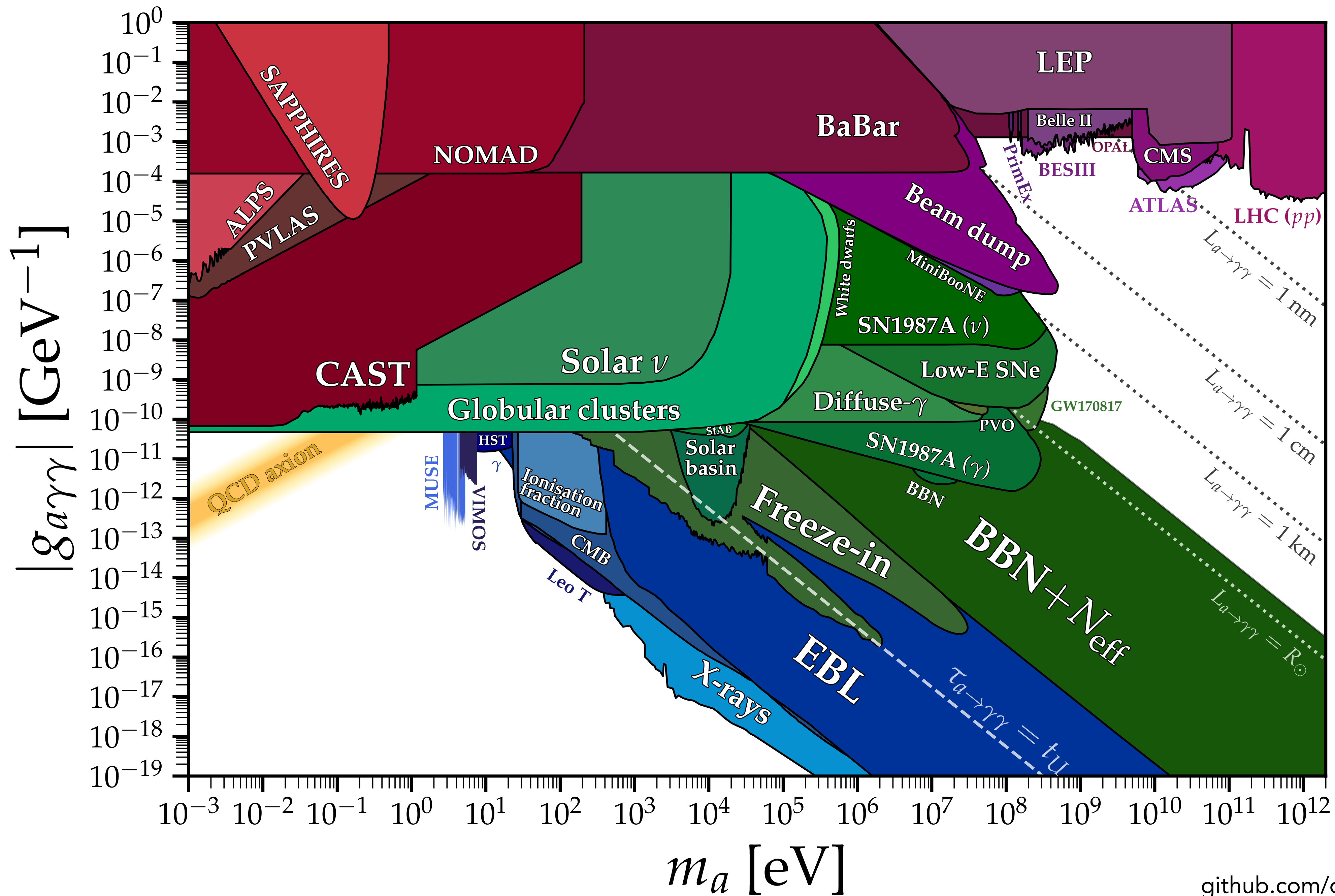


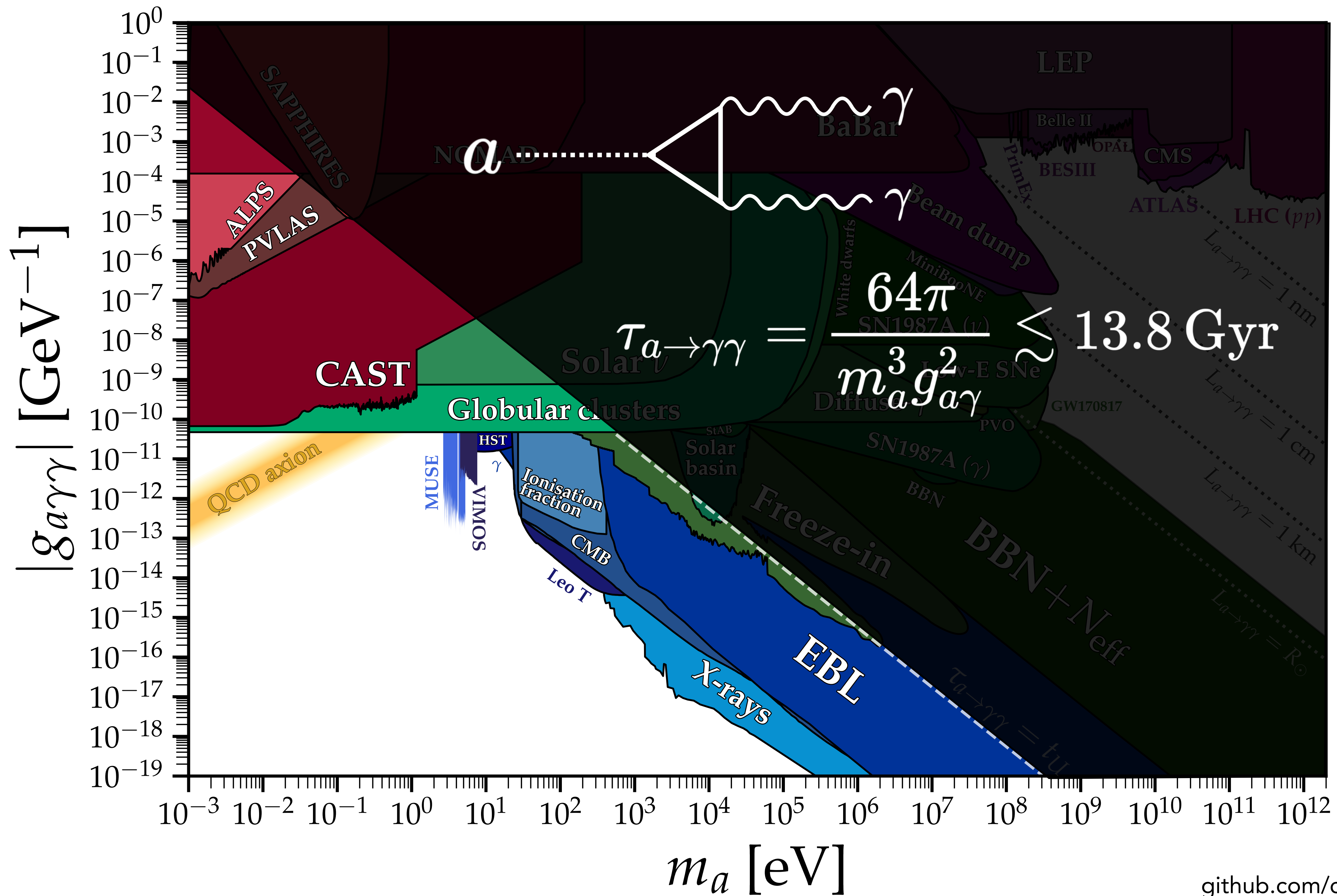
Rogers+ [2007.12705]









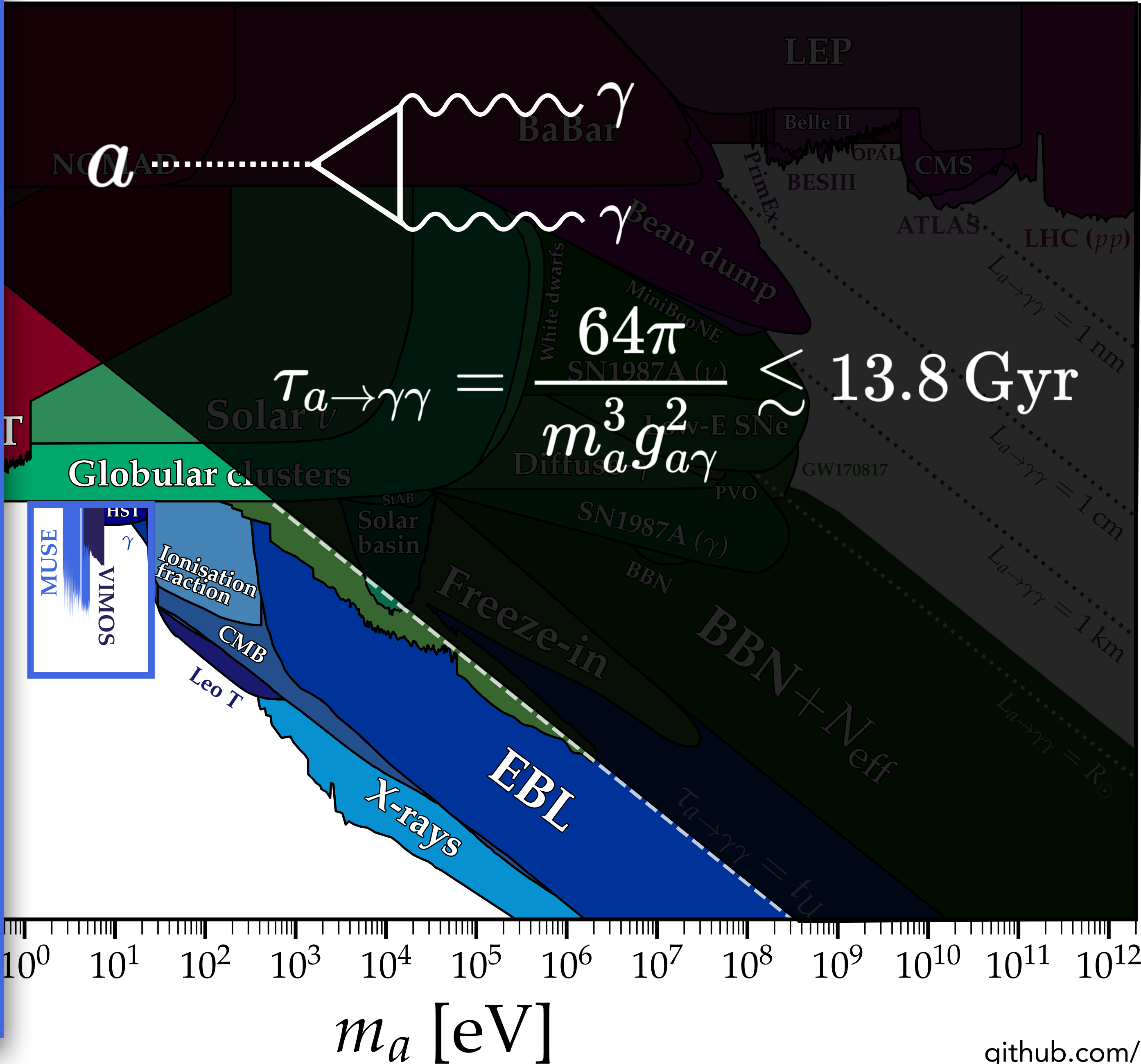
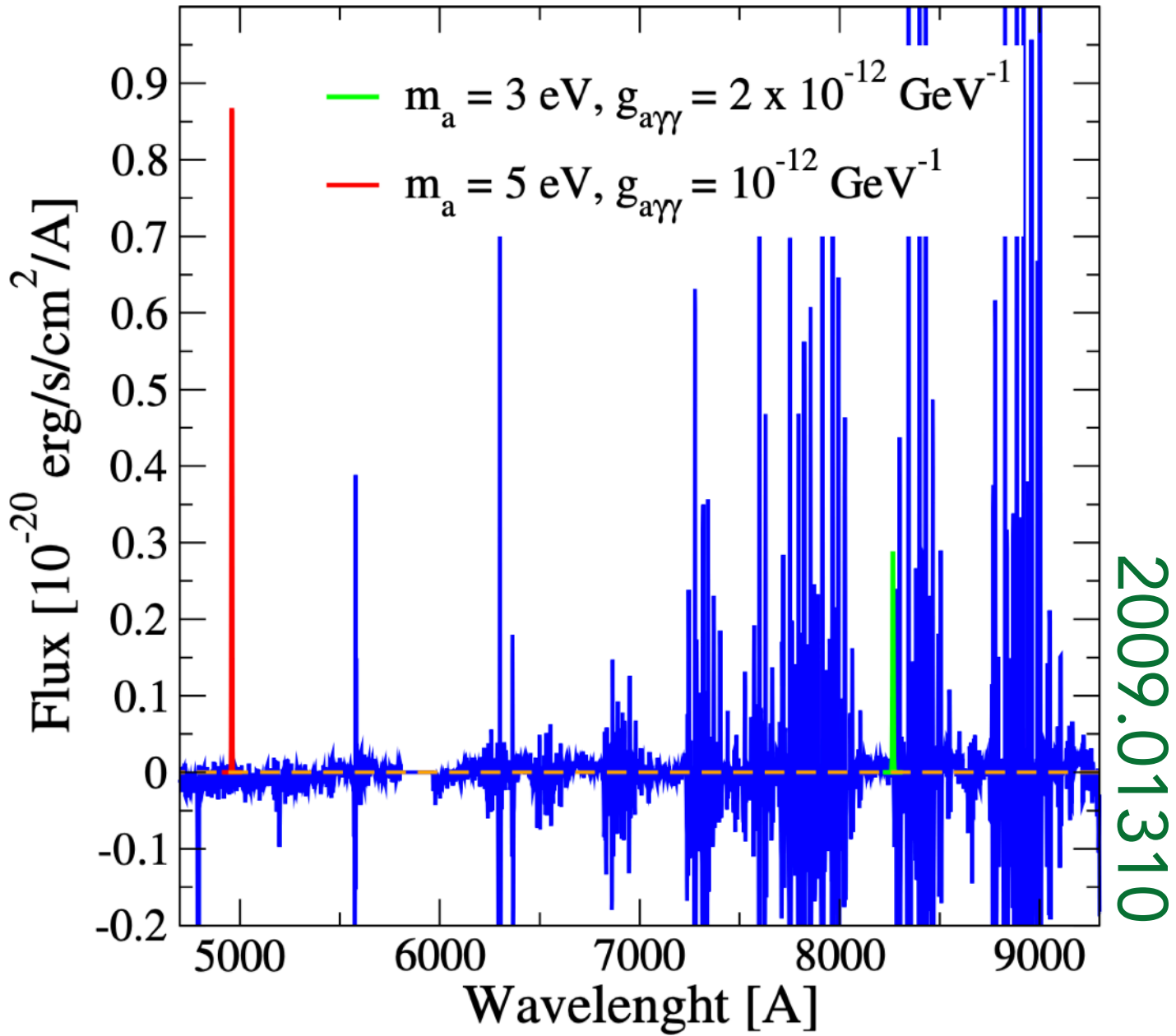


Search for **axion decay line** at $\omega_\gamma = m_a/2$
 in galaxy spectra (e.g. MW dwarf
 spheroidals w/ high DM densities)
 See recent Todorello+ [2307.07403]



Leo-T dwarf

$$M \sim 8 \times 10^6 M_\odot$$



“Near-field cosmology”

→ The structure and formation history of
our own dark matter halo

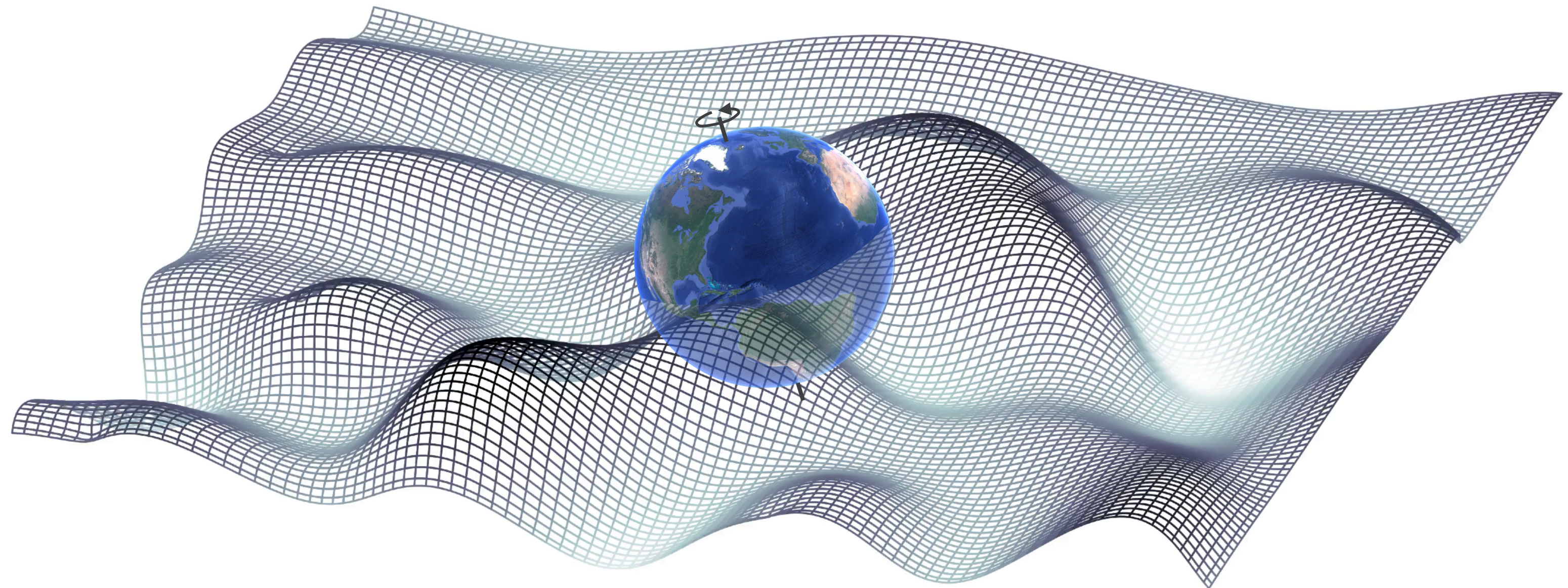
Axions in our galaxy: Wave-like dark matter

We know the local mass density of DM ($\rho_{\text{DM}} \approx 0.4 \text{ GeV/cm}^3$), but not the number density

Occupancy = Number of particles per de Broglie volume: $\mathcal{N} \approx (\rho_{\text{DM}}/m) \times \lambda_{\text{dB}}^3$

Typical DM velocity in galaxy $v = 300 \text{ km/s} = 10^{-3}c$

$$\Rightarrow \mathcal{N} \approx 10^{32} \left(\frac{100 \mu\text{eV}}{m_a} \right)^4$$



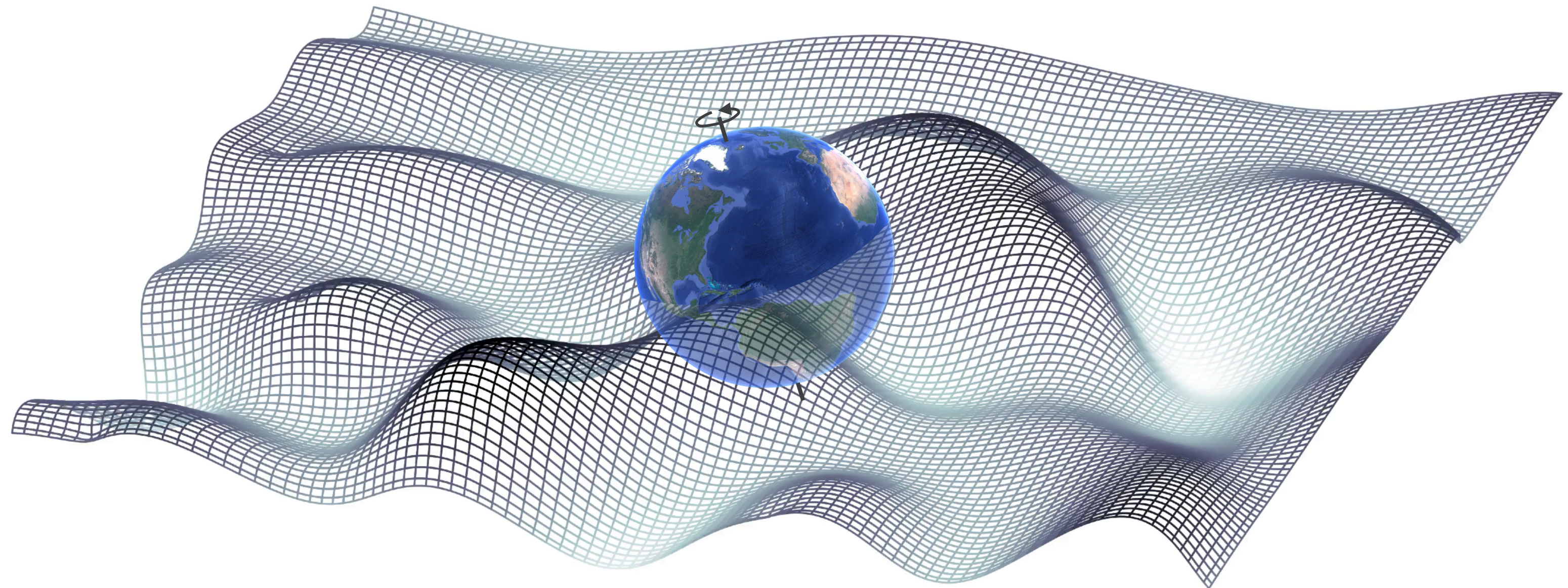
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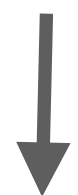
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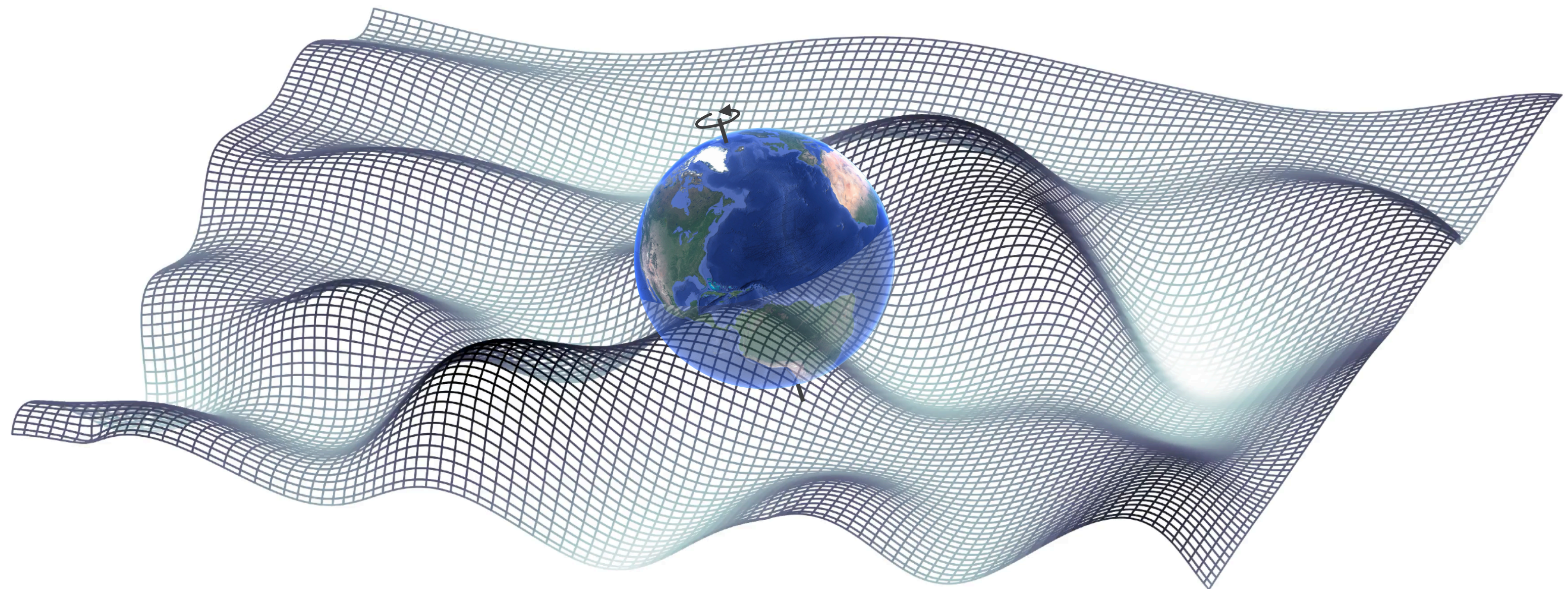
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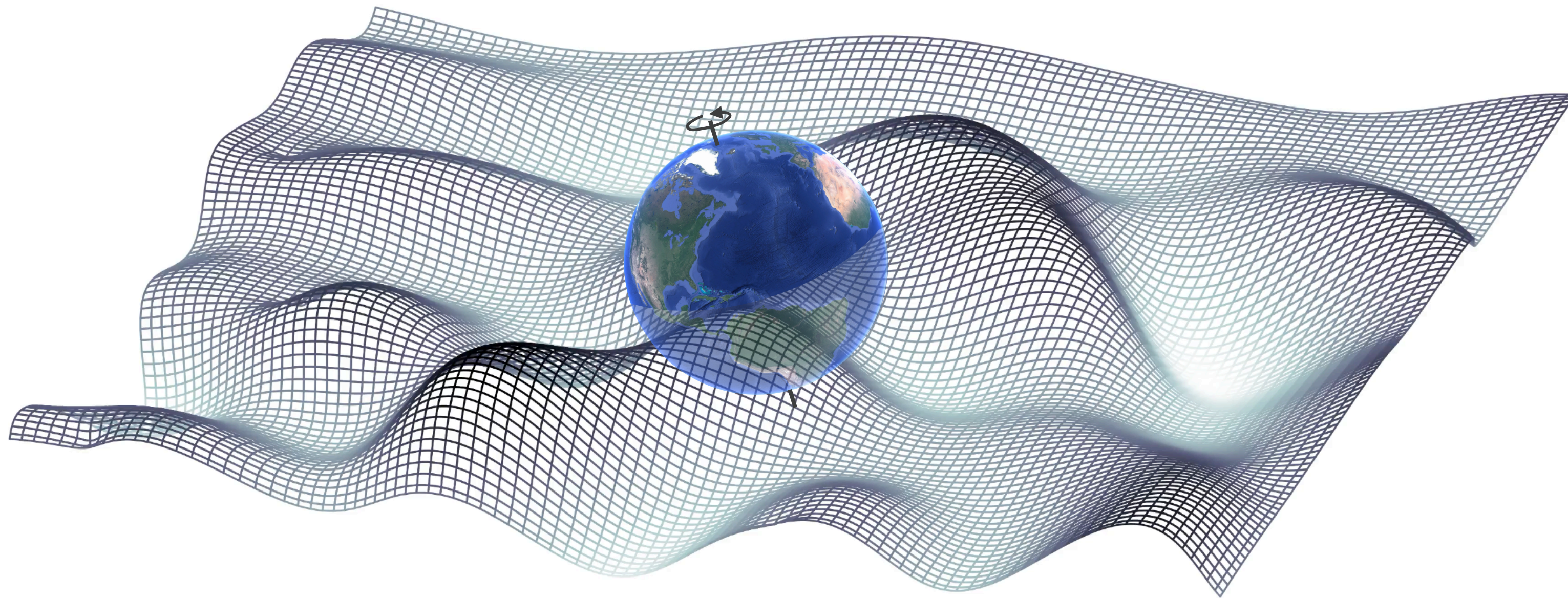
Macroscopically
occupied ground state



Classical field

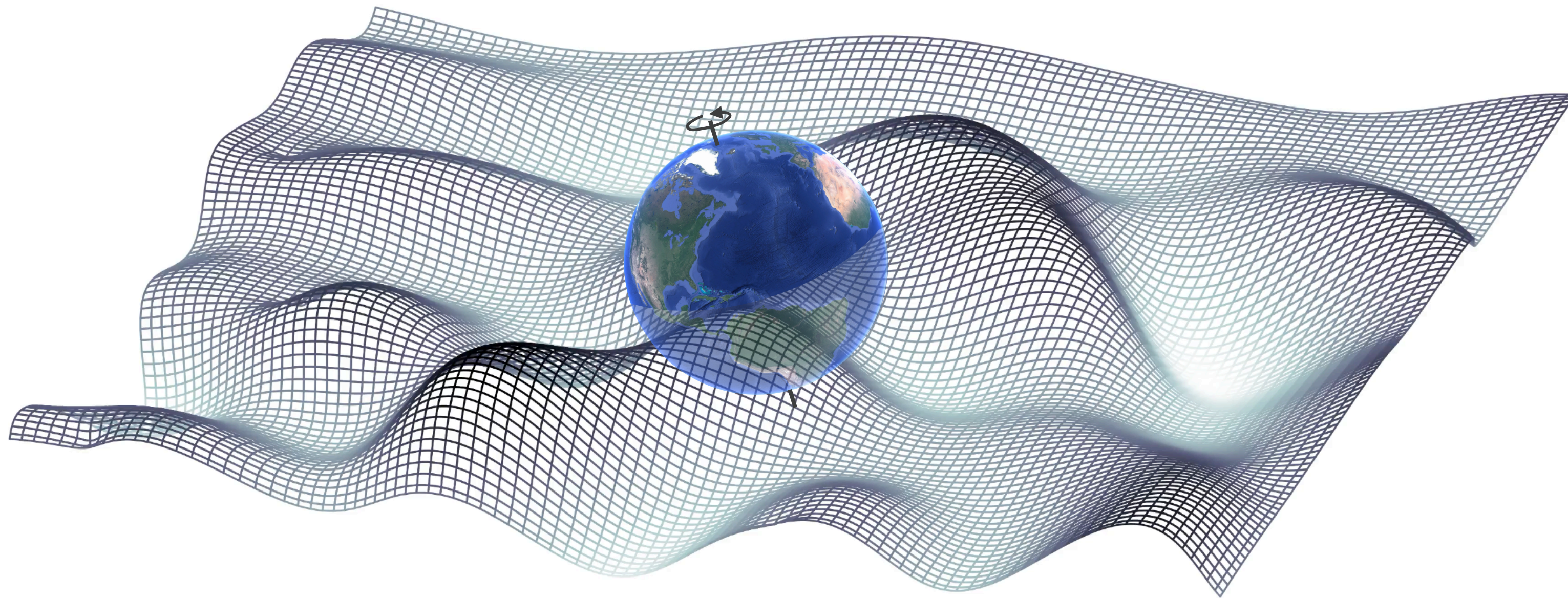


Wave-like dark matter



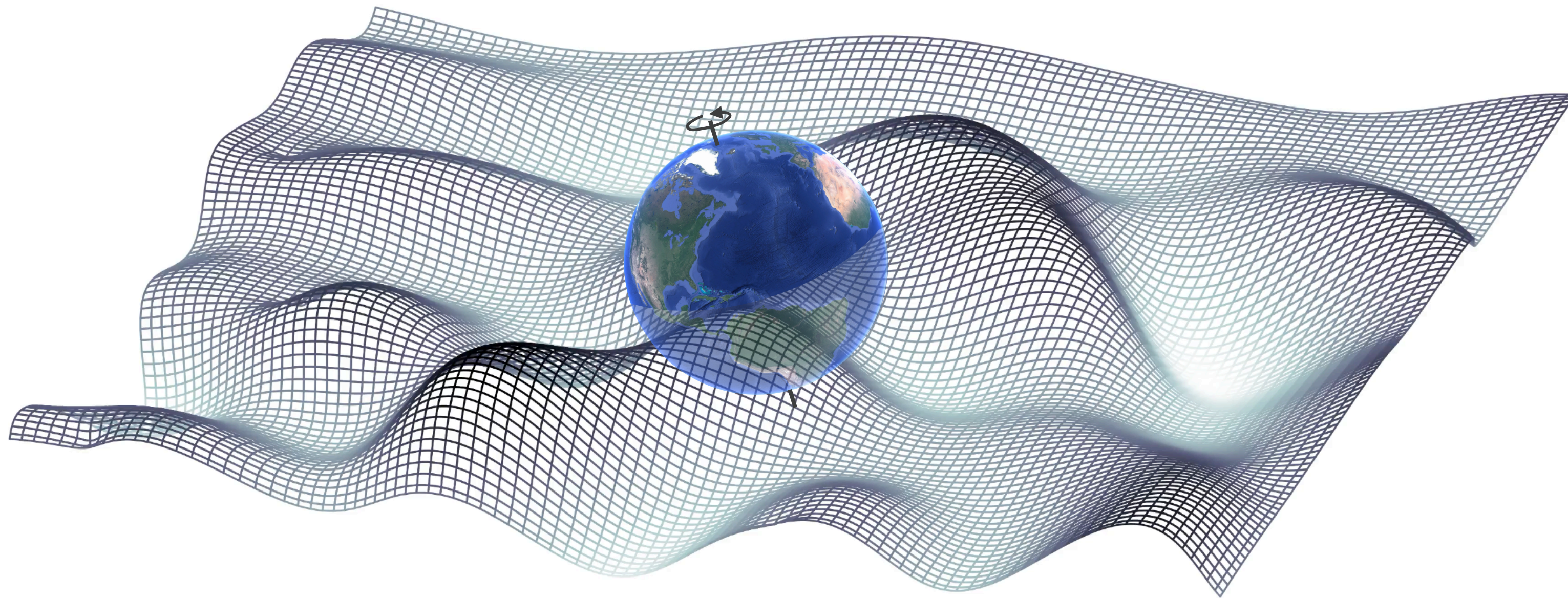
$$\phi(t) \approx A \cos \omega t$$

Wave-like dark matter



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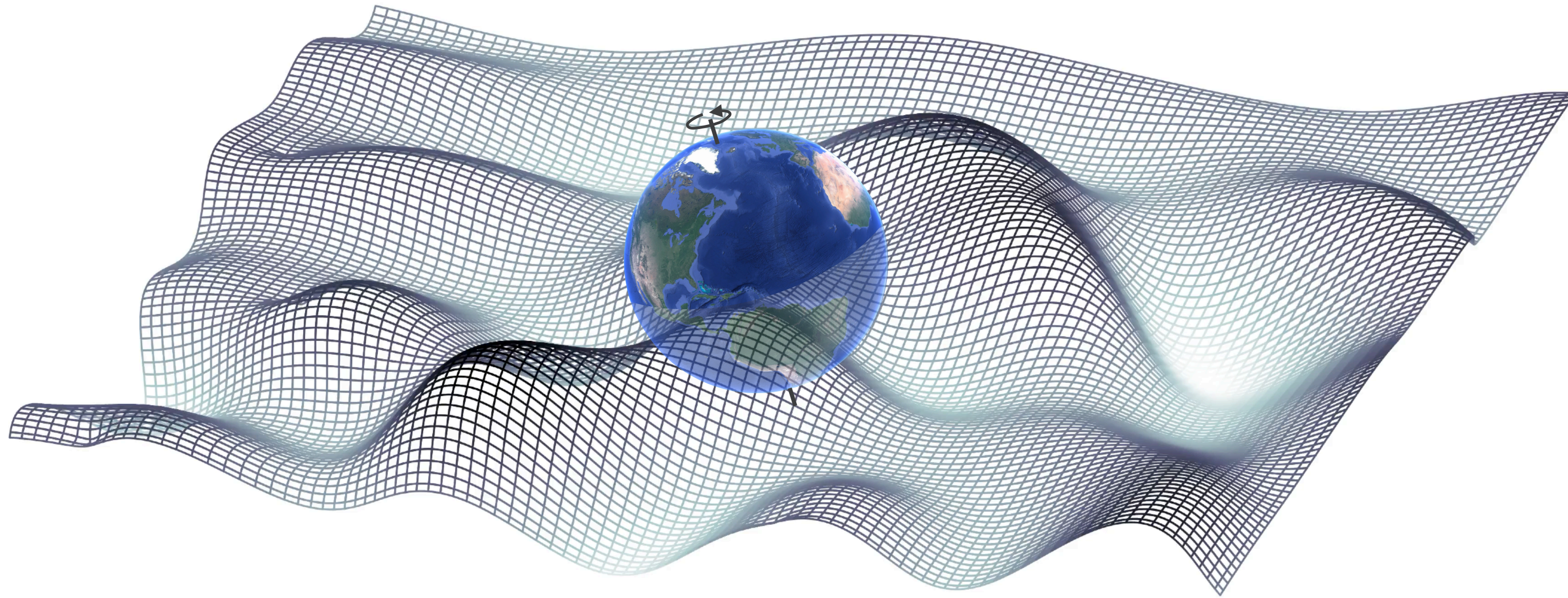
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Amplitude: $A = \frac{\sqrt{2\rho_{\text{DM}}}}{m}$

Wave-like dark matter

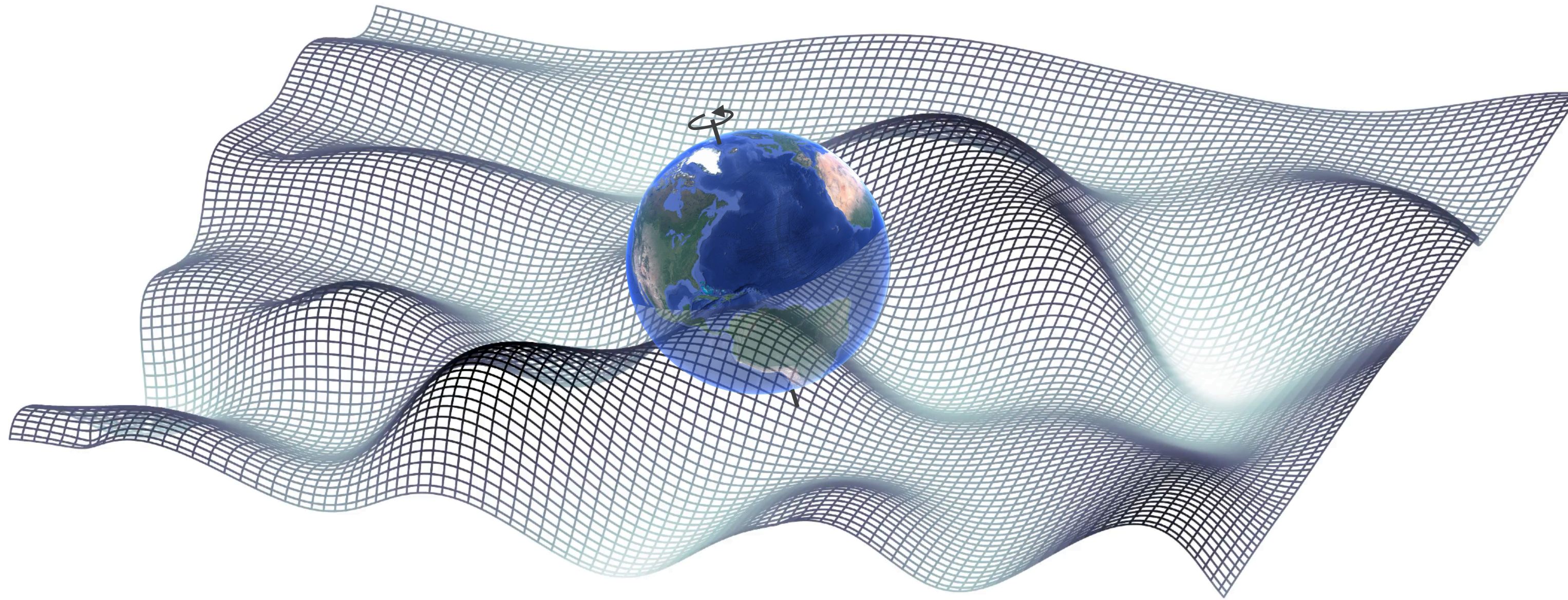


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Amplitude: $A = \frac{\sqrt{2\rho_{\text{DM}}}}{m}$

Frequency: $\omega = m + \frac{1}{2}mv^2$
 $\approx m \underbrace{\left(1 + 10^{-6}\right)}$
Oscillation remains
coherent for 10^6 cycles

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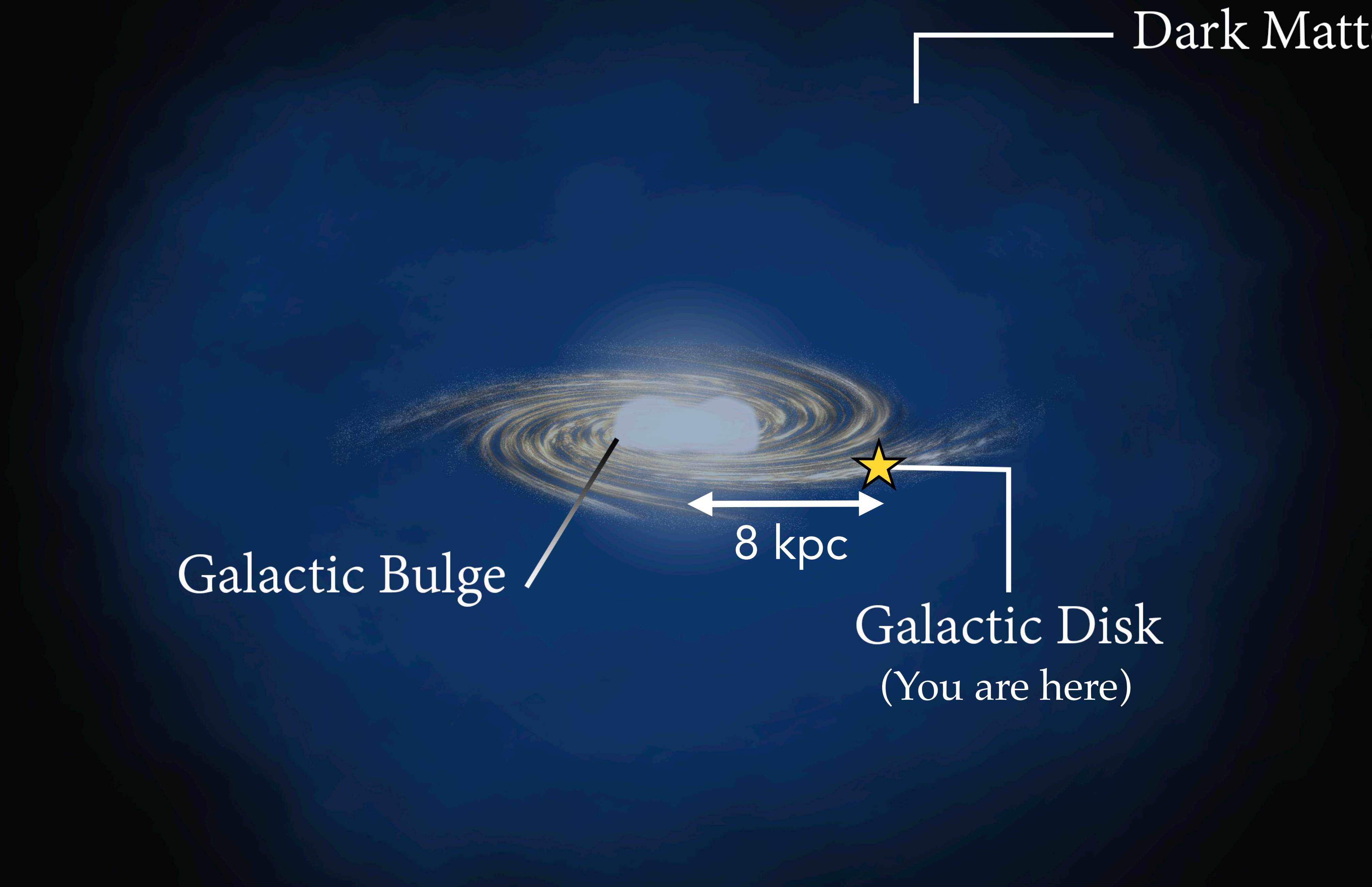
How do we know what to put in for density and velocity?

Dark Matter Halo

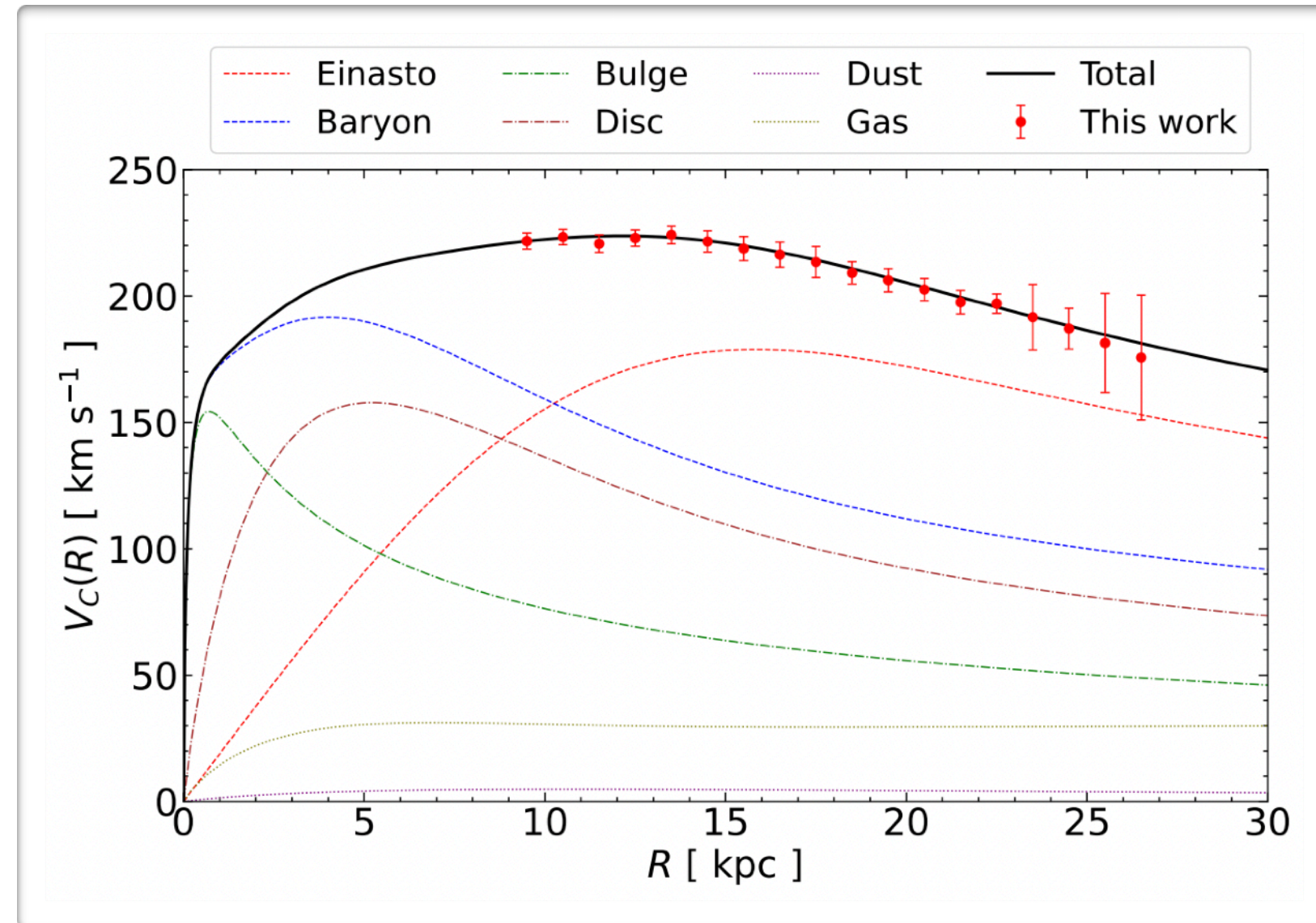
Galactic Bulge

8 kpc

Galactic Disk
(You are here)



[2309.00048]



$$\rho(R = 8 \text{ kpc}) \approx 0.4 \text{ GeV/cm}^3$$

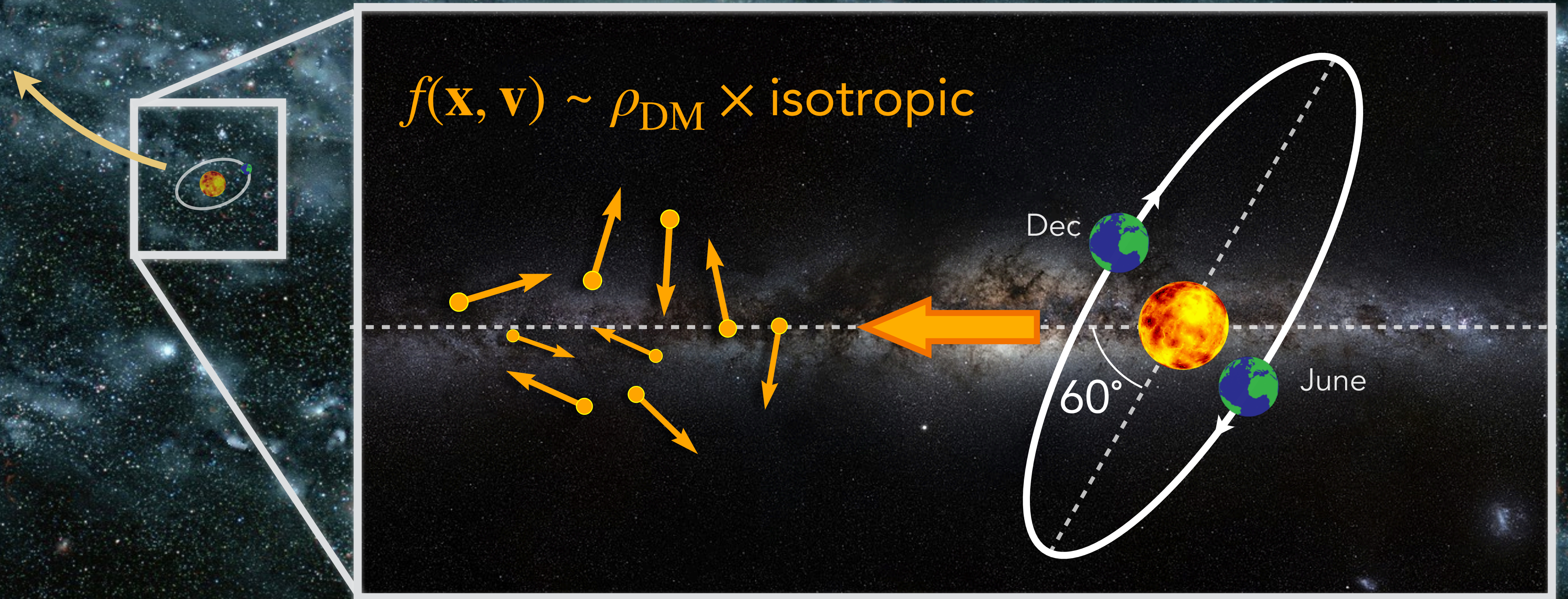
Galactic Bulge

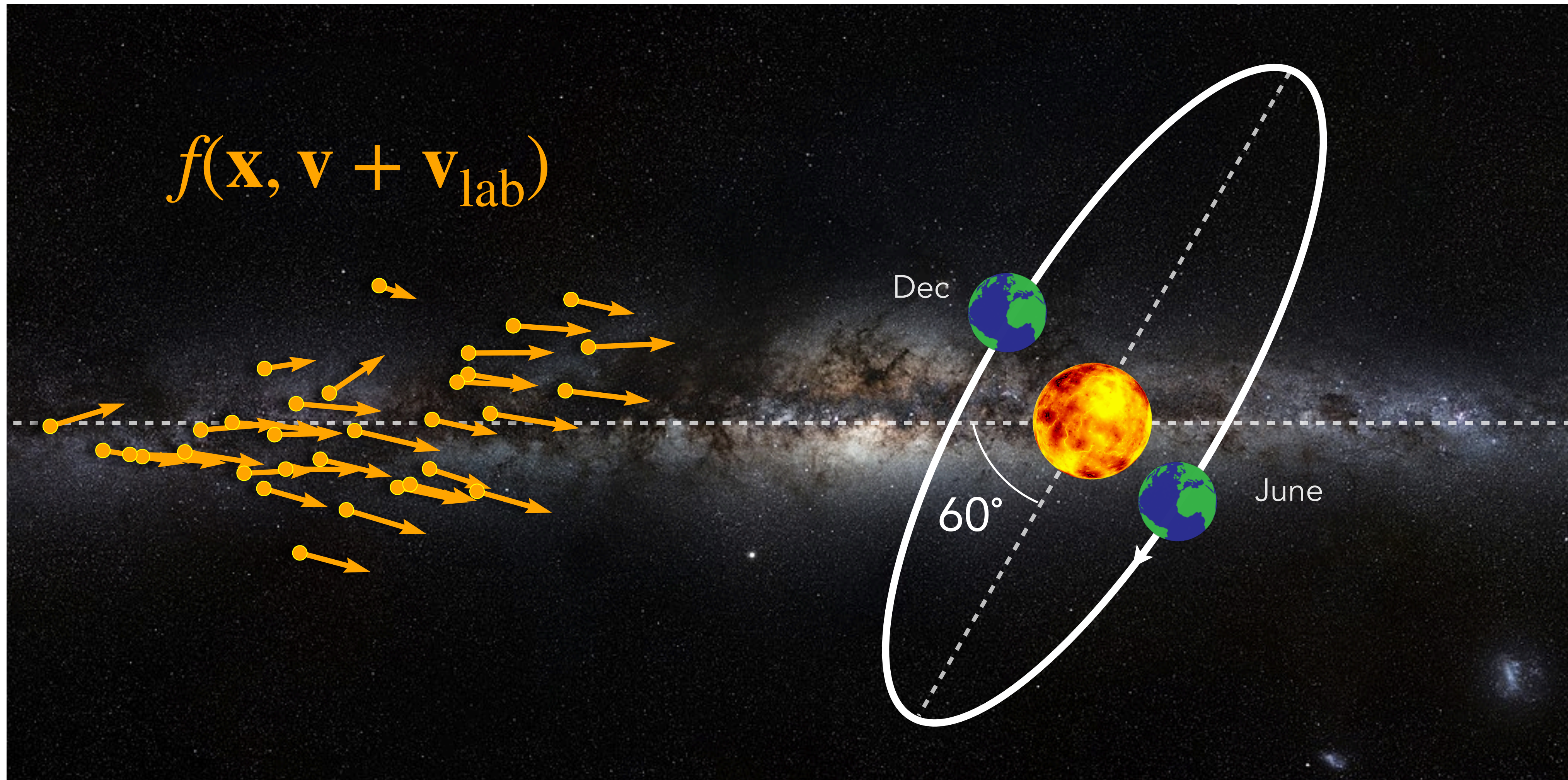
8 kpc

Galactic Disk
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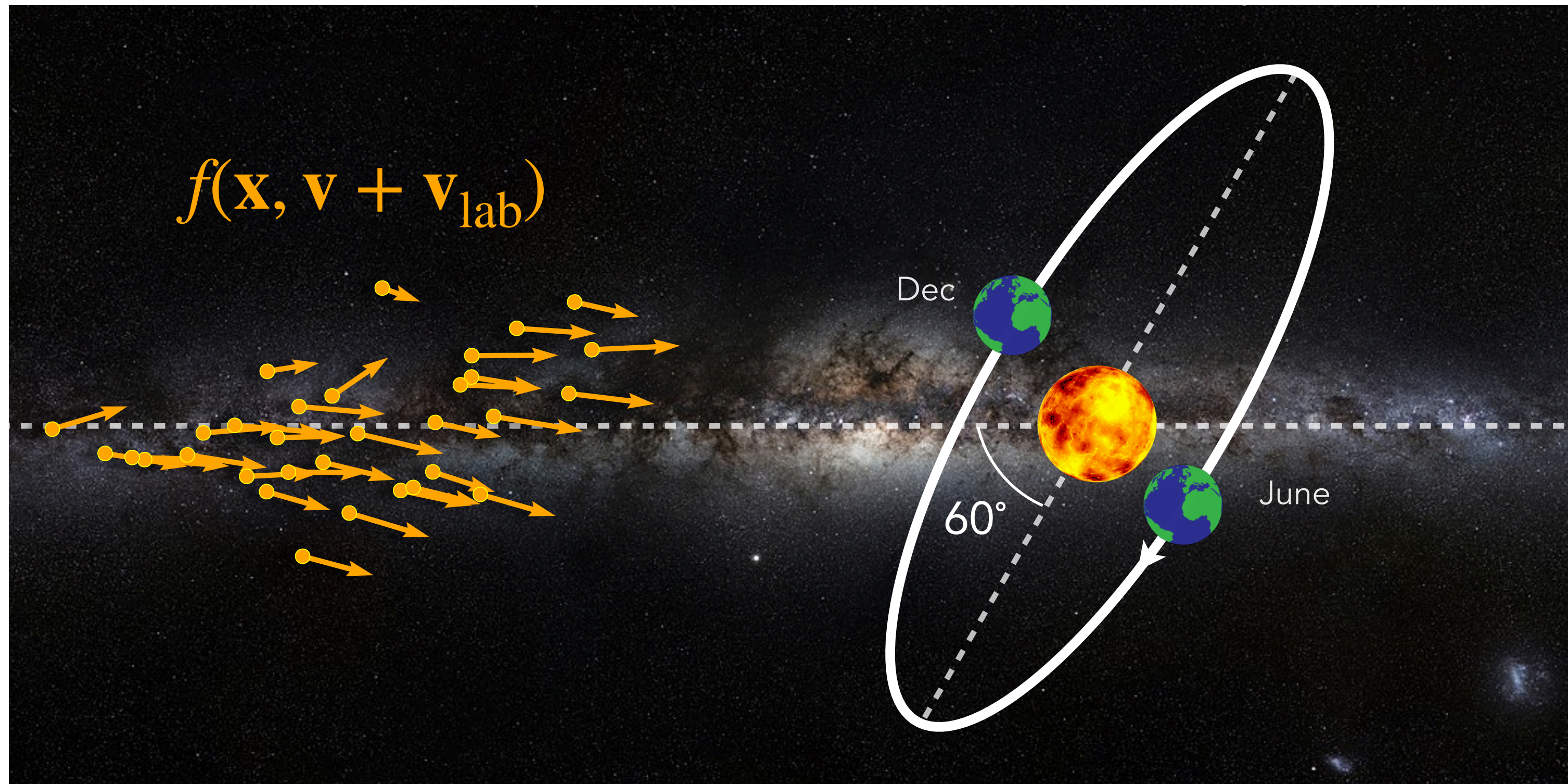
Dark Matter Halo

Distribution function for dark matter in the Solar System





Assuming the dark matter does not co-rotate with the disk, most effects come about from our laboratory's motion **through** the dark matter: $v_{\text{lab}} \sim 300 \text{ km/s}$



Sun/Earth motion leads to characteristic signals independent of DM model

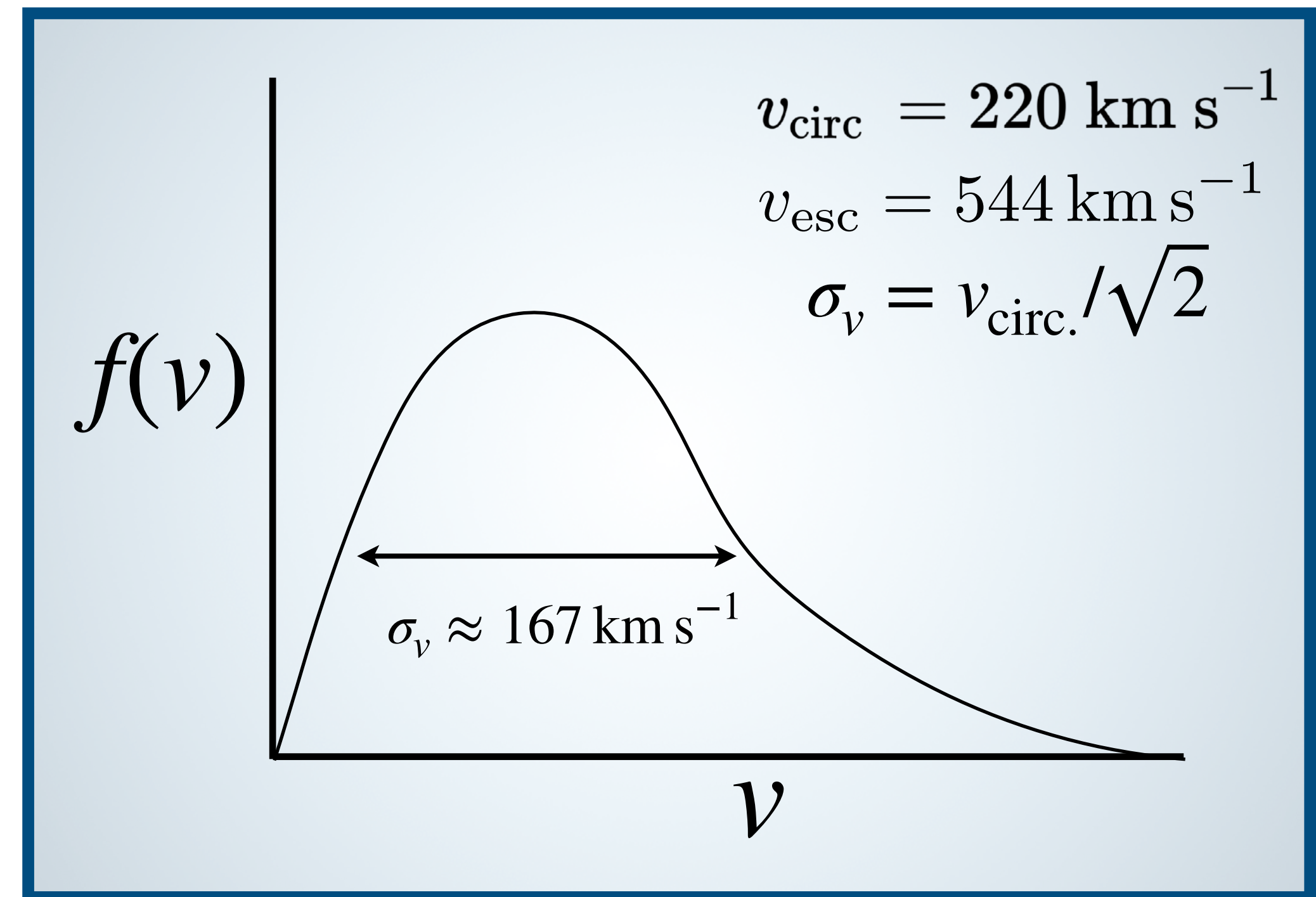
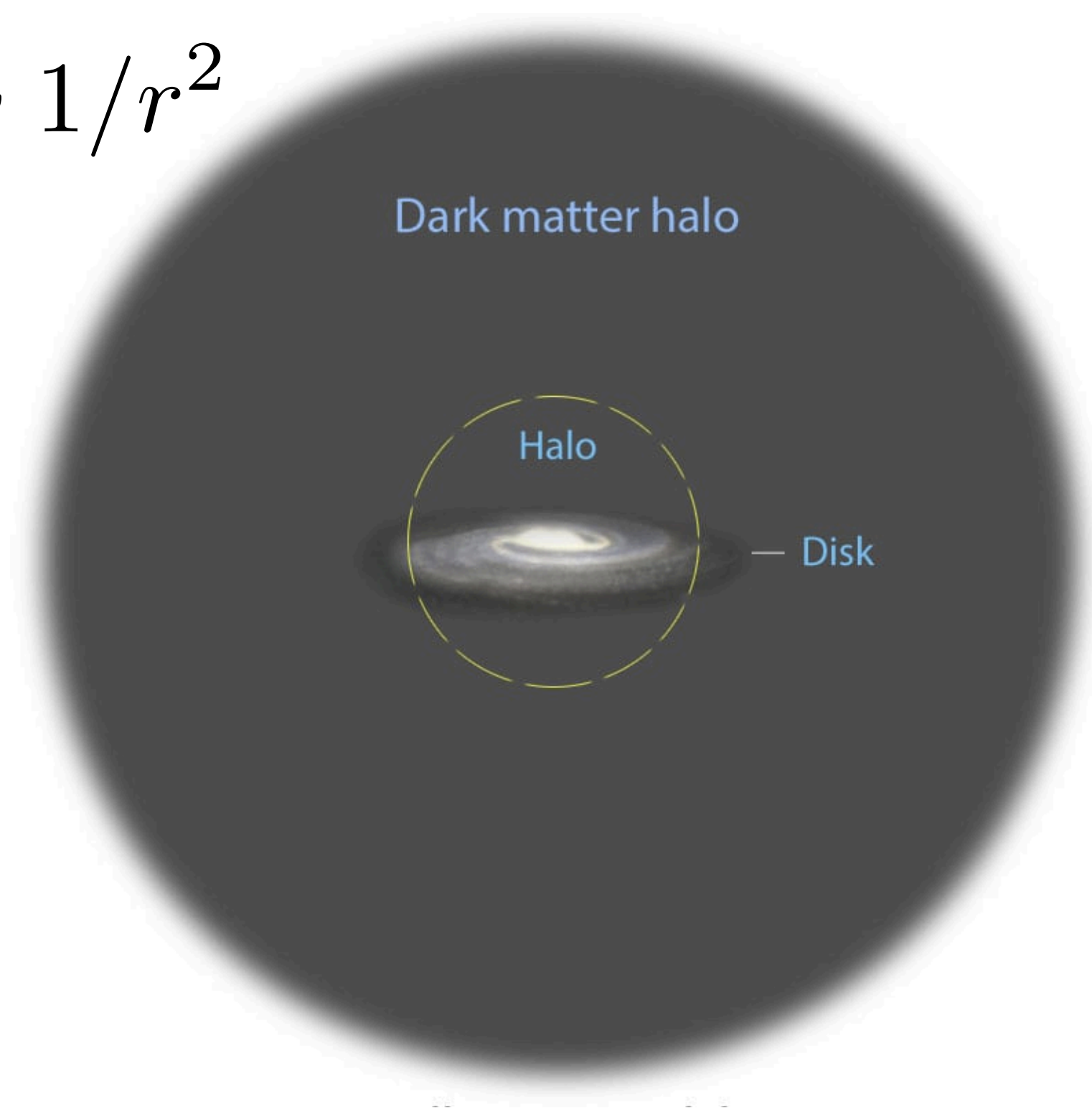
- Annual modulation
- Gravitational focusing by Sun
- Direction-dependence

$$\mathbf{v}_{\text{lab}} = \underbrace{\mathbf{v}_{\text{LSR}} + \mathbf{v}_{\text{pec}}}_{\text{Sun: 260 km/s}} + \underbrace{\mathbf{v}_{\oplus, \text{rev.}}(t)}_{\text{Earth: } \pm 15 \text{ km/s (left-right) } \pm 20 \text{ km/s (up-down)}}$$

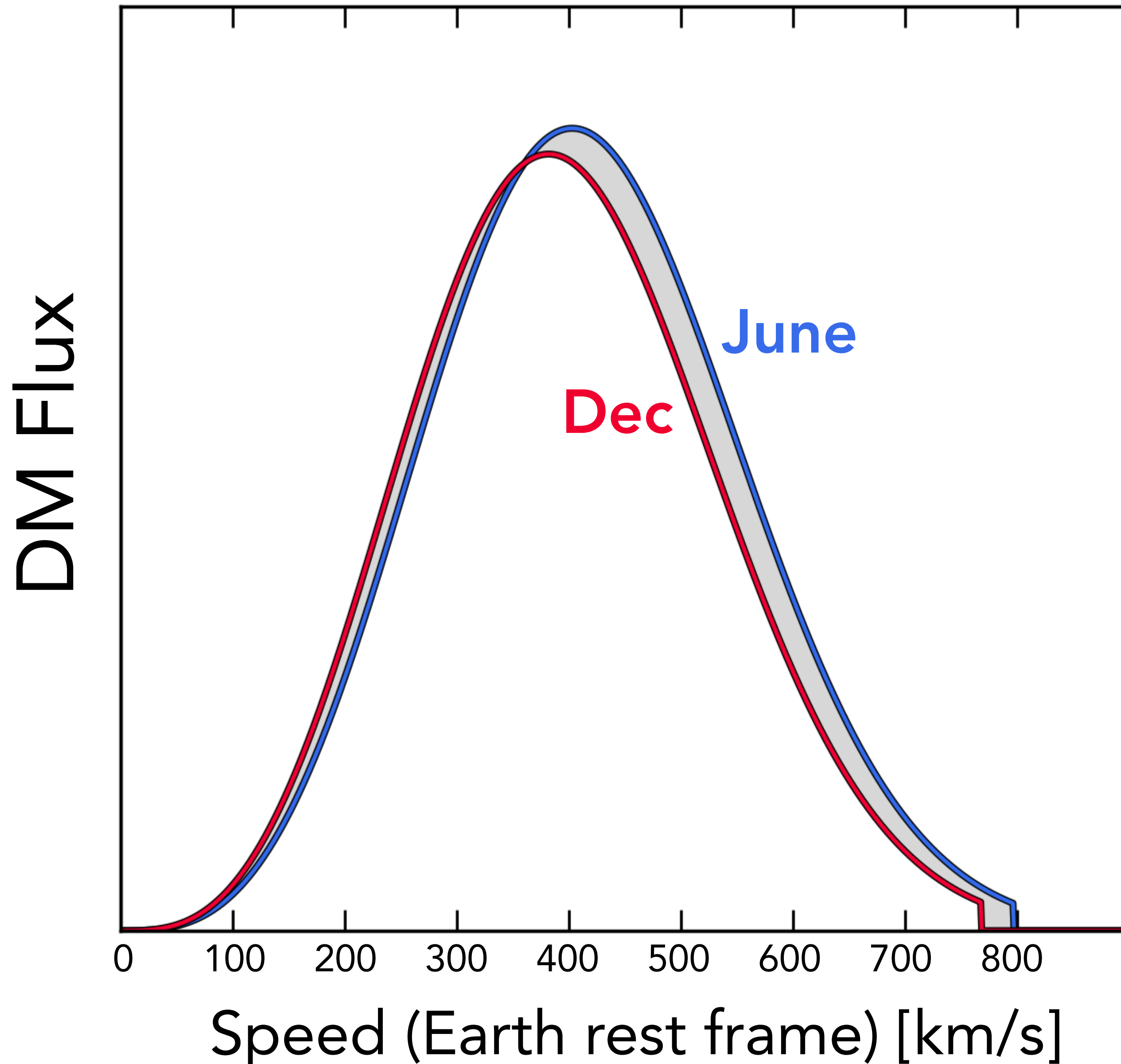
Usual assumption for $f(\mathbf{x}, \mathbf{v})$: the Standard Halo Model (SHM)

- Infinite isothermal sphere $\rightarrow$ Simplest halo model that gives a flat rotation curve
- Truncate at $v > v_{\text{esc}}$ so as to not include unbound particles

$$\rho \sim 1/r^2$$



1. Annual modulation

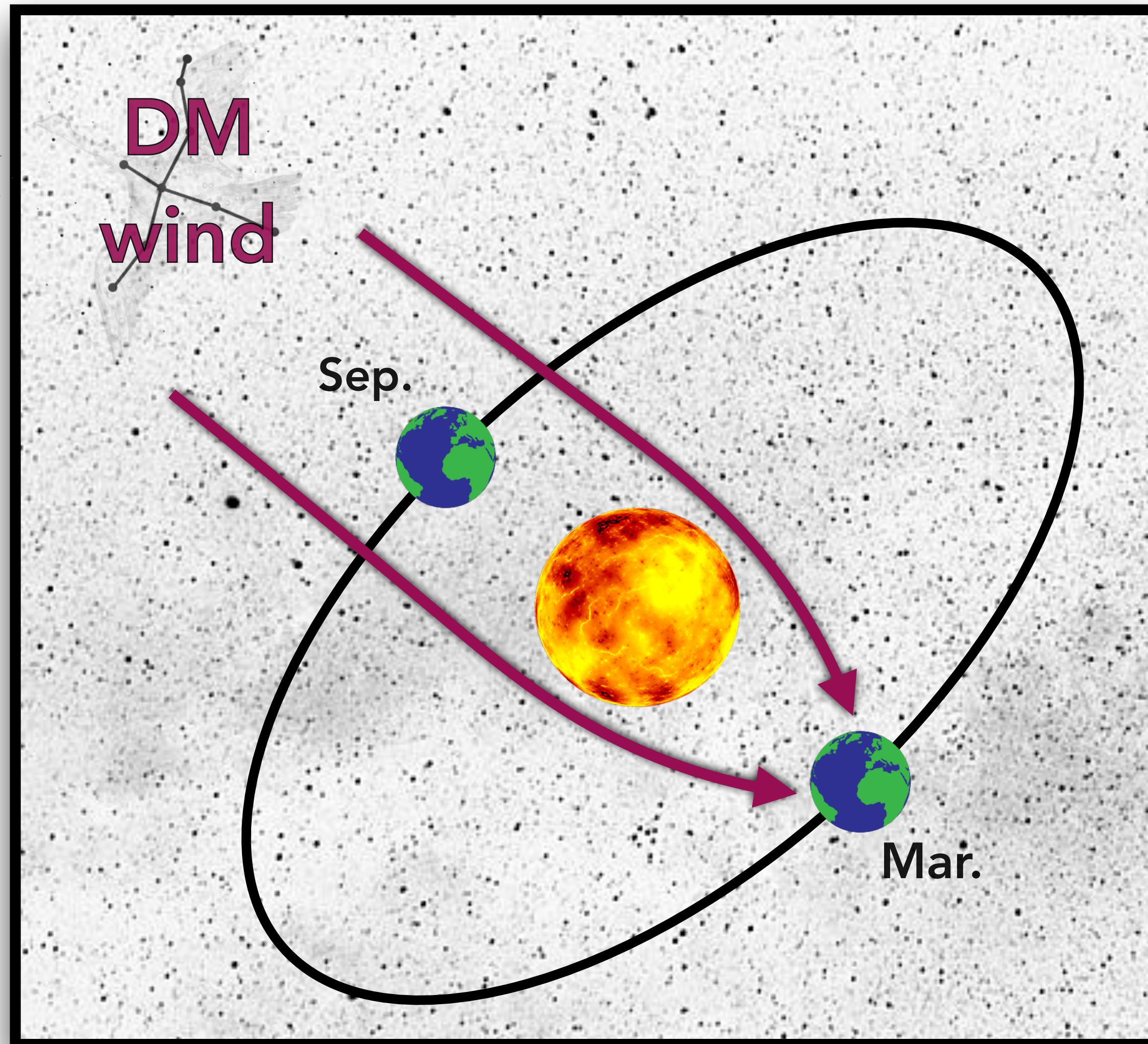


$$\mathbf{v}_{\text{lab}} = \mathbf{v}_{\text{LSR}} + \mathbf{v}_{\text{pec}} + \boxed{\mathbf{v}_{\oplus, \text{rev.}}(t)}$$

$$\text{DM Flux} \propto v f(\mathbf{v} + \mathbf{v}_{\text{lab}})$$

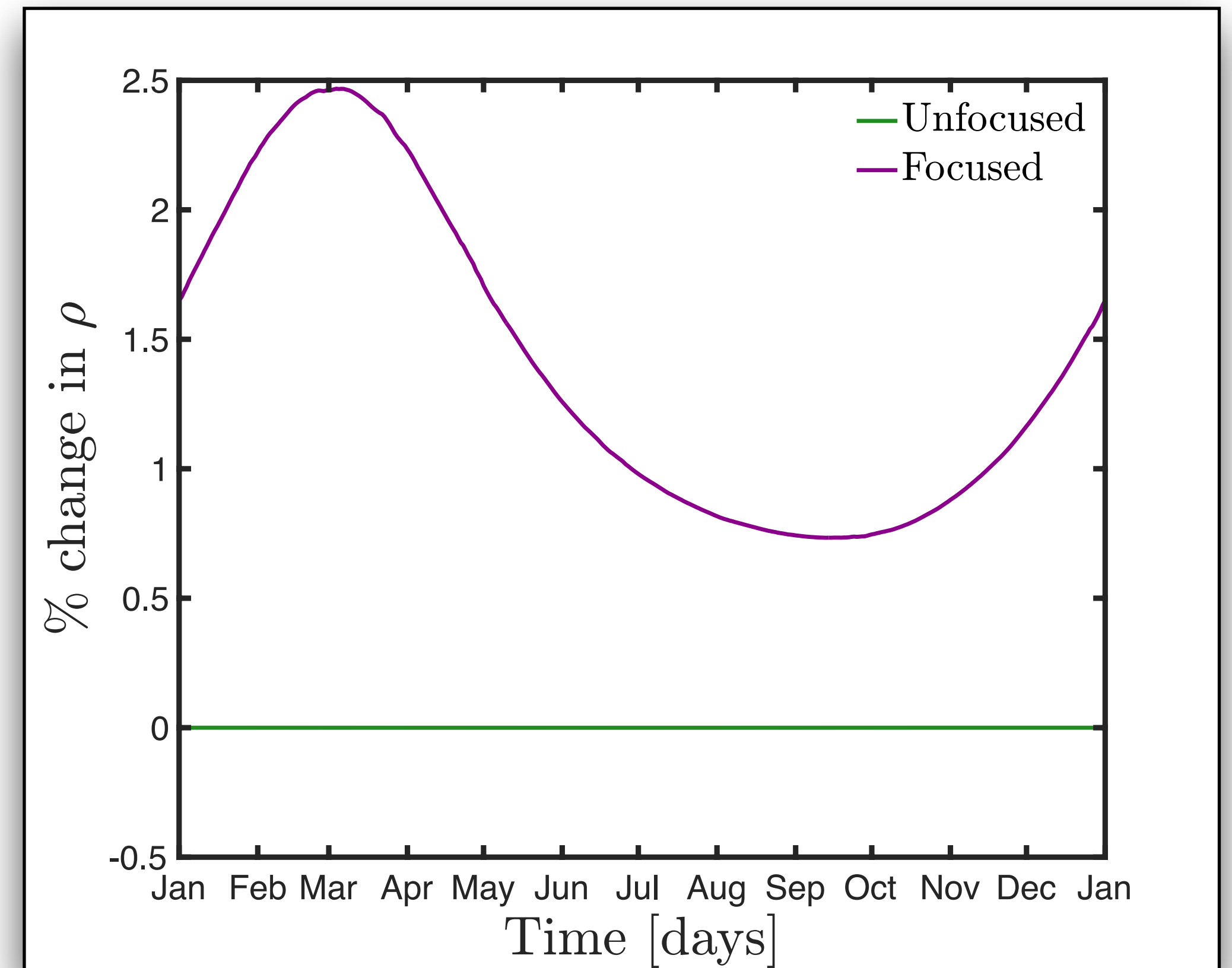
- Integrated flux is maximum during June and minimum in December (few % modulation)
- If sampling over distribution at lower-speeds only, phase is flipped (maximum in Dec.)

2. Gravitational focusing

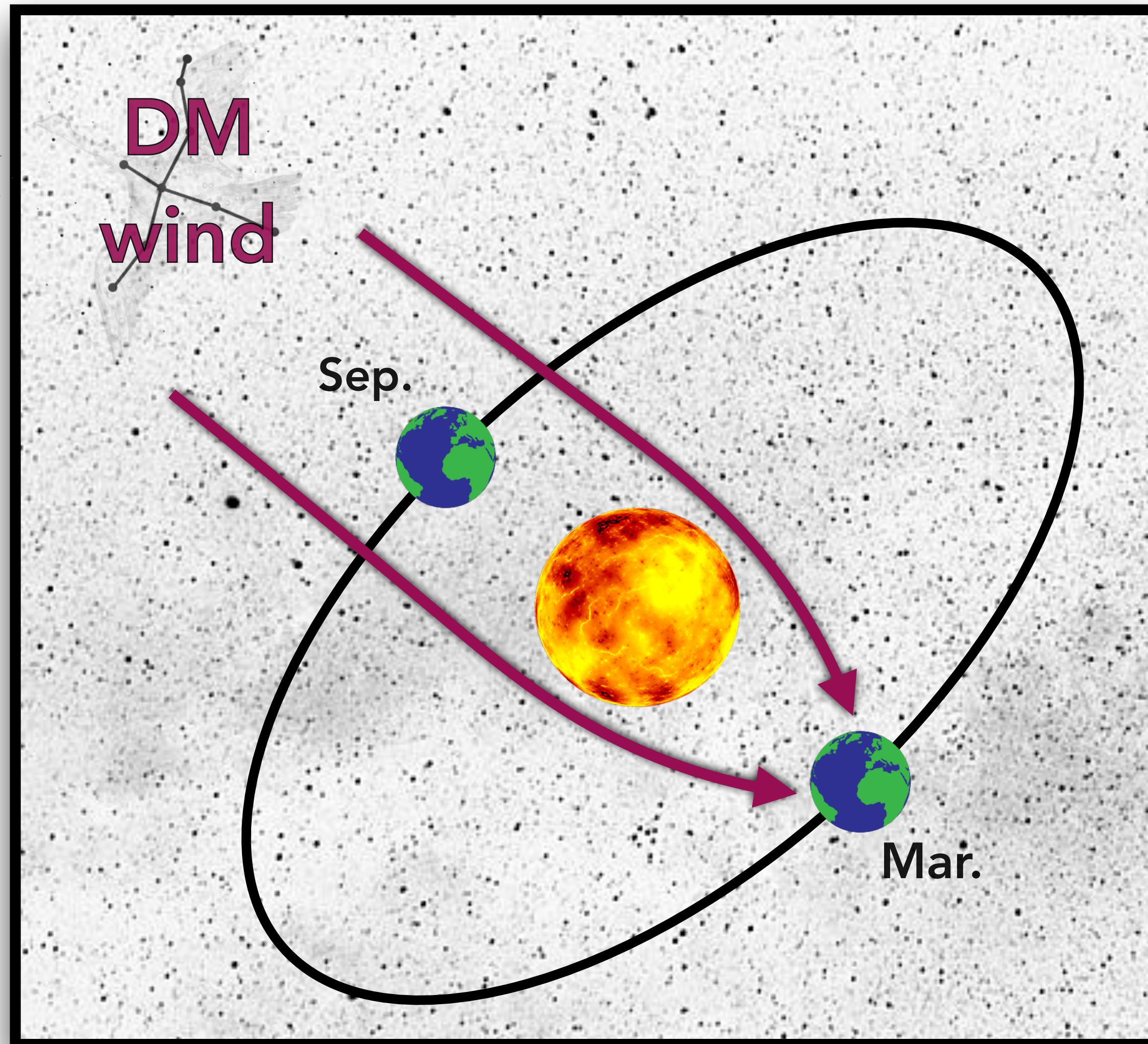


- Additional $\sim 2\%$ modulation in DM **density**
- Distortion to $f(v)$ at small speeds:

$$v < v_{\text{esc}} = \sqrt{2GM_{\odot}/r} \approx 40 \text{ km/s}$$

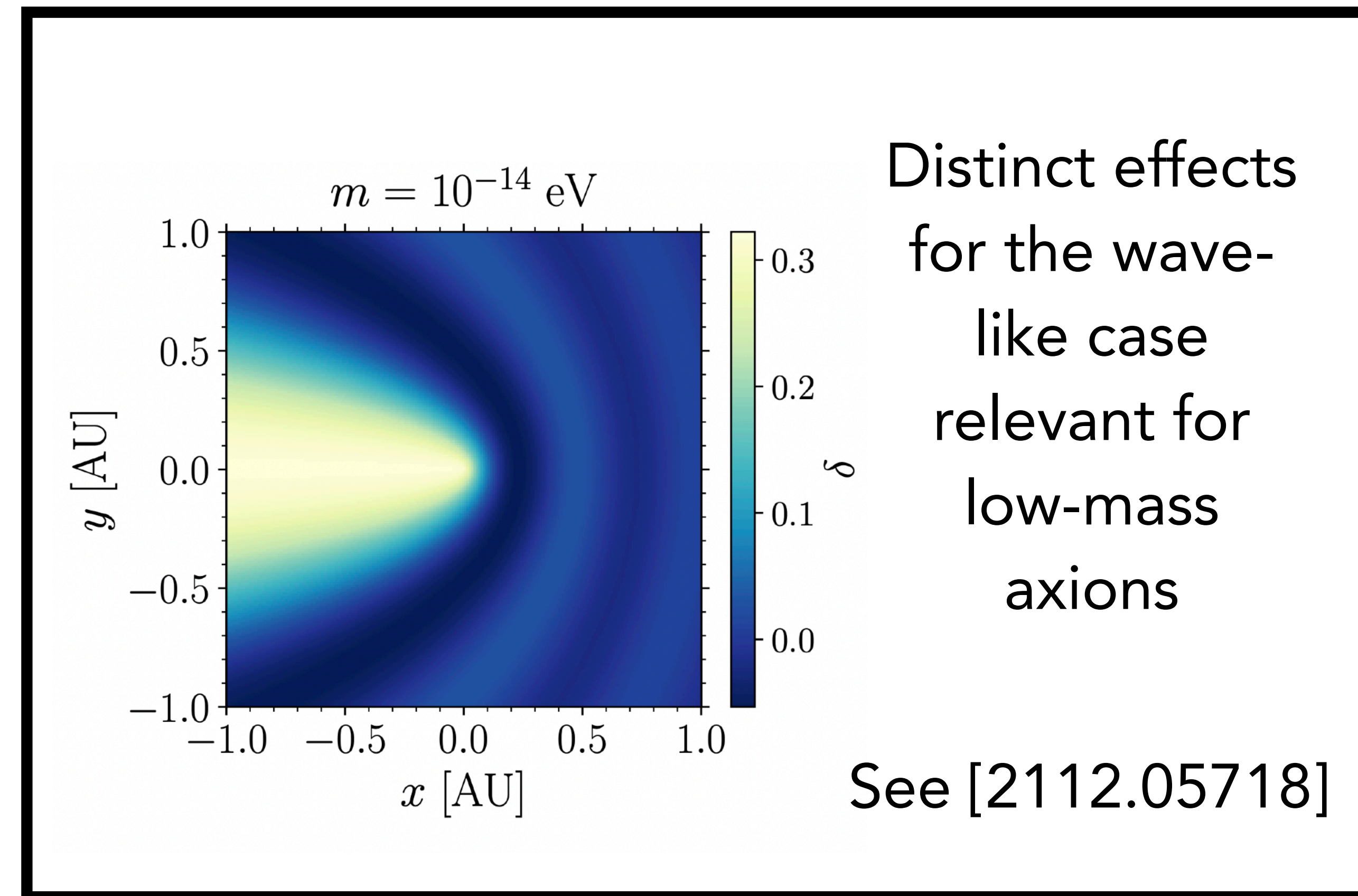


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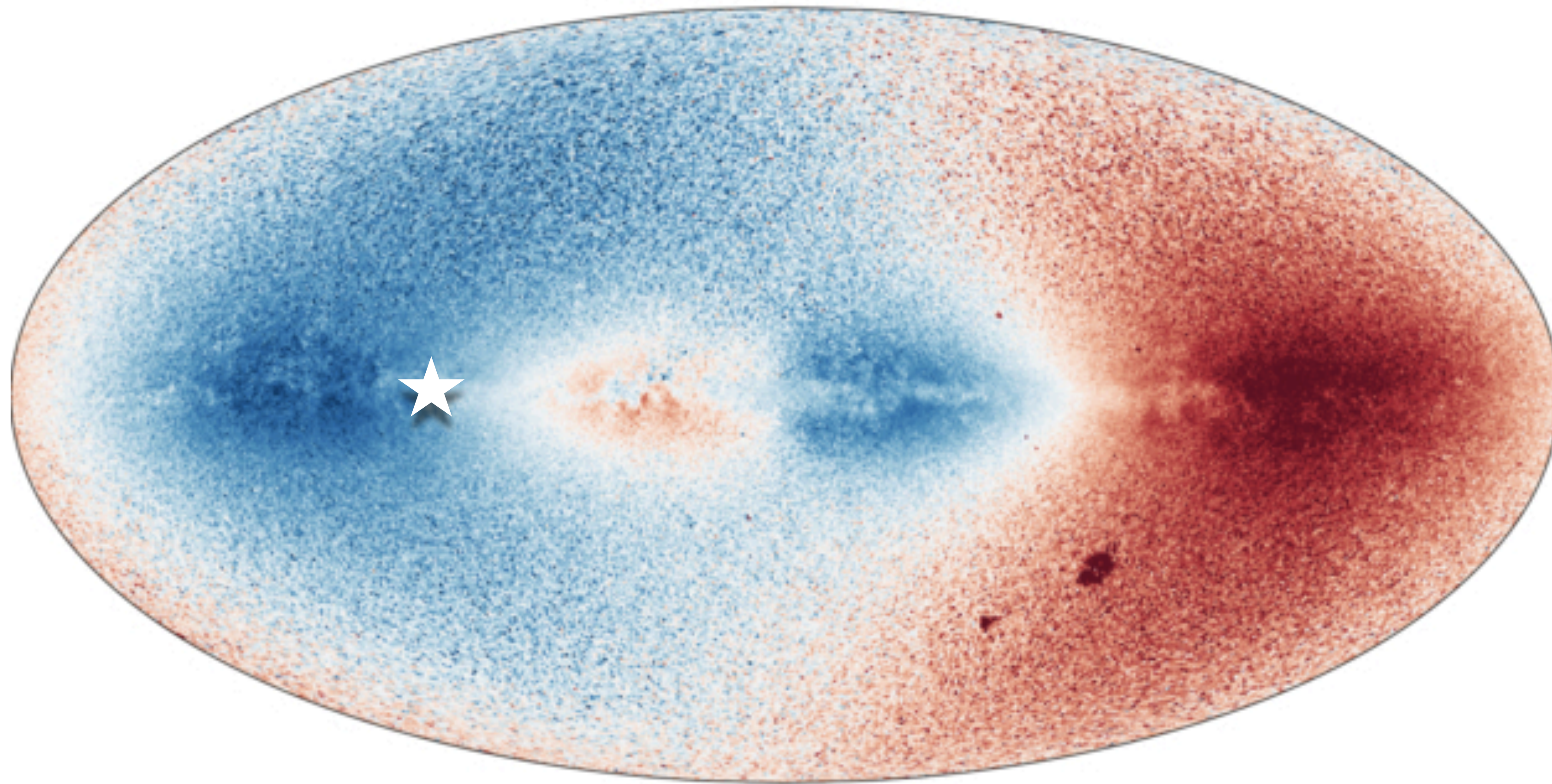


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3. Directionality

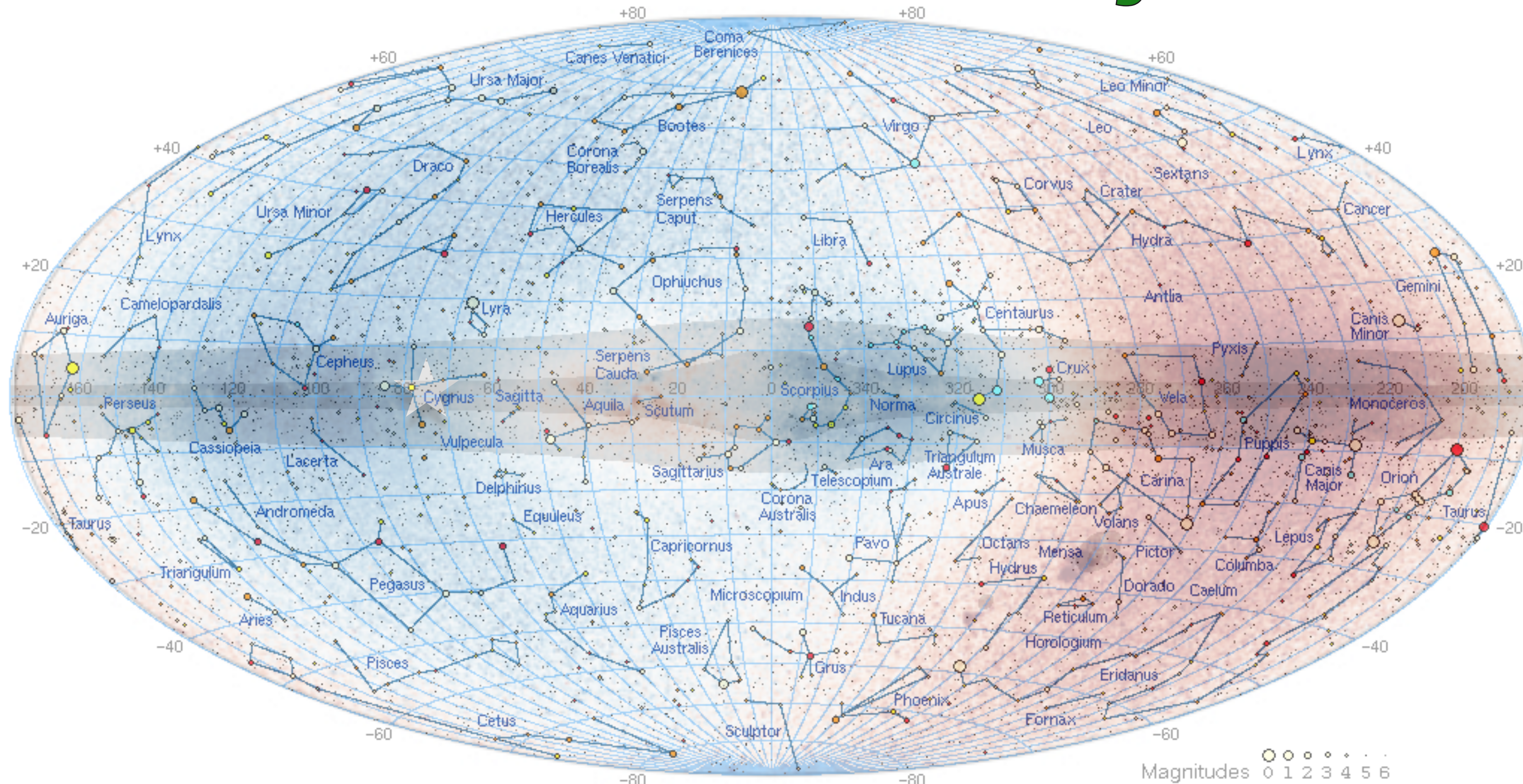


Gaia RVS galactic coordinates skymap of stellar line-of-sight velocities

Blue = moving towards us

Red = moving away from us

3. Directionality

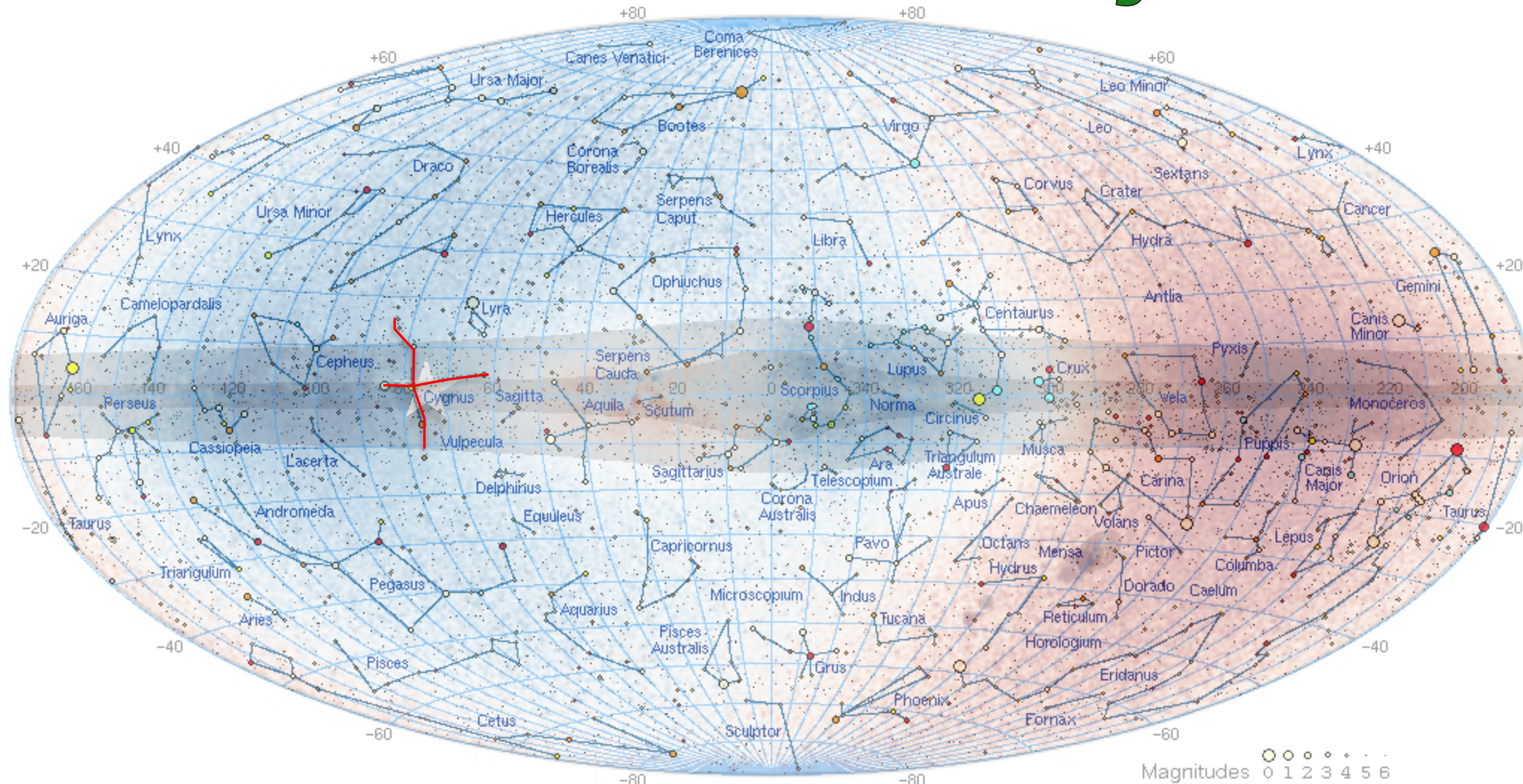


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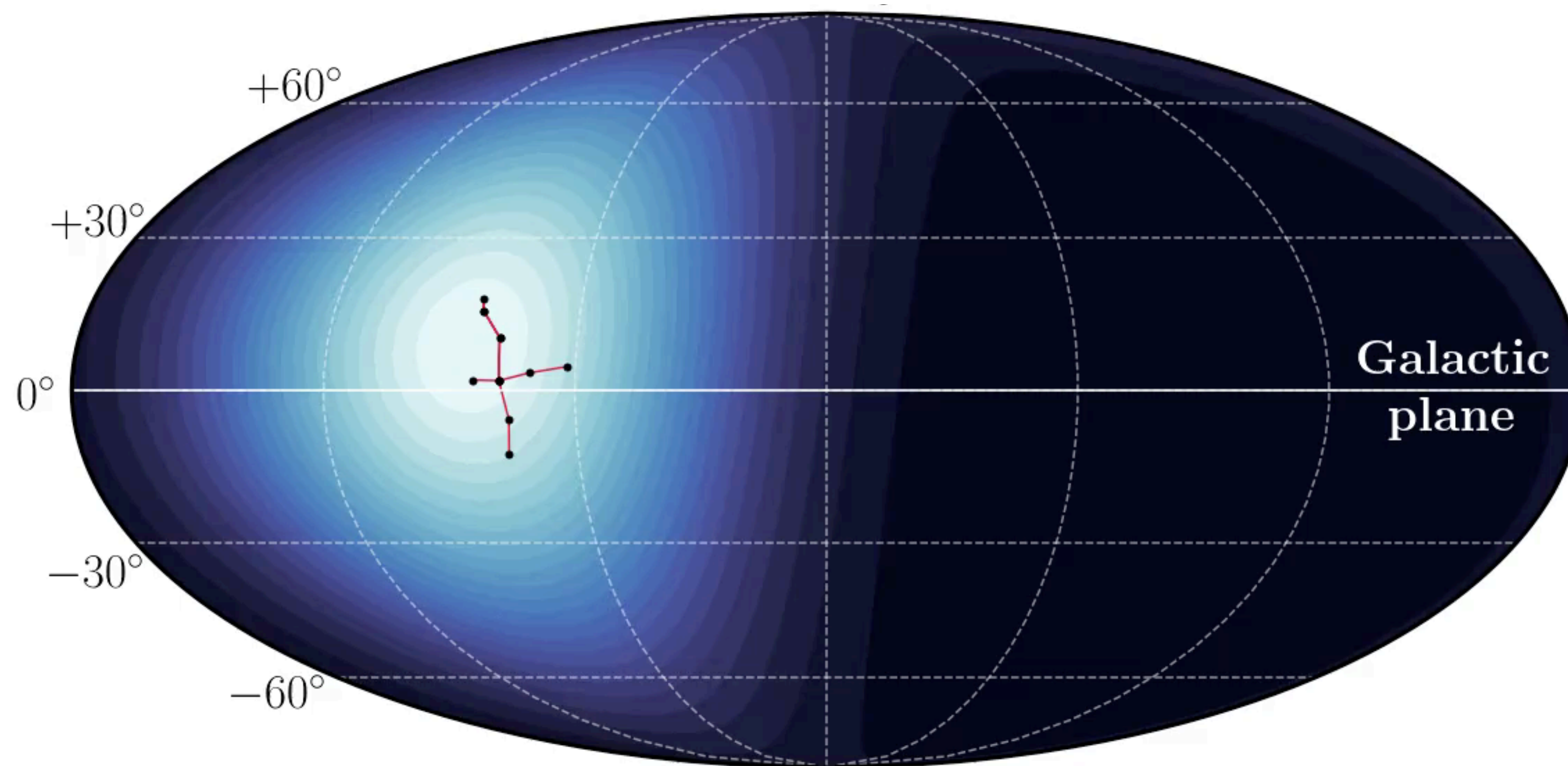


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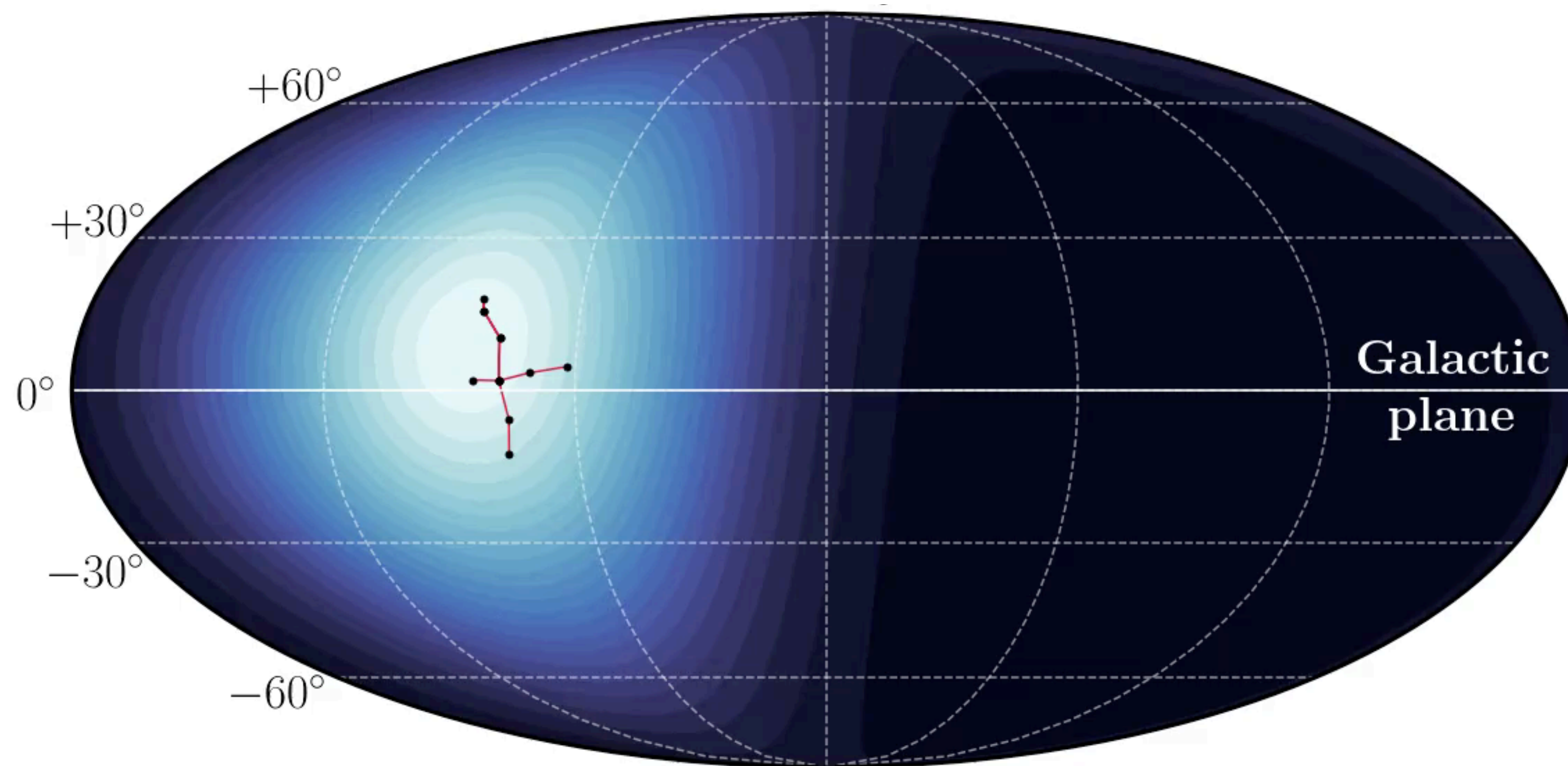
3. Directionality



The dark matter flux on Earth is highly anisotropic towards constellation of Cygnus

$$\Phi_{\text{forward}}/\Phi_{\text{backward}} \sim O(10)$$

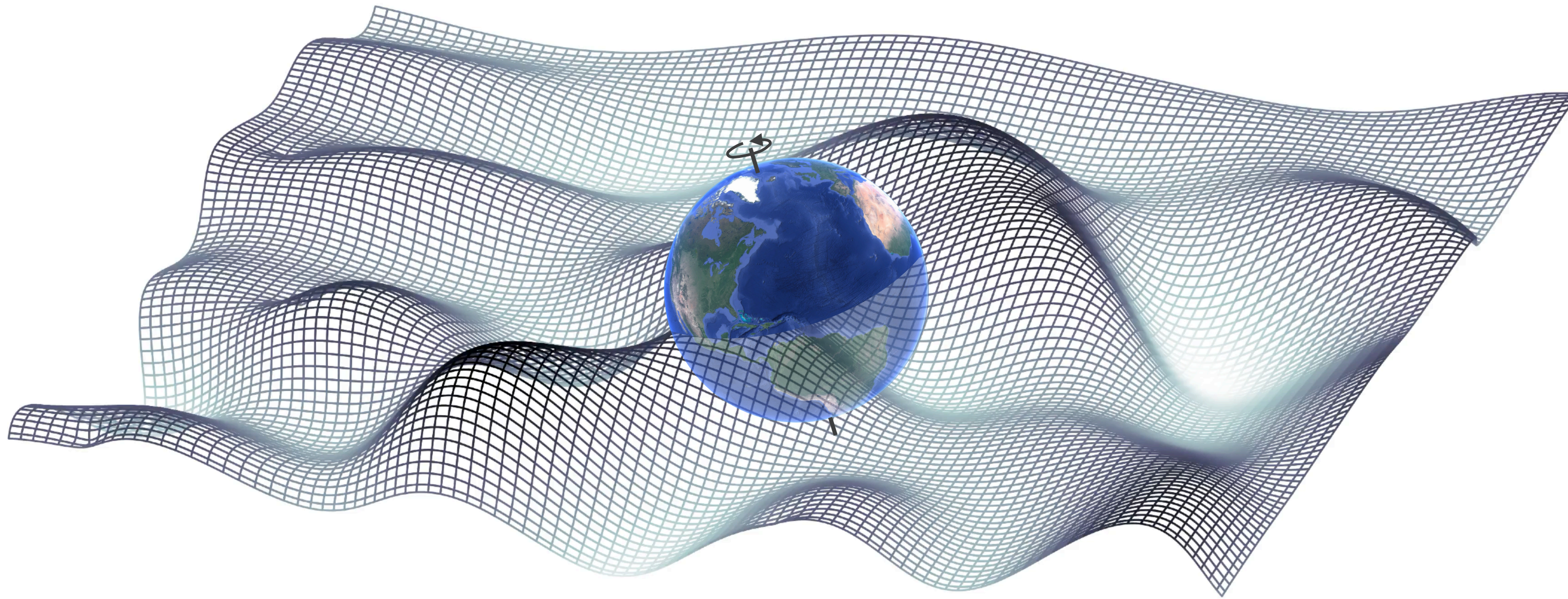
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$$\Phi_{\text{forward}}/\Phi_{\text{backward}} \sim O(10)$$

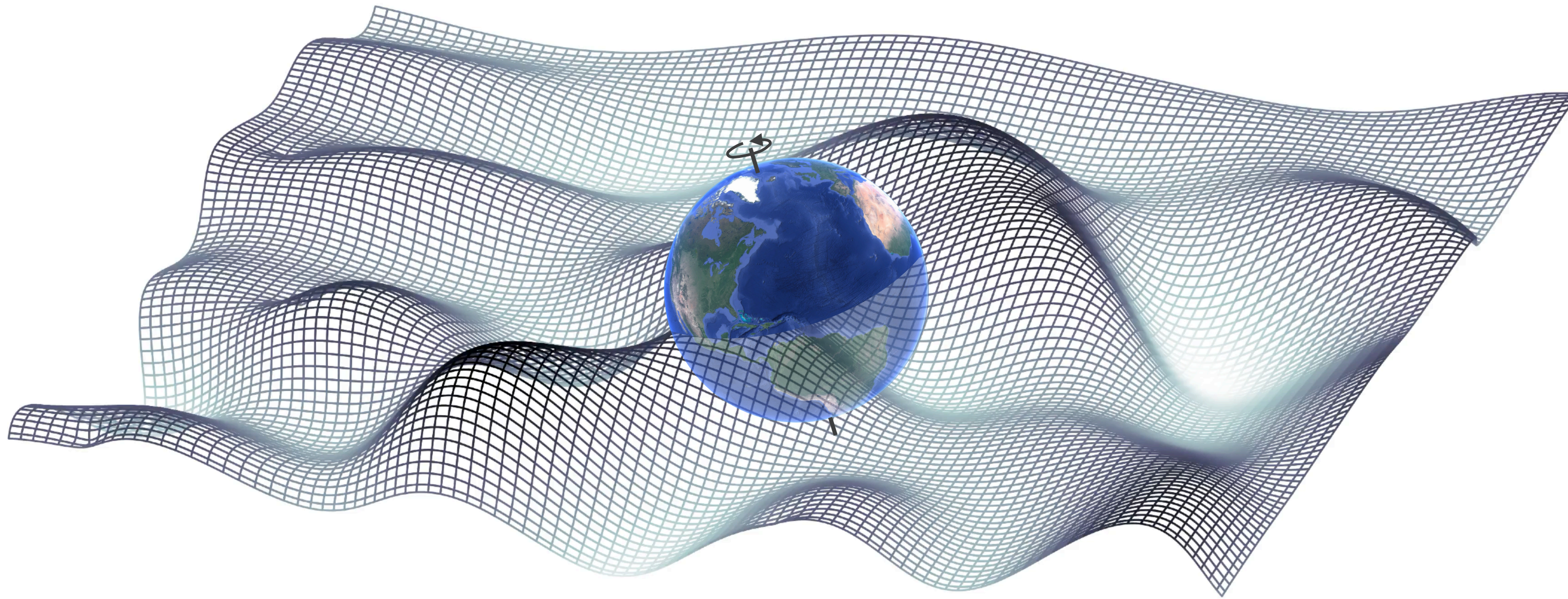
How do the speed distribution, annual modulation, directionality manifest in the wave-like case?



Amplitude: $A = \frac{\sqrt{2\rho_{\text{DM}}}}{m}$

Frequency: $\omega = m + \frac{1}{2}mv^2$
 $\approx m(1 + 10^{-6})$

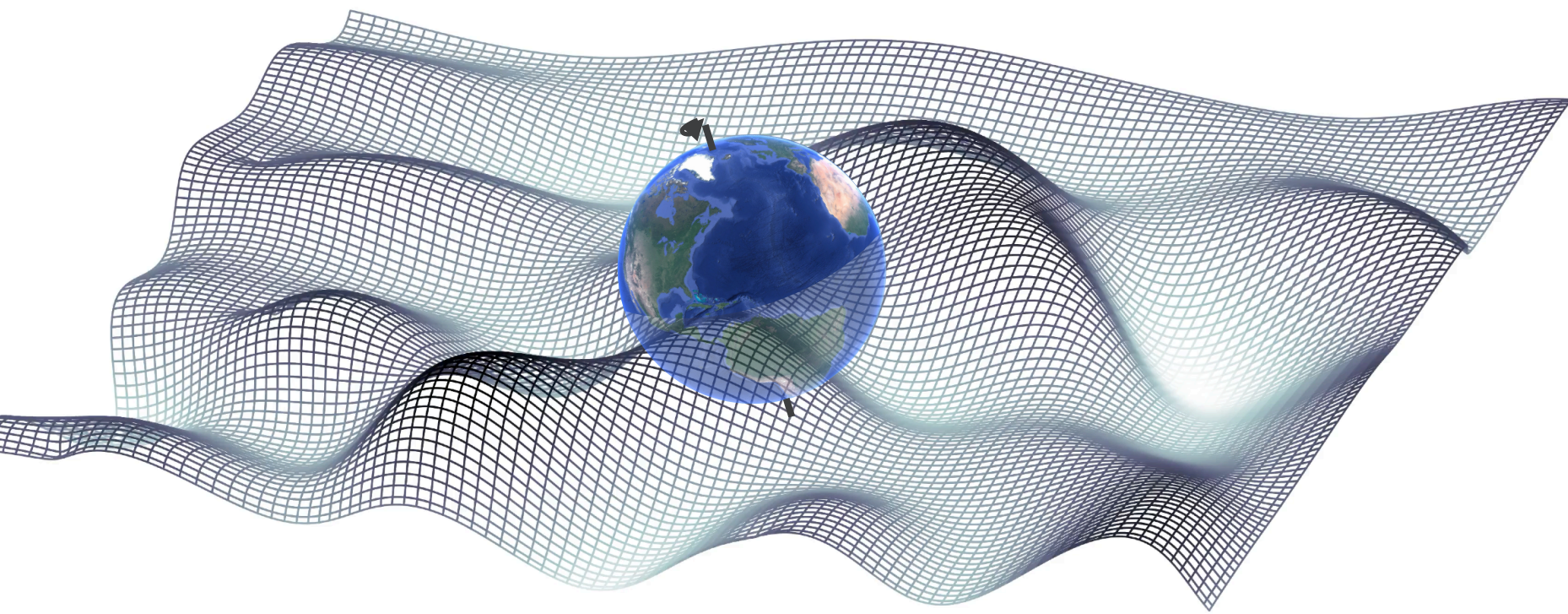
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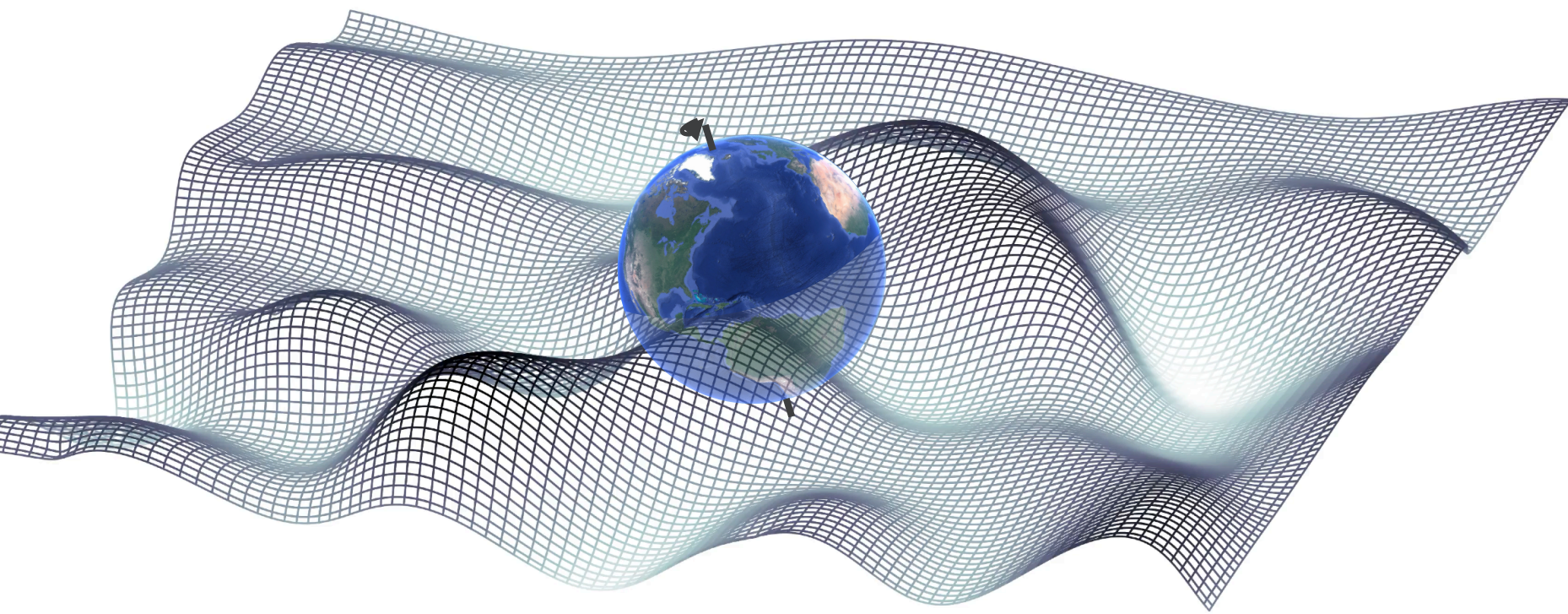
Wave-like dark matter, properly



Superposition of plane waves in some box of volume, V

$$\phi(t, \mathbf{x}) = \sqrt{V} \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \phi(\mathbf{p}) e^{-i(\omega t - \mathbf{p} \cdot \mathbf{x} + \beta(\mathbf{p}))}$$

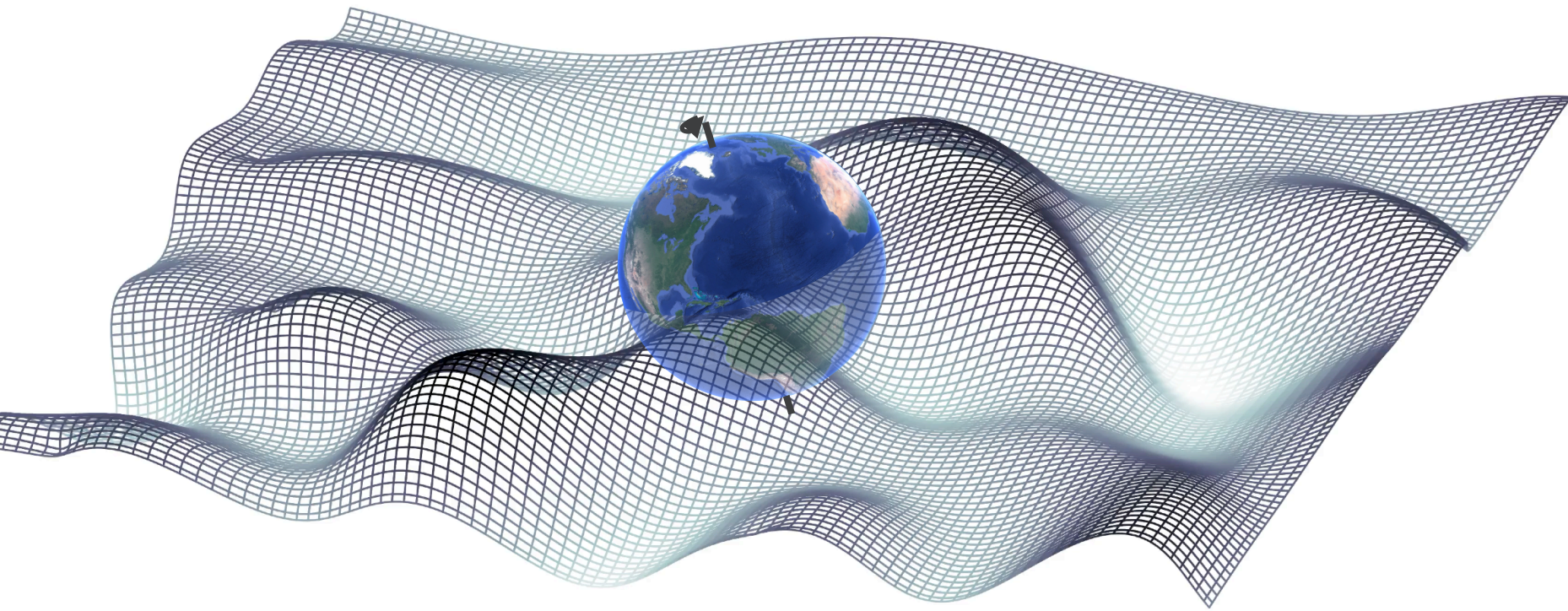
Wave-like dark matter, properly



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Wave-like dark matter, properly



Superposition of plane waves in some box of volume, V

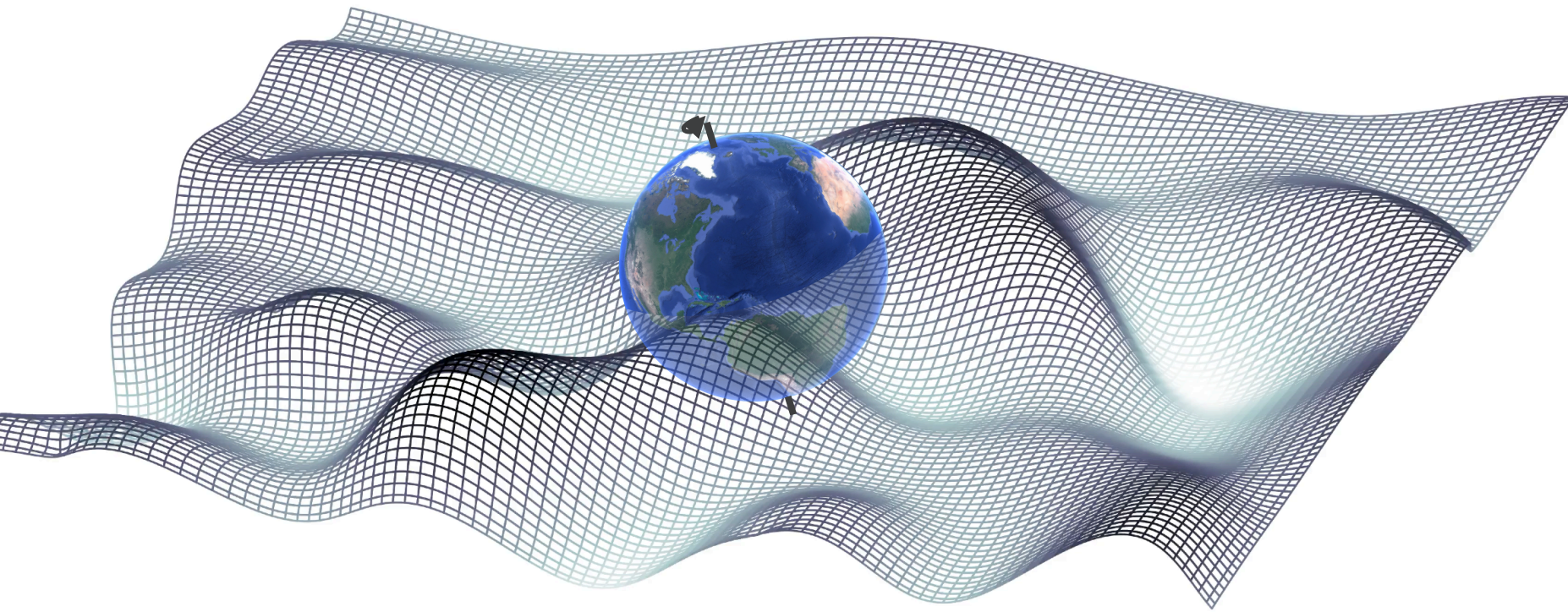
$$\phi(t, \mathbf{x}) = \sqrt{V} \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \phi(\mathbf{p}) e^{-i(\omega t - \mathbf{p} \cdot \mathbf{x} + \beta(\mathbf{p}))}$$

Only when we measure over some **short enough** time/length scale do we have:

$$\phi = \phi_0 \cos(\omega t - \mathbf{p} \cdot \mathbf{x} + \beta)$$

Random amplitude Random draw from the velocity distribution Arbitrary phase

Wave-like dark matter, properly



Superposition of plane waves in some box of volume, V

$$\phi(t, \mathbf{x}) = \sqrt{V} \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \phi(\mathbf{p}) e^{-i(\omega t - \mathbf{p} \cdot \mathbf{x} + \beta(\mathbf{p}))}$$

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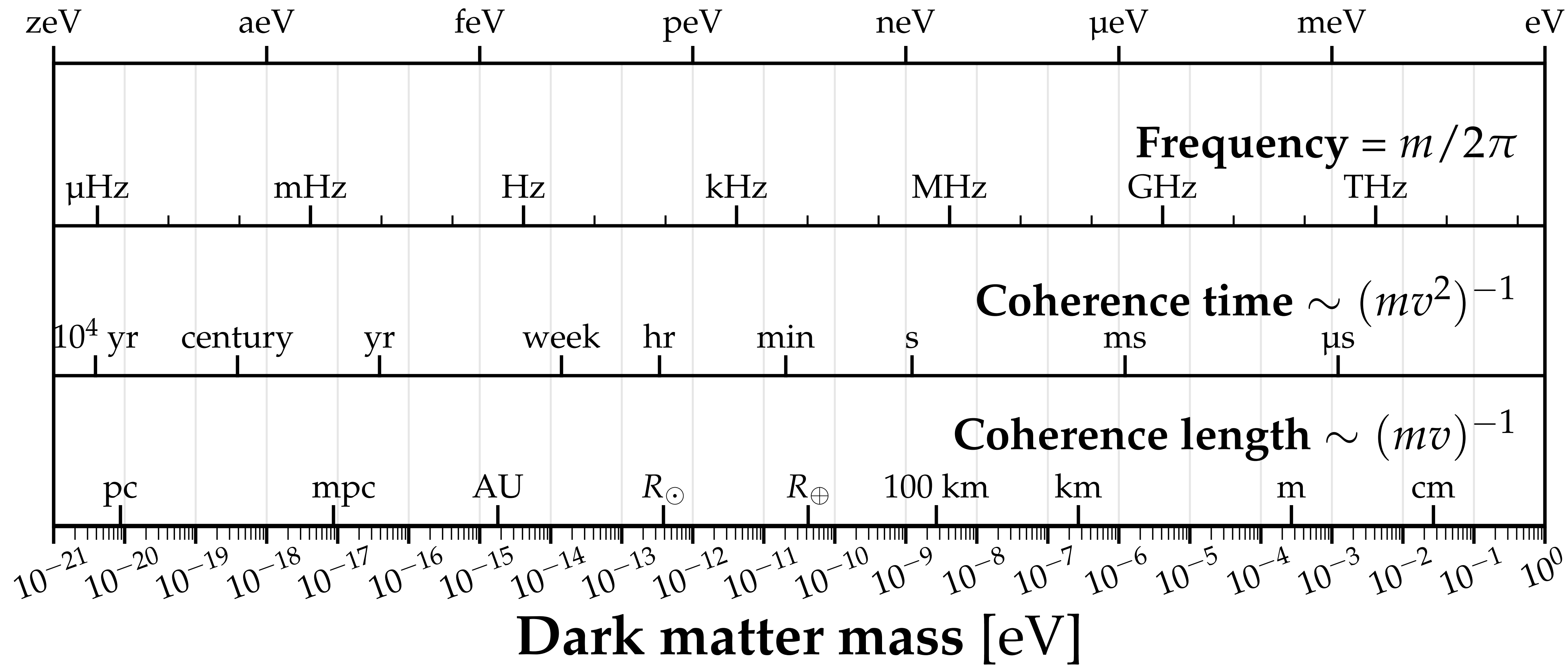
$$\phi = \phi_0 \cos(\omega t - \mathbf{p} \cdot \mathbf{x} + \beta)$$

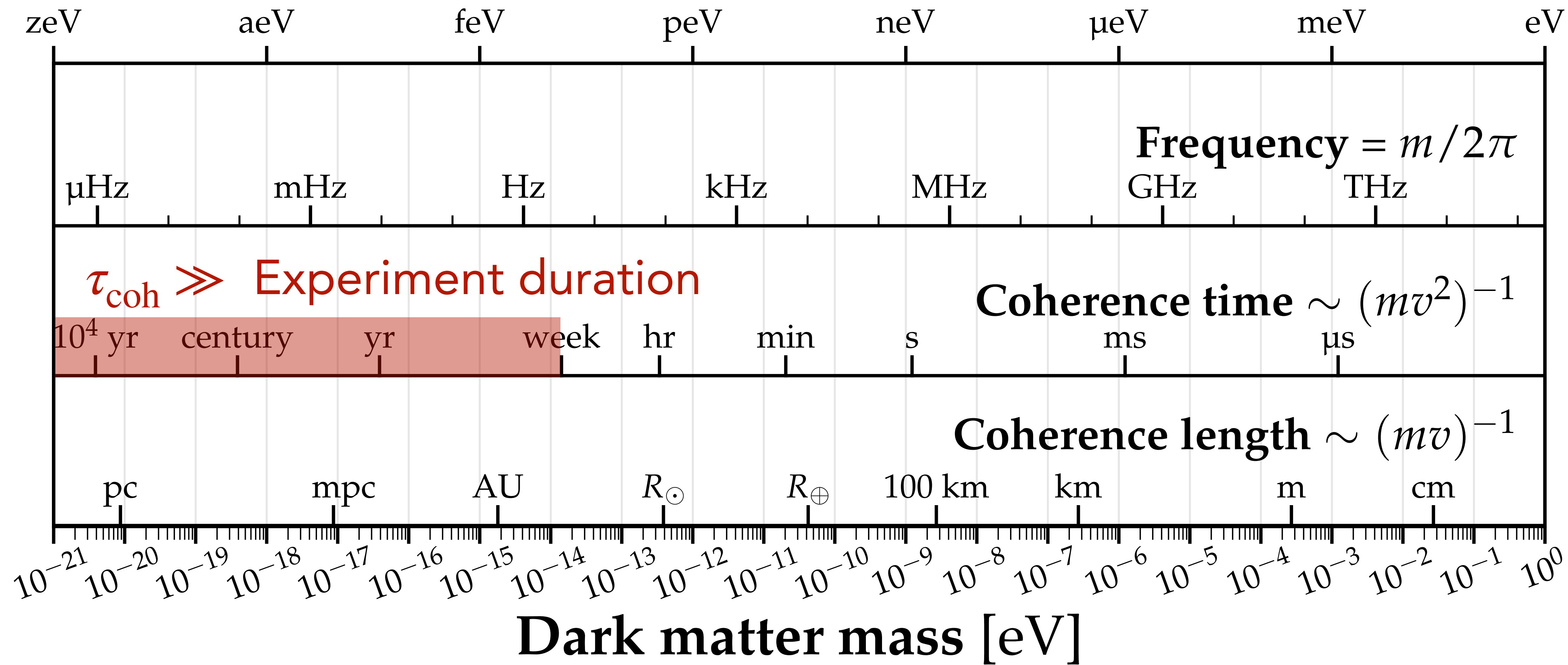
Random amplitude Random draw from the velocity distribution Arbitrary phase

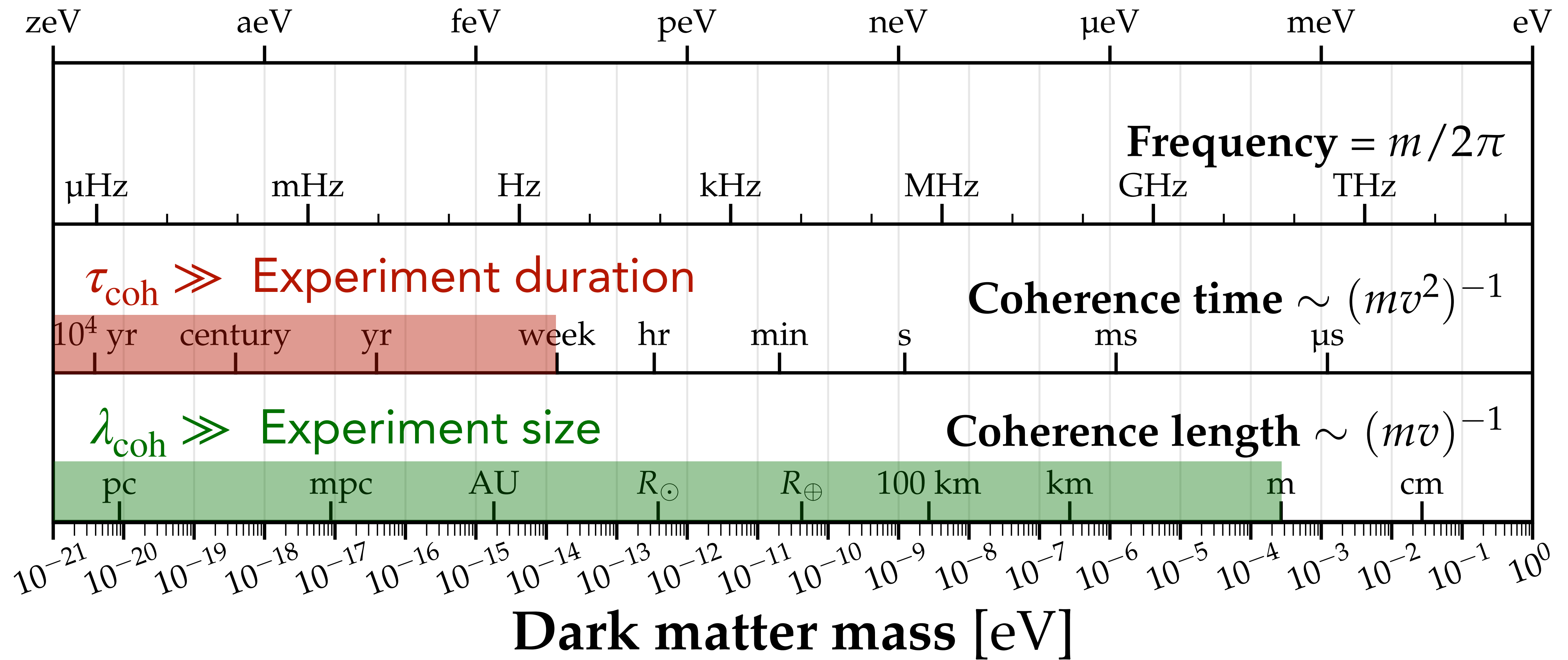
What is considered "short enough"?

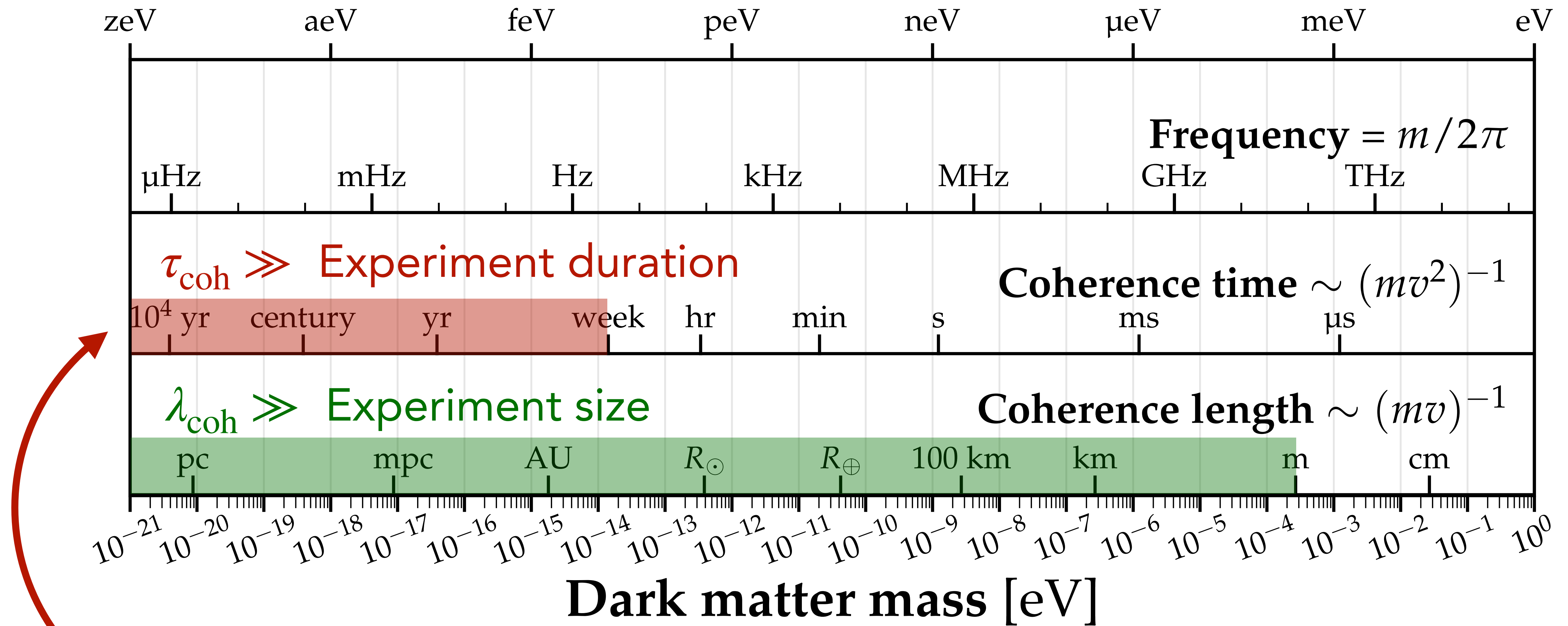
→ Coherence length and coherence time:
the length/timescale over which field will be
out of phase with itself

$$\lambda_{\text{coh}} \sim \frac{2\pi}{mv} \quad \tau_{\text{coh}} \sim \frac{2\pi}{mv^2}$$

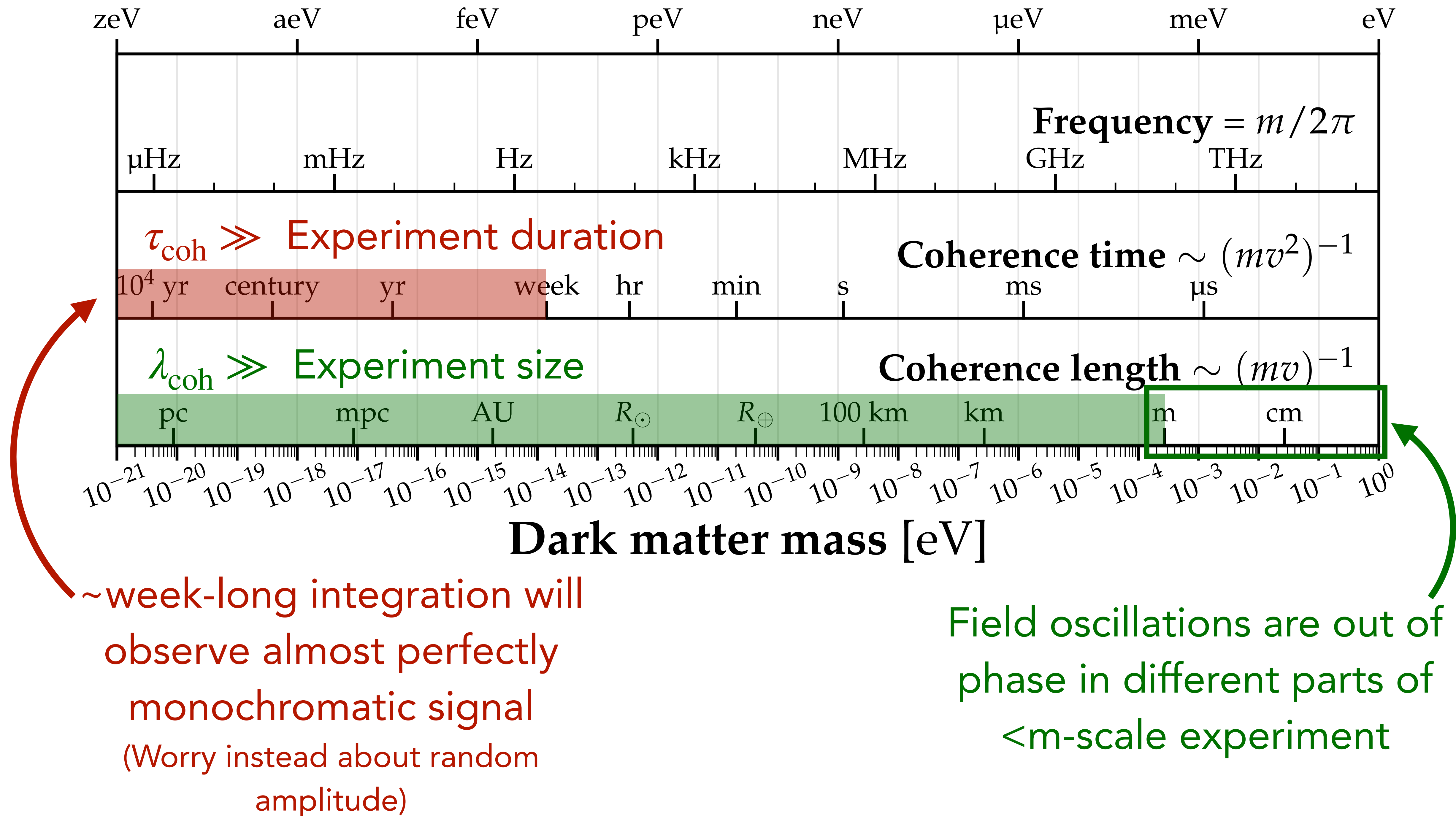






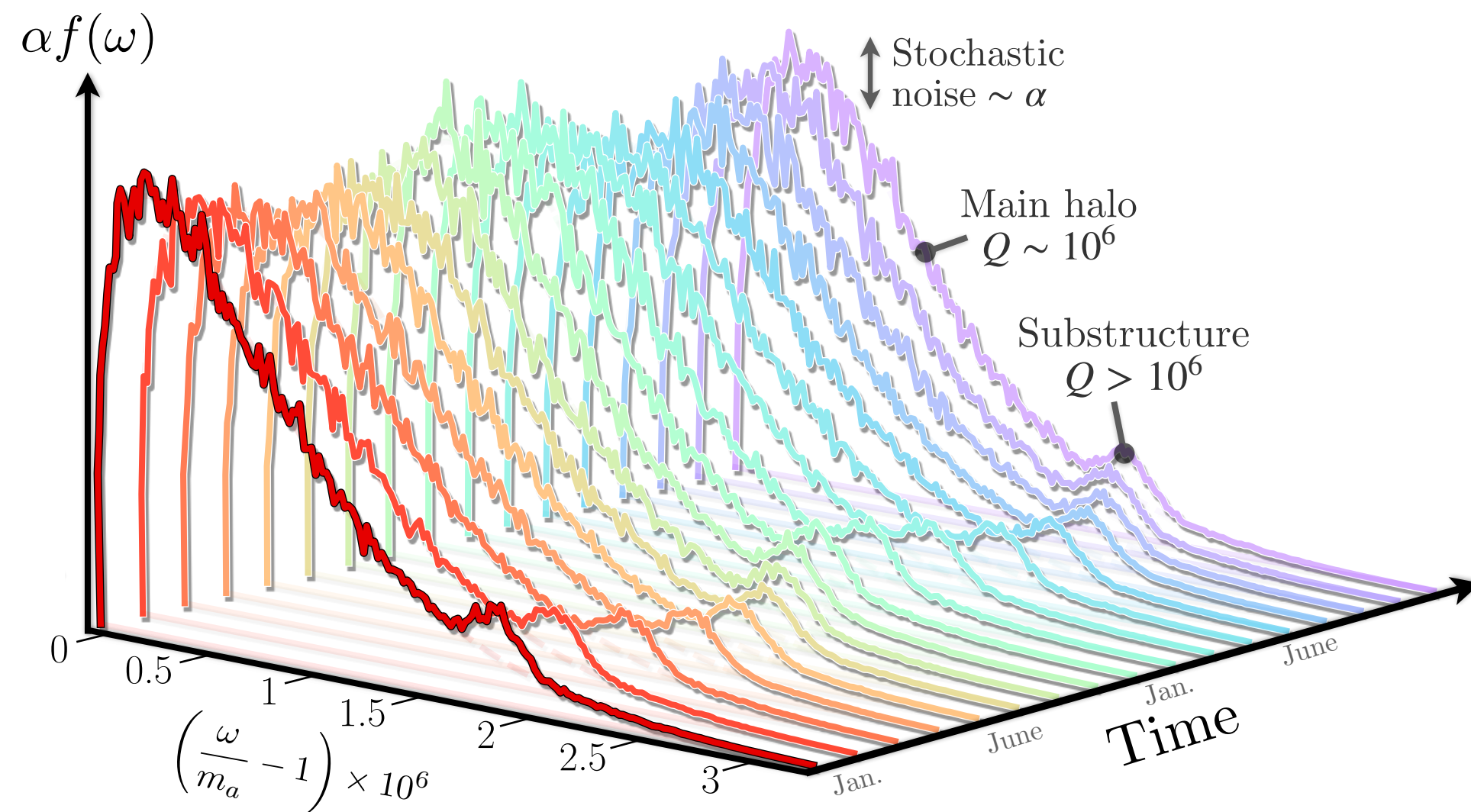


$\sim$ week-long integration will
observe almost perfectly
monochromatic signal
(Worry instead about random
amplitude)



Wave-like dark matter

How do the speed distribution, annual modulation, directionality manifest in the wave-like case?

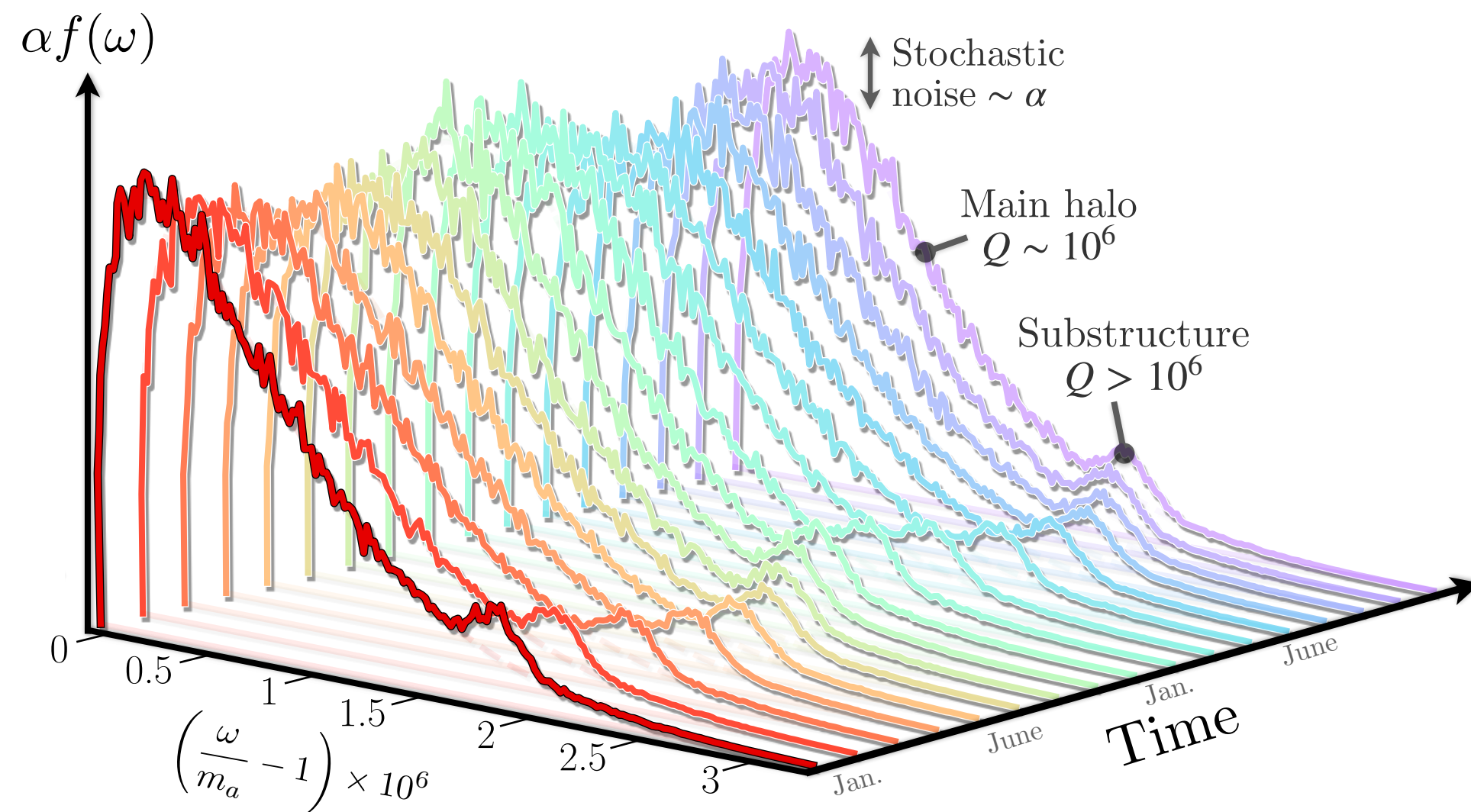


→ Speed distribution leads to a distinctive “lineshape” in frequency that will modulate over the year

$$f(\omega, t) = f(v, t) \frac{dv}{d\omega}$$

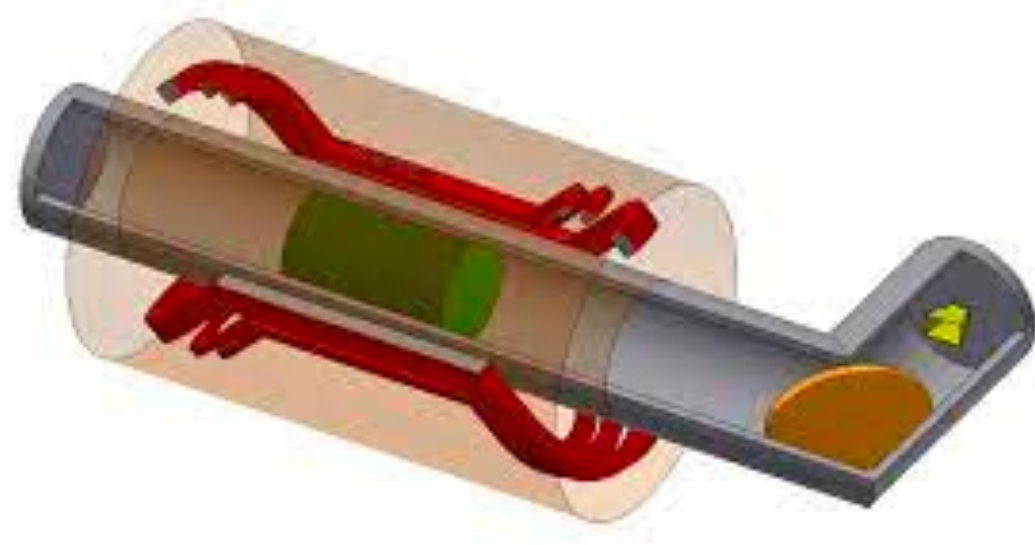
Wave-like dark matter

How do the speed distribution, annual modulation, directionality manifest in the wave-like case?

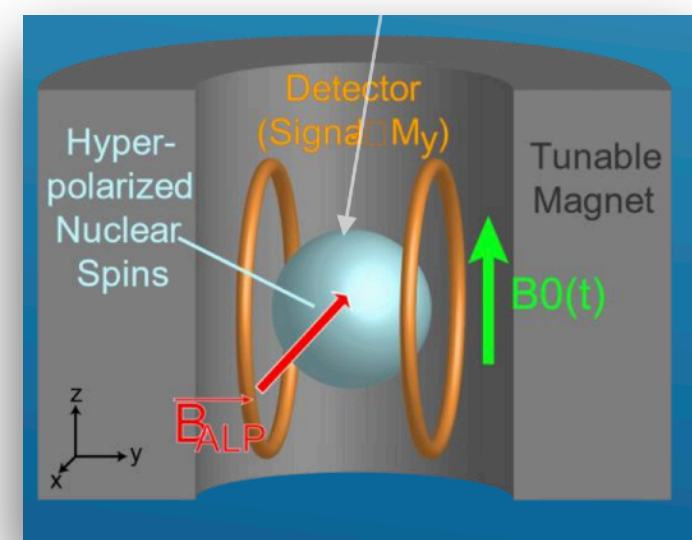


→ Speed distribution leads to a distinctive “lineshape” in frequency that will modulate over the year

$$f(\omega, t) = f(v, t) \frac{dv}{d\omega}$$



MADMAX



CASPER-Gradient

→ Directionality could appear in two forms:

→ Experiments that are larger than coherence length

→ Experiments that measure the field-gradient:

$$\nabla \phi = \sqrt{2\rho} \mathbf{v} \sin(\omega t - m_a \mathbf{v} \cdot \mathbf{x} + \beta)$$

But how much do we trust our assumptions?

Is the DM halo
spherical?

Is the DM speed
distribution
Maxwellian?

Is the DM halo
stationary?

But how much do we trust our assumptions?

Is the DM halo
spherical?



No

Is the DM speed
distribution
Maxwellian?



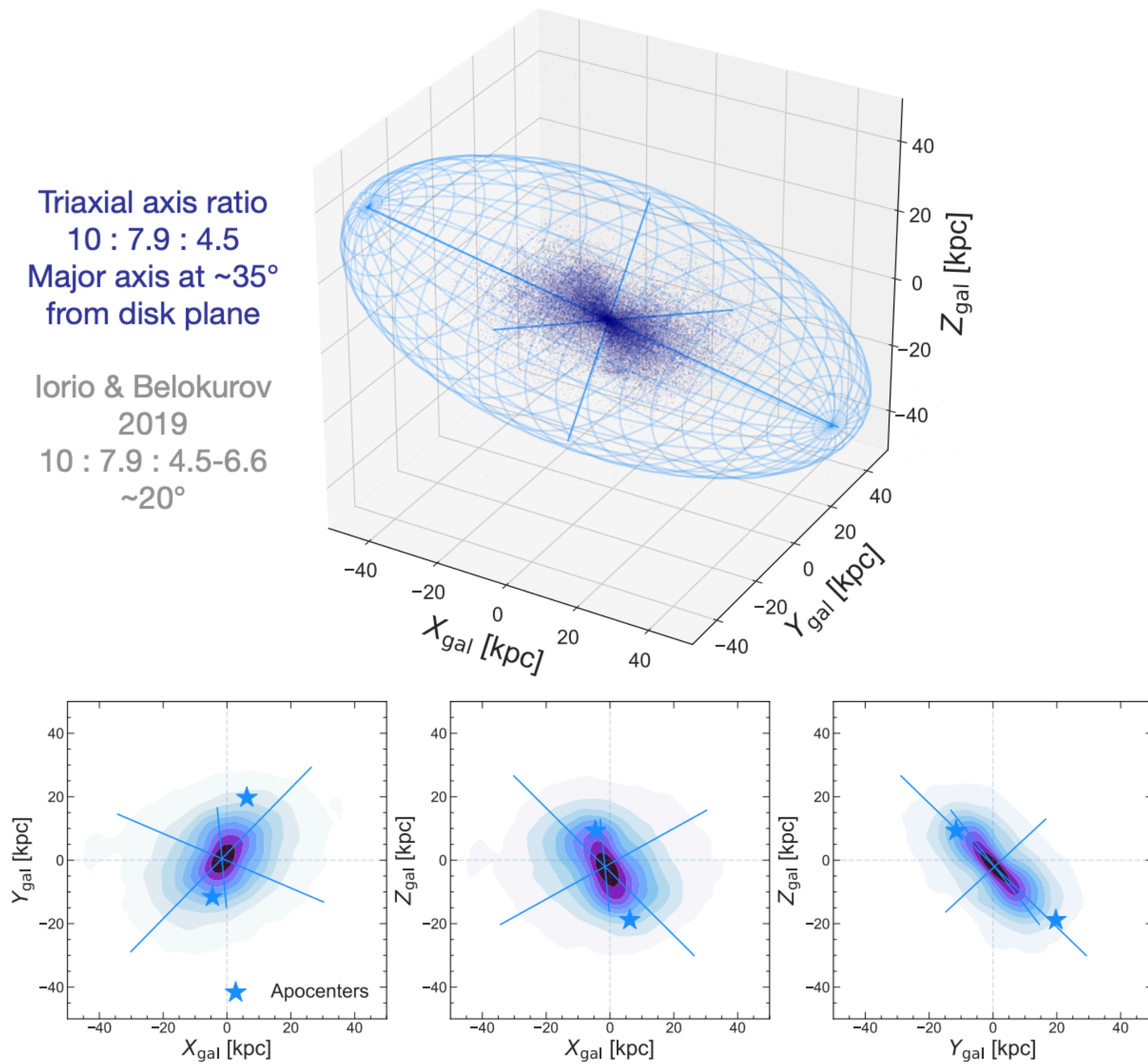
Probably not

Is the DM halo
stationary?



Probably not

Han+ [2208.04327]
 Naidu+[2103.03251]
 (H3 survey)



Iorio & Belokurov
 [1804.11347] (RR Lyraes)

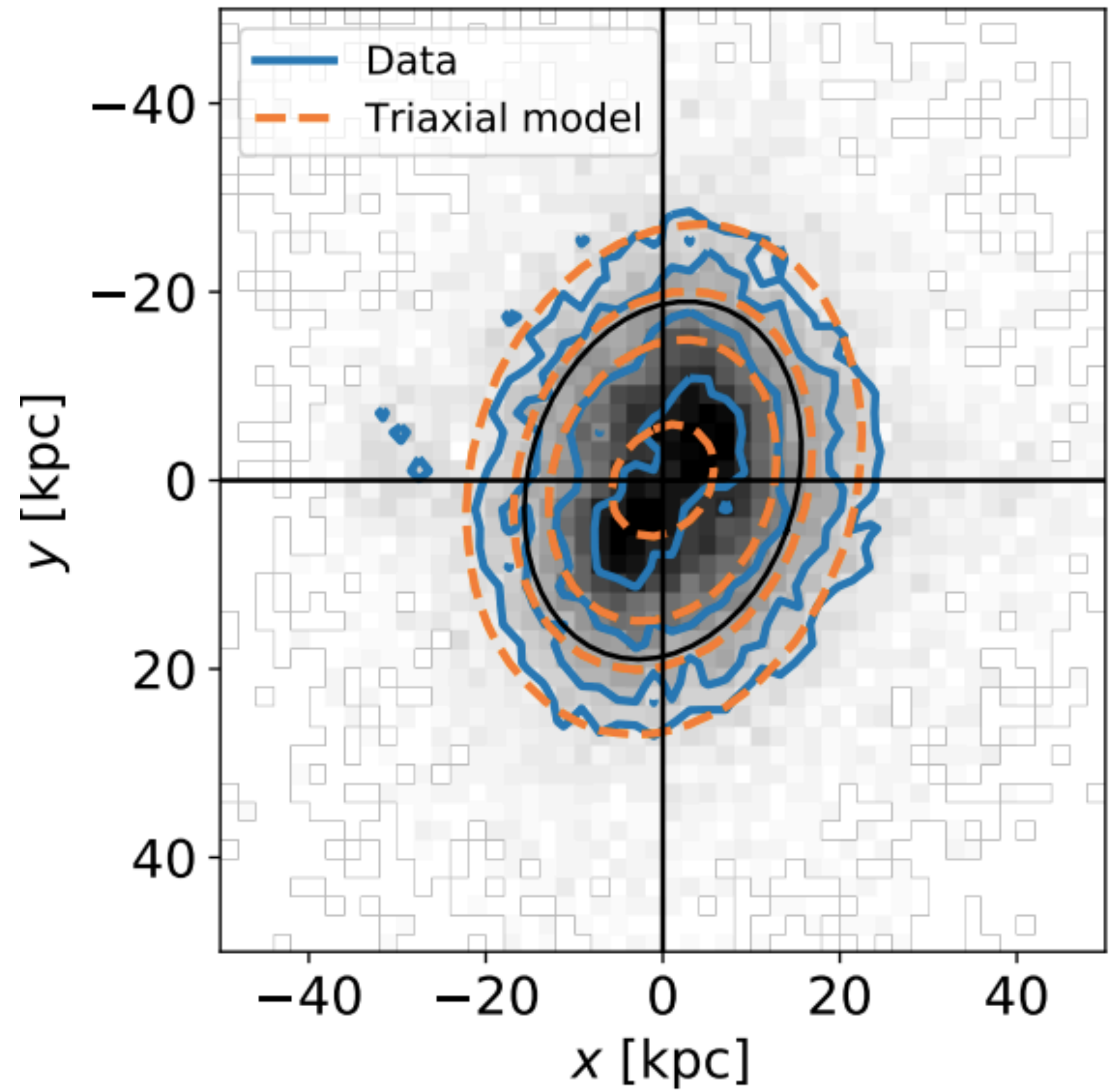


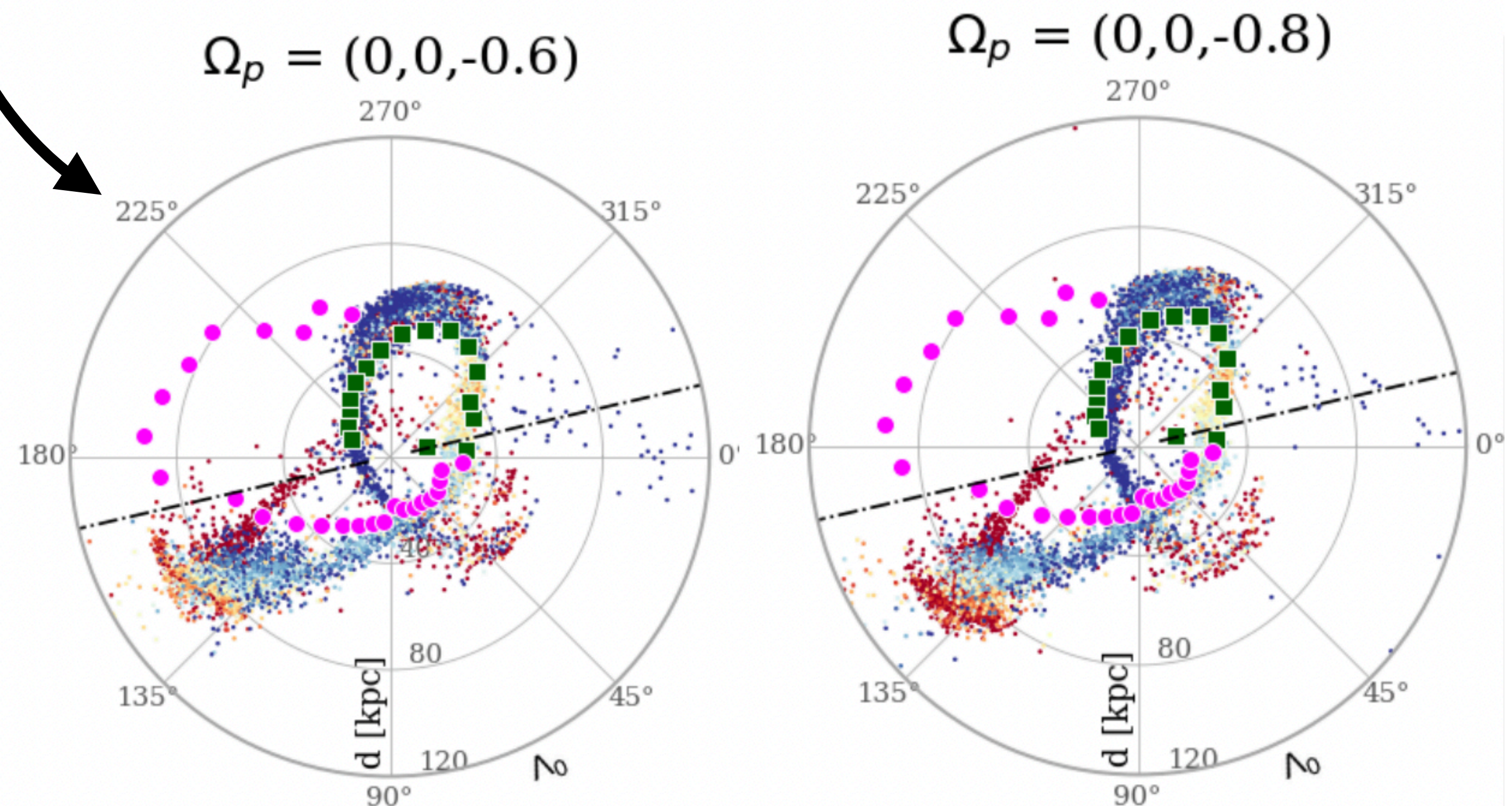
Figure rotation/tumbling of DM halo

Simulations find typical pattern speeds for triaxial halos in the range:

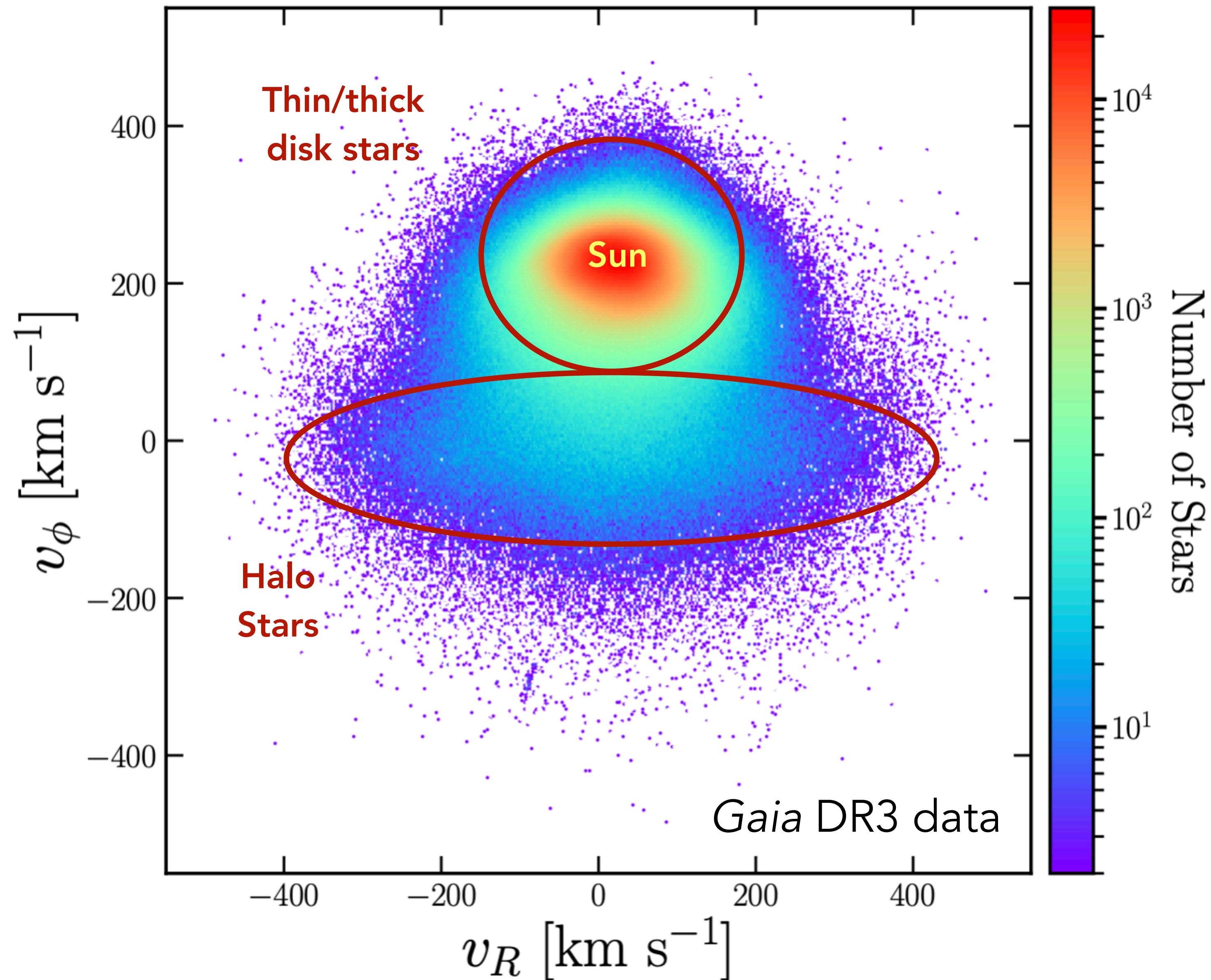
$$\Omega_p \sim 0.15 - 0.6 \text{ km s}^{-1} \text{ kpc}^{-1} \sim 9^\circ - 35^\circ \text{ Gyr}^{-1}$$

→ MW spin cannot be anomalously large or the Sagittarius stream would look measurably different from the way it does (Valluri et al. 2009.09004)

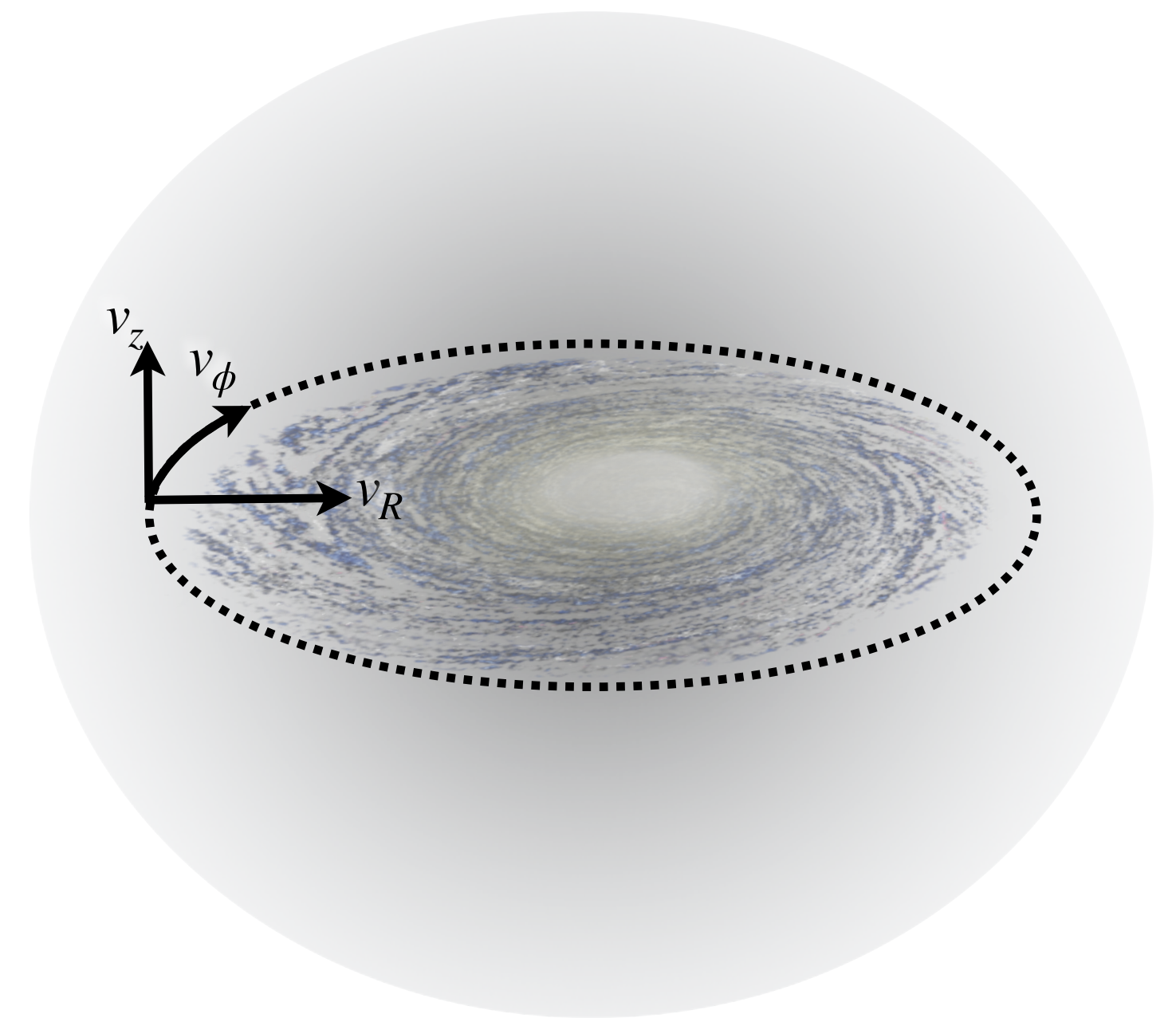
Even extreme figure rotation would not reduce directionality of DM flux in Solar System

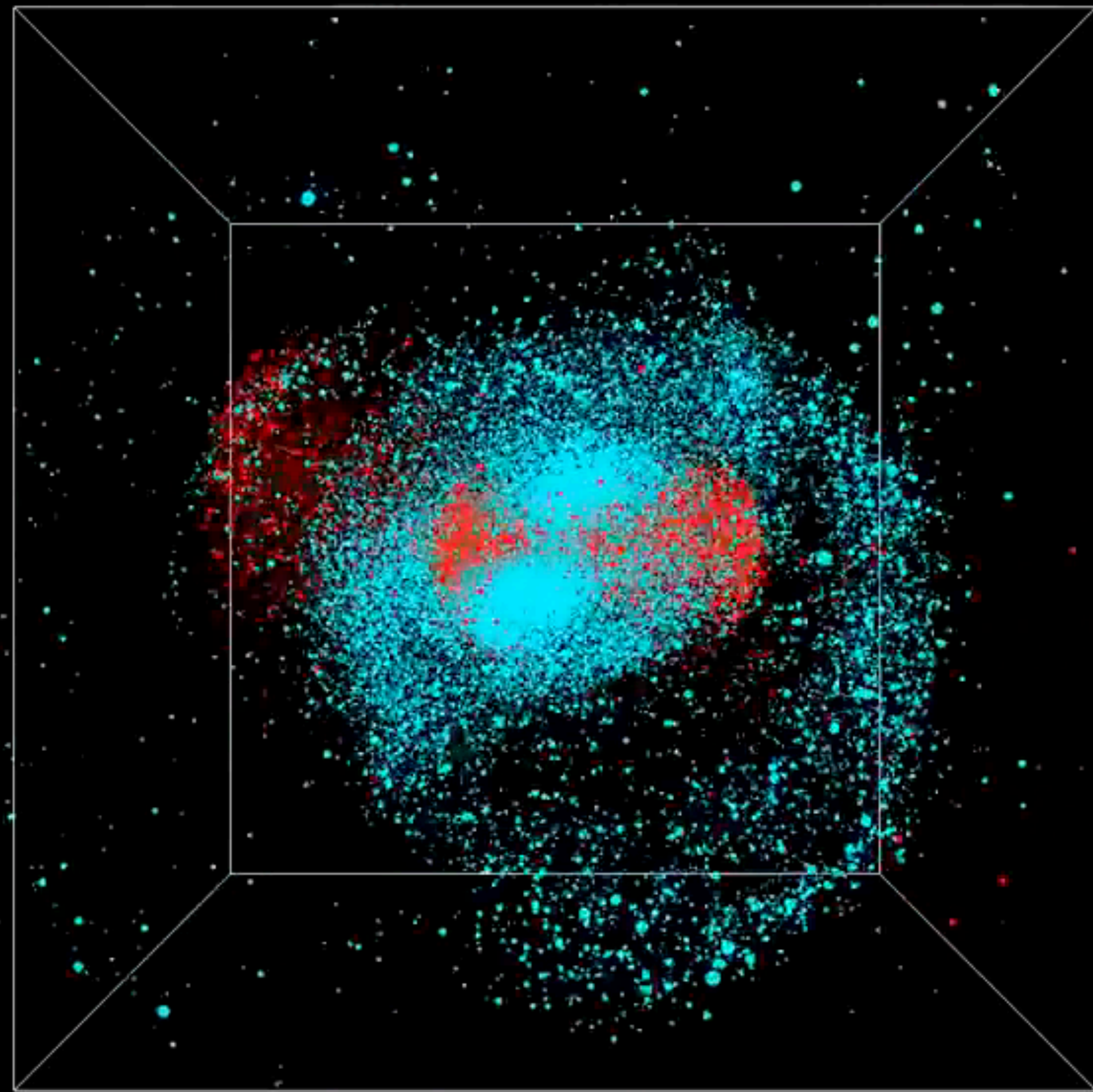


Velocity distribution of the MW stellar halo

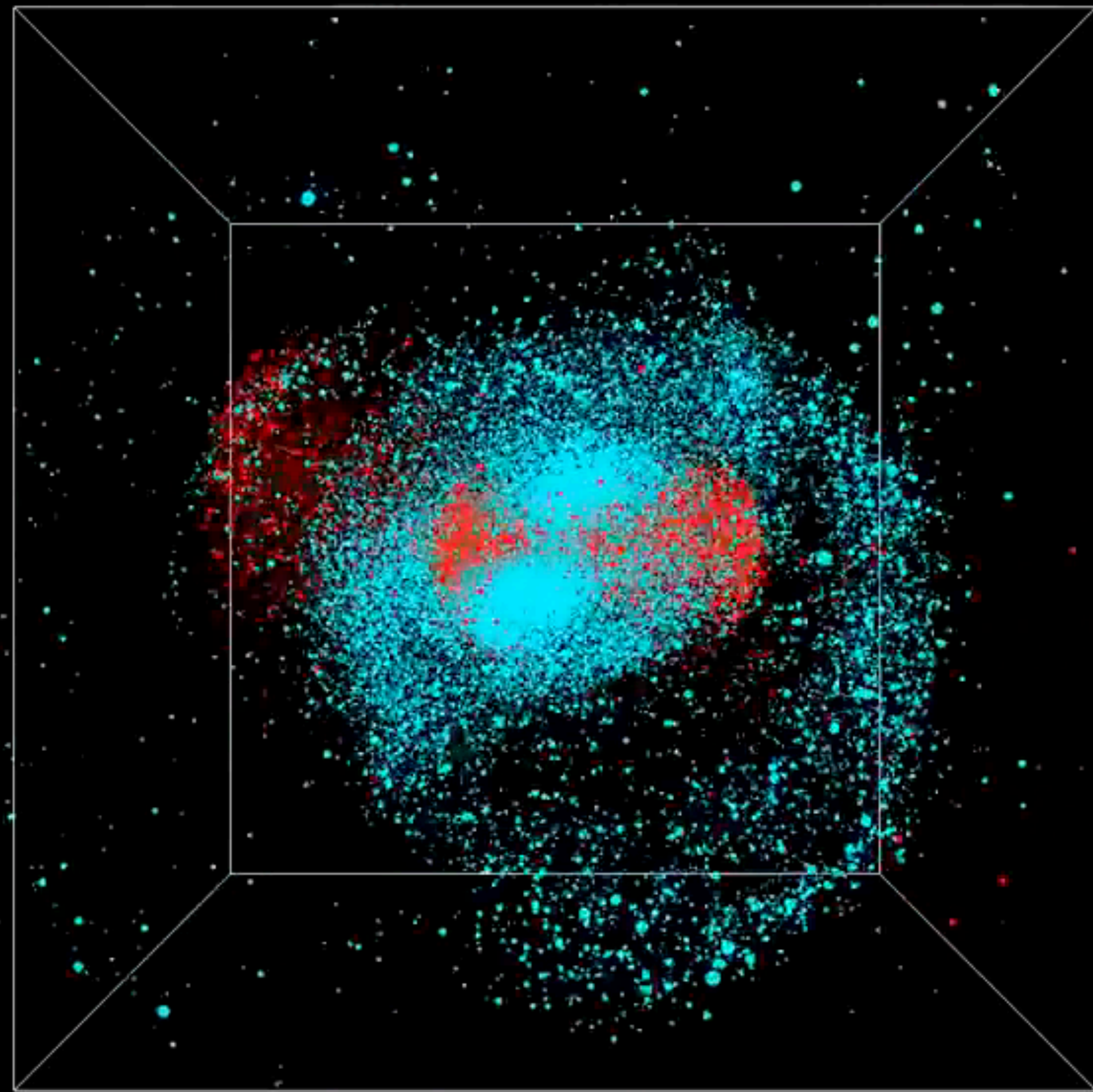


Substantial evidence for recent merger event with a dwarf galaxy filling much of the inner halo
→ **The Gaia-Sausage-Enceladus (GSE)**





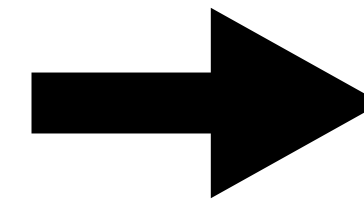
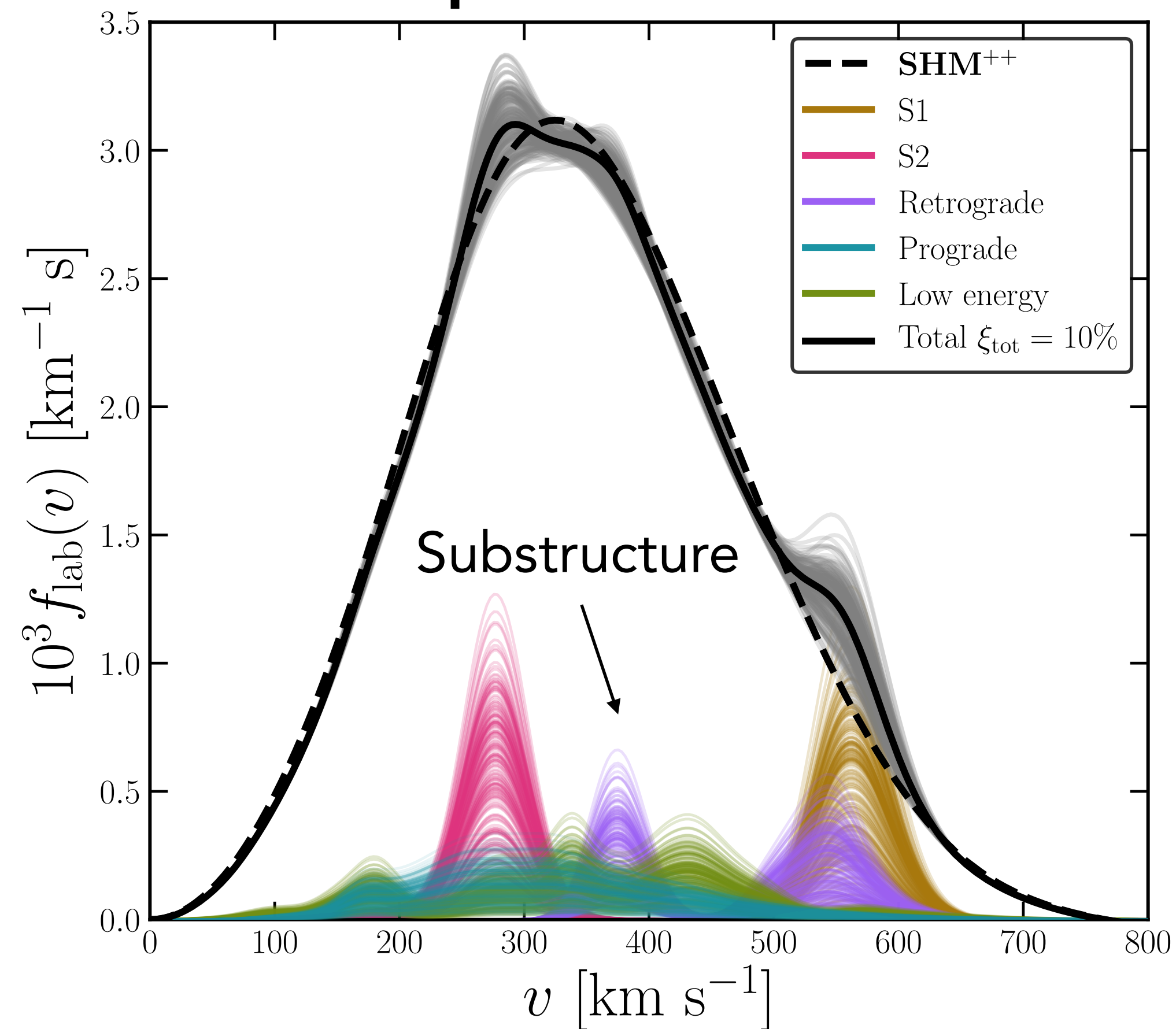
The GSE Merger:
Stars+DM brought in on
highly radial orbits by a
 $10^9\text{-}10^{10} M_{\odot}$ stellar mass
galaxy merging with the
MW 8-10 billion years ago



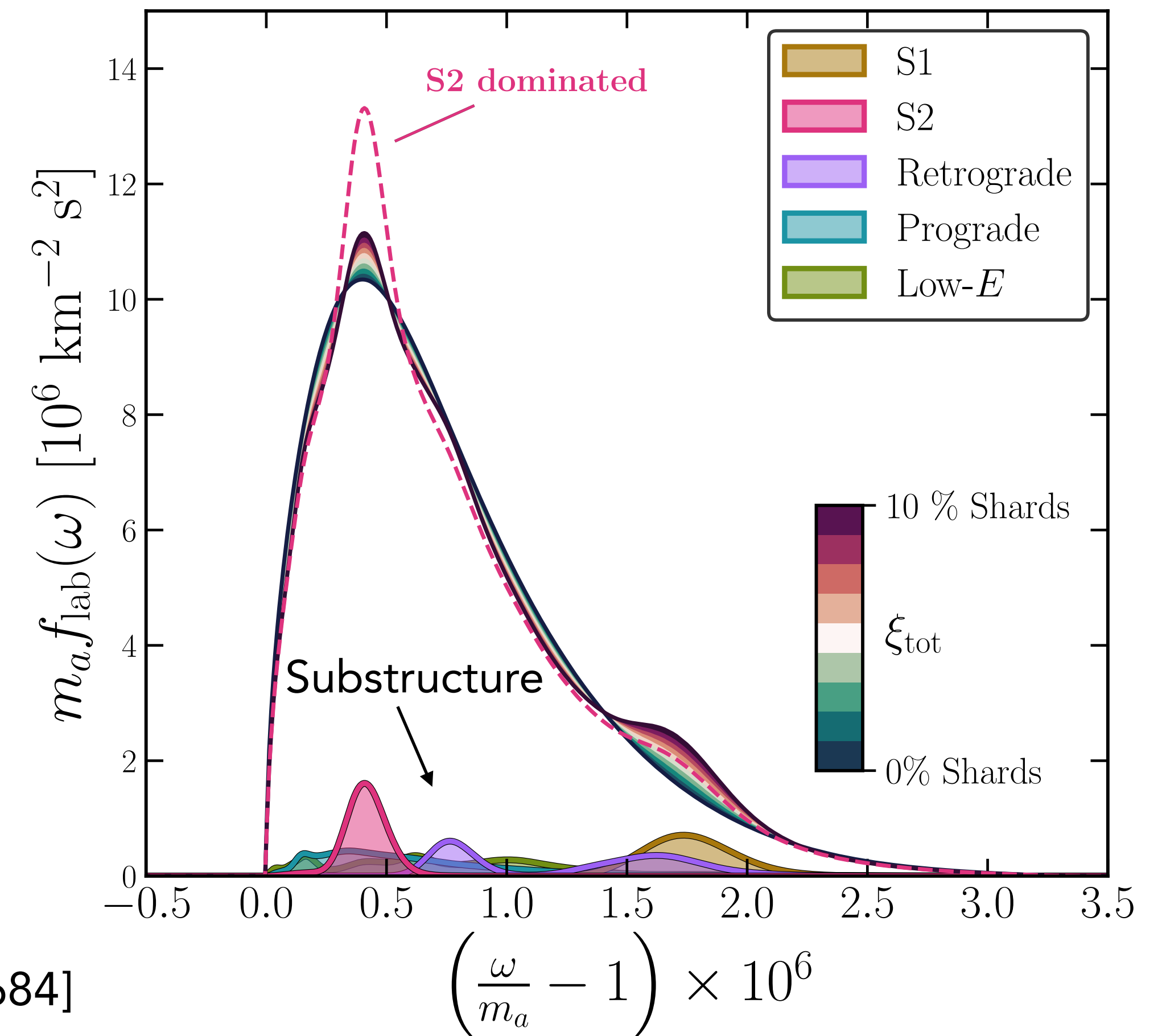
The GSE Merger:
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galaxy merging with the
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Ultimately the relevance of these structures for dark matter is only something that direct detection can reveal. That's why we call our experiments **"Haloscopes"**

DM speed distribution



Wave DM frequency distribution



O'Hare+ [1909.04684]